High Resolution Simulations of Cosmic Texture, by N. Turok and V. Travieso

The movies show the numerical evolution of the $SU(2)$ nonlinear sigma model, describing the simplest kind of texture, in an expanding universe. The initial conditions for the symmetry-breaking $SU(2)$ field simulate a phase transition in the very early universe: the field is randomly chosen and there are no long range correlations.

The $SU(2)$ matrix is written as $n_0 + i(n_1\sigma_1 + n_2\sigma_2 + n_3\sigma_3))$ where σ_i, $i = 1, 2, 3$ are the Pauli matrices and $\vec{n} = (n_0, n_1, n_2, n_3)$ is a four-component unit vector. The equation of motion for the texture field in a flat, FRW cosmology is then just that for the $O(4)$ nonlinear sigma model, namely

$$\ddot{\vec{n}} + 2\frac{\dot{a}}{a}\dot{n} - \nabla^2\vec{n} = \left((\nabla\vec{n})^2 - \dot{\vec{n}}^2\right)\vec{n},$$

in comoving space coordinates $\vec{x}$ and conformal time τ. Since the textures of interest all unwound in the matter-dominated era, long before the dark energy dominated, we use a flat FRW metric with scale factor $a \propto \tau^2$.

The numerical algorithm used to evolve the texture is that of Pen *et al.*, described in Pen, U., Spergel, D. N. and Turok, N., Phys. Rev. **D49**, 692, 1994. without the "spin flip" procedure used there when textures unwind and which, as other authors have noted, is unnecessary. The movies shown represent simulations in 1024^3 boxes, with the $SU(2)$ field chosen randomly at each lattice site in the initial condition. The simulations start at τ of order the lattice spacing and run to τ of order the box size. Note that the evolution equation above has no free parameters, so the statistical properties of the texture are in principle completely predicted in a given spacetime geometry. The overall energy scale of the texture is however set by the symmetry breaking scale ϕ_0 and the stress-energy tensor is quadratic in this parameter.

The energy density in the texture field is the sum of two terms: the kinetic energy density $\frac{1}{2}\phi_0^2 a^{-2}\dot{\vec{n}}^2$ and the gradient energy density $\frac{1}{2}\phi_0^2 a^{-2}(\nabla\vec{n})^2$. In the scaling solution, the spatial average of these quantities scales with time, so that when they are multiplied by $a^2\tau^2$, and when spatial length scales are appropriately rescaled (so comoving lengths are measured in units of the particle horizon τ) the statistical properties of the defect network are invariant. We also plot the topological charge density

$$\epsilon_{abcd}\left(\partial_x n^a \partial_y n^b \partial_z n^c n^d\right)/(2\pi^2)$$

whose spatial integral measures the topological number. When n^a is constant on the surface of some volume V of space, then the integral of the topological density over V is an integer. The only way for this integer to change is for the textures inside the volume to shrink down and unwind, a process in which the topological number changes by one unit.

The code has been parallelized for runs of up to 2048^3 on the COSMOS supercomputer. A detailed analysis of the results of high resolutions results is in preparation.