Flux Superpotential in Heterotic M-theory

Lilia Anguelova

hep-th/0602039

(collaboration with K. Zoubos)

Contents

- Motivation
- Horava-Witten set-up
- Heterotic M-theory
- Flux-induced superpotential
- De Sitter vacua

Motivation

– Heterotic $E_8 \times E_8$ string: unified description of phenomenology (?)

$\longrightarrow$ unresolved problems...

– Strong coupling limit: heterotic M-theory

$\longrightarrow$ de Sitter, assisted inflation solutions

– Moduli stabilization:

In type II: fluxes and nonperturb. effects
(pert. effects $\rightarrow$ Kähler moduli)

In het. M-theory: so far only nonp. effects

But $G_{(4)} \neq 0$ always!

$\Rightarrow$ flux-induced superpotential

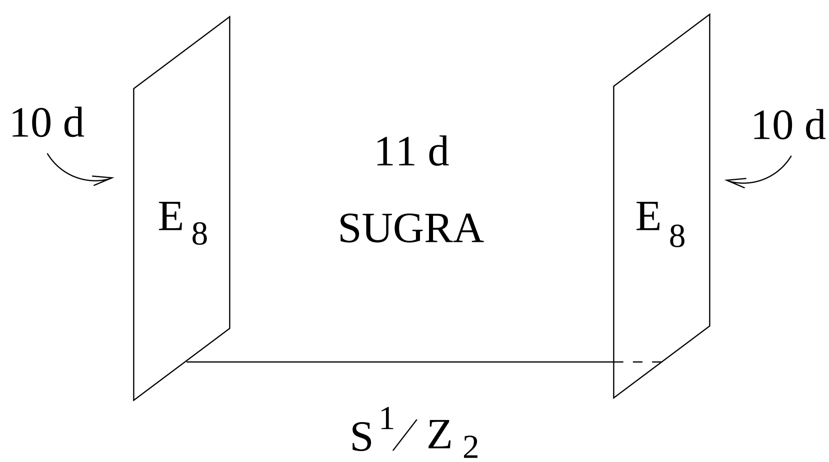
Horava-Witten

– M-theory on $\mathbb{R}^{1,9} \times S^1/\mathbb{Z}_2$:

11d SUGRA: g_{IJ}, Ψ_I^α, C_{IJK} $(G = dC)$

Massless chiral gravitinos on $\mathbb{R}^{1,9}$
$\Rightarrow$ gravitational anomalies

To cancel them: E_8 gauge fields on each boundary



Description of strongly coupled heterotic $E_8 \times E_8$ string theory

– Compactification to $d = 4$:

Compactify 10d on $CY(3)$

But: $dG \sim \kappa^{2/3}\delta(x^{11})dx^{11}(\text{tr}F^2 - \frac{1}{2}\text{tr}R^2)$

$\Rightarrow$ Background with $G \neq 0$

$\rightarrow$ warping of $M_4 \times CY(3) \times S^1/\mathbb{Z}_2$

$\rightarrow$ 6d manifold X: generically non-Kähler

$\Rightarrow$ 7d: 6d $SU(3)$ str. fib. over $S^1/\mathbb{Z}_2$!

E_8 gauge anomaly cancellation

$\Rightarrow$ $\lambda^2 \sim \kappa^{4/3}$, λ - gauge coupling

$$S_{11d} = \frac{1}{\kappa^2}(S^{(0)} + \kappa^{2/3}S^{(1)} + \kappa^{4/3}S^{(2)} + ...)$$

– Linear solution:

To 1st order in $\kappa^{2/3}$:

$$\delta ds^2_{11} = \underbrace{b\eta_{\mu\nu}dx^\mu dx^\nu}_{4d} + \underbrace{h_{mn}dx^m dx^n}_{6d} + \underbrace{k(dx^{11})^2}_{\text{interval}}$$

Witten: $\exists$ solution of

$$\delta_{susy}\Psi_I = 0, \quad dG = \text{sources}, \quad D^I G_{IJKL} = 0$$

(Gauntlett and Pakis: Einstein eqs follow)

$$\rightarrow \; V_X(x^{11}) \sim V_0 - c(x^m)\, x^{11}$$

$\Rightarrow$ singularity at $V_X(x_0^{11}) = 0$

Need: nonlinear solution; new effects (higher derivative corrections, M5-branes...)

Heterotic M-theory

– Field content:

- universal moduli

$$ds^2_{\text{dir. pr.}} = ds^2_4 + e^{2a} g_{mn} dx^m dx^n + e^{2c} (dx^{11})^2$$

a - CY volume, c - interval size

$$\rightarrow \quad b = b(a,c), \;\; h_{mn} = h_{mn}(a,c), \quad k = k(a,c)$$

$N = 1$ susy: chiral superfields S, T

$$S = e^{6a} + i\sigma_S, \;\; T = e^{(c+2a)} + i\sigma_T,$$
$$d\sigma_S = *_4 dB \;\; B_{\mu\nu} = C_{\mu\nu 11}, \, C_{mn11} = \sigma_T J_{mn}$$

- non-universal moduli

$h^{2,1}$ complex, $h^{1,1}$ Kähler,
dim(gauge group) C^I's: charged matter

– Effective $4d$ action:

$N = 1$ SUGRA + chiral and vector multiplets

- $0^{\rm th}$ order: $\{\Phi^i\}$ - all chiral superfields

  kinetic term: $2K_{i\bar{j}}\,\partial_\mu\Phi^i\,\partial^\mu\bar{\Phi}^{\bar{j}}$,

  $K_{\rm univ.} = -3\ln(T+\bar{T}) - \ln(S+\bar{S})$

- $\kappa^{2/3}$ order: gauge field action

  No corrections induced by metric and C-field distortion

- $\kappa^{4/3}$ order:

  - gauge fields and charged matter (C^I)

  - $\mathcal{O}(\kappa^{4/3})$ distortion of the background

  - $\mathcal{O}(\kappa^{4/3})$ terms in 11d action: R^4

  - flux superpotential: $W \sim G$, $U \sim W^2$

Flux-induced superpotential

– $SU(3)$ structure:

- 6d $SU(3)$ structure:

  comp.: $J \wedge \Omega = 0$, $J \wedge J \wedge J \sim \Omega \wedge \bar{\Omega}$

  $$dJ = -\tfrac{3}{2}\,\mathrm{Im}(W_1 \bar{\Omega}) + J \wedge W_4 + W_3$$
  $$d\Omega = W_1\, J \wedge J + J \wedge W_2 + \Omega \wedge W_5$$

  $W_1, ..., W_5$ - torsion classes

  Complex manifold $\Rightarrow$ $W_{1,2} = 0$
  (req. in het. str.)

- 7d $SU(3)$ structure: $\tilde{J}$, $\tilde{\Omega}$, v - vector

  v: $i_v \tilde{J} = 0$, $i_v \tilde{\Omega} = 0$; $\tilde{J}$, $\tilde{\Omega}$ - comp.

  $\Rightarrow$ 6d $SU(3)$ str. man. fibered over v

– Spinors: $\exists$ two $SU(3)$ singlets

$$\theta,\ \theta^*: \quad \nabla_a^{(T)}\theta \equiv (\nabla_a - \tfrac{1}{4}\tau_{abc}\gamma^{bc})\theta = 0\,, \qquad a = (\{m\}, 11)$$

- Gravitino:

$$\Psi_\mu = \psi_\mu \otimes (a_L\theta + a_R^*\theta^*) + \psi_\mu^* \otimes (a_L^*\theta^* + a_R\theta)$$

$$a_L = a_R^* \quad \longrightarrow \quad G_2 \text{ structure}$$

- Bilinears:

two kinds: $\theta^\dagger\gamma_{a_1\ldots a_p}\theta \quad , \quad \theta^T\gamma_{b_1\ldots b_q}\theta$

$$\tilde{J}_{ab} = -i\theta^\dagger\gamma_{ab}\theta \;\; , \; \tilde{\Omega}_{abc} = -i\theta^T\gamma_{abc}\theta$$
$$v_a = \theta^\dagger\gamma_a\theta$$

- Mass term:

$$\tfrac{1}{2}e^{K/2}(W\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu^* + c.c.)$$

Ψ_a – does not contribute; $K = -3\ln\mathcal{V}_{(7)}$

On shell: torsion $\leftrightarrow$ fluxes

Off shell: independent!

– Dimensional reduction of 11d action:

$$ds^2_{11} = ds^2_4 + ds^2_7$$

- Flux term: $\bar{\Psi}_M \Gamma^{MNPQRS} \Psi_N G_{PQRS} \Rightarrow$

$$\begin{aligned} W^{(flux)} &= \frac{1}{2}\int a_L^* a_R G \wedge \tilde{J} \wedge v \\ &+ \frac{1}{4}\int [(a_R)^2 G \wedge \tilde{\Omega} + (a_L^*)^2 G \wedge \bar{\tilde{\Omega}}] \end{aligned}$$

- Kinetic term: $\bar{\Psi}_M \Gamma^{MNP} D_N \Psi_P \Rightarrow$

$$\begin{aligned} W^{(geom)} &= \frac{i}{4}\int (a_R)^2\, v \wedge d\tilde{J} \wedge \tilde{\Omega} \\ &+ \frac{i}{4}\int (a_L^*)^2\, v \wedge d\tilde{J} \wedge \bar{\tilde{\Omega}} \\ &- \frac{i}{16}\int a_L^* a_R\, [d\tilde{\Omega} \wedge \bar{\tilde{\Omega}} + d\bar{\tilde{\Omega}} \wedge \tilde{\Omega}] \\ &- \frac{i}{4}\int a_L^* a_R\, dv \wedge v \wedge \tilde{J} \wedge \tilde{J} \\ &- \frac{1}{6}\int (a_L^* da_R - a_R da_L^*) \wedge \tilde{J} \wedge \tilde{J} \wedge \tilde{J} \end{aligned}$$

- Warp factors:

$$ds^2_{11} = e^{2b}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2f}g_{mn}dx^m dx^n + e^{2k}(dx^{11})^2$$

$b, f, k = \text{function}(x^m, x^{11})$

$$\tilde{J} = e^{2f}J\,, \quad \tilde{\Omega} = e^{3f}\Omega\,, \quad v = e^k dx^{11}$$

$$|a_L|^2 + |a_R|^2 = e^b$$

- Weakly coupled het. $E_8 \times E_8$:

$$W \sim \int (H + idJ) \wedge \Omega$$

In weakly coupled limit: $G = v \wedge H$, $a_L \to 0$

- G_2 structure: $\Phi = \text{Re}\,\tilde{\Omega} + \tilde{J} \wedge v$

$$W = \frac{1}{4}\int \left(G \wedge \Phi - \frac{i}{2} d\Phi \wedge \Phi\right)$$

$a_L^* = a_R \quad \to \quad$ Not good for het. $E_8 \times E_8$!

de Sitter vacua

Becker, Curio, Krause: dS vacua can ∃ in heterotic M-theory

– Scalar potential: $N = 1$, $d = 4$ SUGRA

$$U = e^K \left(K^{A\bar{B}} D_A W D_{\bar{B}} \bar{W} - 3|W|^2 \right) + U_D \,,$$

$$D_A W = (\partial_A + K_A) W \,, \; U_D \sim \sum_a (\bar{C} T^a C)^2$$

$$W = W_{tree} + W_{non-pert} \;, \; W_{tree} = \Lambda_{pqr} C^p C^q C^r$$

$$W_{non-pert} = W_{OM} + W_{GC}$$

- Kähler and complex structure moduli:

  As for weakly coupled het. theory

- Charged matter:

  $\langle C \rangle \neq 0$, but very small

- Orbifold length:

  Main new issue ! $\rightarrow$ ∃ $U > 0$ local min.

– Nonlinear solution: all orders in $\kappa^{2/3}$

Warp factors only ; g_{mn} - Calabi-Yau

Curio-Krause: $k(x^{11}) = -b(x^{11})$

$$V_{CY}(x^{11}) = \int d^6x \sqrt{g}\, e^{6f(x^{11})} = V_v(1 - \mathcal{S}\, x^{11})^2$$

Singularity at $x^{11} = 1/\mathcal{S}$ $\Rightarrow$ $x^{11} < 1/\mathcal{S}$

$\mathcal{V}(\mathcal{L}, \mathcal{V}_v)$ - average CY volume

Moduli: $(a,\ c) \rightarrow (\mathcal{V}_v,\ \mathcal{L})$, $\mathcal{V}_v$ - fixed

$$S = \mathcal{V} + i\sigma_S, \quad T = \mathcal{L}\mathcal{V}^{1/3} + i\sigma_T$$

$K = -\ln(\mathrm{Re}\, S) - 3\ln(\mathrm{Re}\, T)$

$W_{non-pert} = W(S,T)$ - depends on σ_S, σ_T

– Nonzero W_{flux}: Enough for dS vacuum?

$$W_{flux} = W^{(flux)} + W^{(geom)} =$$

$$= \frac{1}{4} \int e^b [G \wedge \tilde{\Omega} + iv \wedge d\tilde{J} \wedge \tilde{\Omega}] \, , \; v = e^k dx^{11}$$

$$W_{flux}(G_{CK}) = 0 \quad \rightarrow \quad \text{Mink. vacuum}$$

Susy breaking: $\delta G = v \wedge H \sim v \wedge \bar{\tilde{\Omega}}$

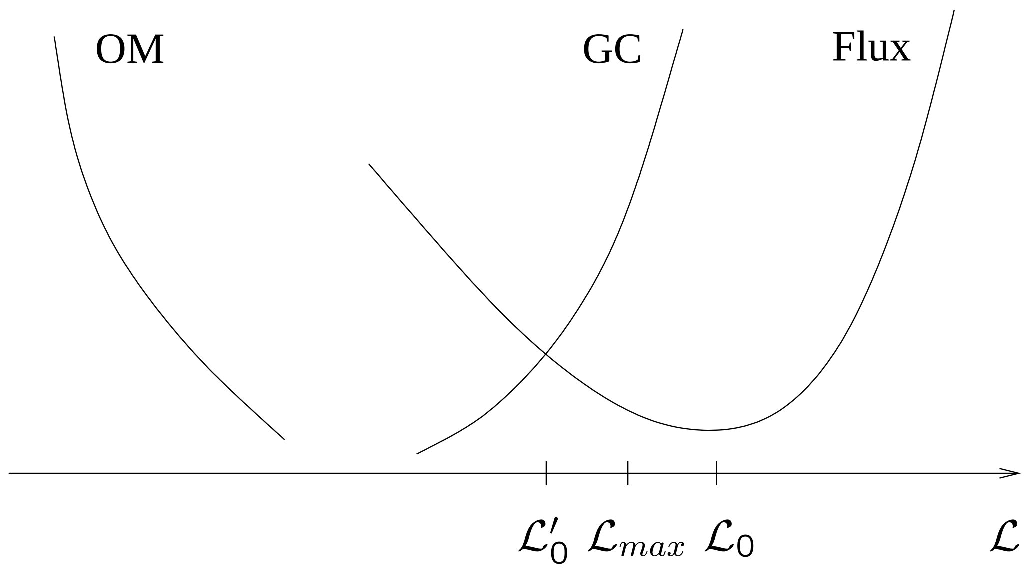
$$\Rightarrow \; W_{flux} \neq 0 \, , \quad \text{Solve} \quad \frac{\partial U(\mathcal{L})}{\partial \mathcal{L}} = 0$$

Result: $\exists$ local min. with $U > 0$

But: occurs at $\mathcal{L}_0 > \mathcal{L}_{max} \equiv 1/\mathcal{S}$

still: $\mathcal{L}_0$, $\mathcal{L}_{max}$ - very close

$\rightarrow$ Gaugino condensation may help...


OM
GC
Flux
$\mathcal{L}'_0$
$\mathcal{L}_{max}$
$\mathcal{L}_0$
$\mathcal{L}$

Summary

– 4d effective action of heterotic M-theory:

- Flux-induced superpotential

– De Sitter vacua

– Open issues:

- C-field moduli...
- Boundary contr. to superpotential...
- Combining W_{flux} and $W_{non-pert} \dots$
- Higher derivative corrections...