A Theory of Dark Energy

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It is an honour and privilege to speak at MBG-60.

Michael's intellectual courage and persistence in bringing an initially apparently unpromising idea to a fruitful and (apparently) highly sucessful conclusion are an inspiration for us all.

It is said that imitation is the sincerest form of flattery and in that spirit I offer an initially apparently unpromising idea ....

The latest WMAP data release confirms that

- Over 70% of the current universe is made up of matter with $P \approx -\rho$ ('Dark Energy ')

- The scalar spectral index $n_s < 1$, indicating that the simple Harrison -Zeldovich scale-invariant spectrum is inadequate to explain the data and that some sort of primordial inflationary era, with dark energy is required.

This data continues to pose a challenge to theorists. While simple models based on an (extremely) finely tuned cosmological constant at late times and an inflaton scalar field at early times may eventually turn out to be the most convincing explanation it still seems worth exploring other avenues.

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*At worst it may provide a new foil against whch to test the claims of high precision cosmology

After all, if the astronomers are right, this is the first time in over 300 years (since Newton (gravitation) , Roemer and Bradley (finite speed of light)) that a fundamental law of nature has been established purely by astronomical rather than terrestrial means.

What follows is not necesssarily advocated as the actual solution, and certainly suffers from fine-tuning problems. Rather it is intended to illustrate how physics quite different from that of conventional scalar fields can mimic the effects of a cosmological constant. Since it involves (a vector) tachyon in an essential way, it may interest string theorists. It also has a purely geometrical description in terms of non-riemannian geometry which may be related to Born-Infeld Gravity.

The basic idea is that the dark energy is matter which couples to a vector field A_μ . The total charge of the dark matter is non-zero, which is compatible with isotropy only of the vector has a mass squared $= \mu^2$. If the vector mass is tachyonic, $\mu^2 < 0$, then charge and hence dark matter is not conserved and in fact the system tends to a state in which the dark matter density is constant , which mimics the behaviour of a cosmological constant Λ.

The model in detail We suppose that the universe has three main components

- A conserved, pressure-free component, with conserved energy momentum tensor

$$T^{(1)}_{\mu\nu} = \rho_1 U^{(1)}_\mu U^{(1)}_\nu \,, \qquad (\rho_1 U^{(1)\,\mu})_{;\mu} = 0 \,. \tag{1}$$

- A second not necessarily conserved, pressure-free component with energy momentum tensor

$$T^{(2)}_{\mu\nu} = \rho_2 U^{(2)}_\mu U^{(2)}_\nu \,. \tag{2}$$

  The second component is supposed to carry a current

$$J^\mu = \frac{e}{m} \rho_2 U^{(2)\,\mu} \,, \tag{3}$$

where the constant $\frac{q}{m}$ is the average charge to mass ratio of the second component and which is coupled to

- A vector field A_μ , satisfying

$$\nabla_\nu F^{\mu\nu} = J^\mu - \mu^2 A^\mu \,, \tag{4}$$

with

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \,. \tag{5}$$

and energy momentum tensor

$$T^{(3)}{}_\mu^{\ \nu} = F_{\mu\sigma}F^{\nu\sigma} - \frac{1}{4}\delta_\mu^\nu F_{\tau\sigma}F^{\tau\sigma} + \mu^2 (A_\mu A^\nu - \frac{1}{2}\delta_\mu^\nu A_\sigma A^\sigma) \,. \tag{6}$$

As a consequence of (45) we have

$$\nabla_\mu J^\mu = \mu^2 \nabla_\mu A^\mu \tag{7}$$

and

$$\nabla_\mu (T^{(2)\,\mu\nu} + T^{(3)\,\mu\nu}) = \\ \rho_2 U^{(2)\,\nu}{}_{;\mu} U^{(2)\,\mu} + U^{(2)\,\nu} \nabla_\mu (\rho_2 U^{(2)\,\mu}) - J_\sigma F^{\nu\sigma} + \mu^2 (A^\nu \nabla_\mu A^\mu)\,. \tag{8}$$

Now by (1) the first pressure free component moves on a geodesic

$$U^{(1)\,\nu}{}_{;\mu} U^{(1)\,\mu} = 0\,. \tag{9}$$

Since the second component might be expected to have almost the same four velocity and is also pressure free, it seems reasonable to suppose that it also moves along a geodesic,

$$U^{(2)\,\nu}{}_{;\mu} U^{(2)\,\mu} = 0. \tag{10}$$

Now the isotropy of the universe, suggests that although the second component carries a net charge, we must assume that $F_{\mu\nu} = 0$. Even if $F_{\mu\nu}$ is not strictly zero it certainly seems reasonable to assume that $F_{\mu\nu} J^{\nu} = 0$. It follows that consistency of our equations demands that

$$\nabla_{\mu}(\rho_2 U^{(2)\,\mu}) = \mu^2 \nabla_{\mu} A^{\mu}, \tag{11}$$

$$U^{(2)\,\nu} \nabla_{\mu}(\rho_2 U^{(2)\,\mu}) = -\mu^2 (A^{\nu} \nabla_{\mu} A^{\mu}). \tag{12}$$

If we assume the London condition

$$\frac{q}{m} \rho_2 U^{(2)\,\nu} = \mu^2 A^{\mu} \tag{13}$$

we find that the density of the second component must be constant

$$\rho_2 = -\frac{m^2 \mu^2}{e^2}. \tag{14}$$

The total energy momentum tensor then becomes

$$T_{\mu\nu} = \rho_1 U^{(1)}_\mu U^{(1)}_\nu - \frac{\Lambda}{8\pi G} g_{\mu\nu} \tag{15}$$

with

$$\Lambda = 4\pi G \rho_2 = -\frac{4\pi G m^2 \mu^2}{e^2}. \tag{16}$$

If the density of the second component is taken to be positive This is precisely a mixture of a pressure free fluid and and cosmological term which is observed. Of course we have to assume that μ^2 is negative, which implies that the vector field A_μ is tachyonic.

To give but one example,take a standard $k = 0$ Friedmann-Lemaitre metric with

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2 \,, \tag{17}$$

$$U^{(1)\,\mu} = U^{(2)\,\mu} = \delta_t^\mu \tag{18}$$

$$\rho_2 = \frac{\rho_2(0)}{a^3(t)}\,. \tag{19}$$

One easily checks that indeed

$$F_{\mu\nu} = 0\,. \tag{20}$$

There is a close connection between what has been done here and the idea of Hoyle's creation field $C(x)$ or what is sometimes called a phantom field Essentially the longitudinal photon plays the role of a phantom. If on sets

$$\partial_\mu C = |\mu| A_\mu \,, \tag{21}$$

one recovers the equations describing the creation of matter in Hoyle's version of the Steady State Theory. (see GWG hep-th/0302199). Since μ^2 is taken to be negative, then the field C has negative energy. This is a good reason for taking the second component to be as yet unobserved matter, since if observed matter couples to $C(x)$ it might well exhibit dramatically unstable behaviour.

There are also similarities with attempts to construct theories of the Aether due to Dirac. These attempts have been revised recently by Jacobson and others and some of their models resemble what has been done above.

A theory like this may be a useful foil for testing for possible violation of Lorentz-invariance.

Other applications might be to theories in which the photon has a small mass or in which the fine structure varies in space and time.

One could imagine that the $U(1)$ field is that of standard photon of electromagnetism, but in many ways this is unattractive. The photon is part of the electro-weak theory, which in turn may well turn out be part of a grand unified theory. Breaking gauge-invariance 'by hand 'is likely to lead to anomalies and other quantum-mechanical inconsistencies.*

*On the other other hand, if the Compton wavelength of the vector field is of the order of the Hubble radius, it quite possible that any such effects would be unobservable at laboratory scales.

One could imagine that the net charge density of the dark energy arises in at least two ways.(i) It might be the case that all charges are quantised in terms of a single fundamental unit, as appears to be the case for electromagnetism and there is, for some reason, a net excess of one sign over the other. (ii) Another possibility is that there is no fundamental unit of charge, and that the gauge group is not $U(1)$ but rather $\mathbf{R}$. This possibility corresponds for example to old speculations of Bondi and Lyttleton in which the charge on the proton is not not exactly equal to that on the positron . Again this possibility is not attractive for ordinary electric charge since it is embedded in the electroweak theory where non-commensurate charges might conflict with anomaly cancellation Moreover if the electro-weak theory is contained in a simple grand unified group such as $SO(10)$ then non-commensurate charges do not occur.

If one does adopt the second possibility it has striking feature that something like dark hydrogen will always repel dark hydrogen and dark anti-hydrogen will always repel dark anti-hydrogen, dark hydrogen will attract dark anti-hydrogen

This follows because if q_p and $-q_e$ are the charges of the proton and the electron, then as long as the separation r is much smaller than $\frac{1}{|\mu|}$, and we assume that $q_{\bar{p}} = -q_p$ and $q_{\bar{e}} = -q_e$, then three forces are respectively

$$\frac{1}{4\pi}\frac{(q_p - q_e)^2}{r^2} \quad , \tag{22}$$

$$\frac{1}{4\pi}\frac{(q_p - q_e)^2}{r^2} \quad , \tag{23}$$

$$-\frac{1}{4\pi}\frac{(q_p - q_e)^2}{r^2} \quad . \tag{24}$$

Another way to say this is that ‘neutral ’dark material will tend to clump while non-neutral dark material tends to disperse because of self repulsion.

Because the charge neutrality of ordinary atoms and molecules has been experimentally checked to very high precision, it seems that one should not regard the vector field A_μ as a conventional gauge field of the standard model or one of its extensions, but as something having a rather different origin.

A Brane perspective

When N D-branes approach one another,to form N superposed D-banes a process of symmetry enhancement takes place in which the $N^2 - N$ extra massless states appear which combine with the N $U(1)$ gauge fields on the the N world volumes to make a $U(N)$ gauge theory on the single superposed world sheet. Conversely, as the N-branes separate, a Higgs mechanism takes place and the non-abelian gauge group $U(N)$ breaks down maximally to $U(1)^N$. The massless states correspond to strings with end points on pairs of branes.

The gauge theory is confined to the branes and is governed by a Born-Infeld action, whose precise form is not very well understood in the non-abelian case. Presumably those vector fields which acquire a mass are governed, in some approximation by some sort of Proca-Born-Infeld action in which one adds to the usual BI action a gauge non-invariant mass term

$$-\frac{m_V^2}{2} A_\mu A^\mu, \tag{25}$$

the vector mass being given by

$$m_v = TL \tag{26}$$

where L is the separation of the branes and $T = \frac{1}{2\pi\alpha'}$ is the string tension. Note that the branes have to be much closer than the string length $l_s = \sqrt{2\pi\alpha'}$ in order that there be symmetry enhancment.

By contrast the gravitational interactions of D-branes are believed to be governed by the usual Einstein-Hilbert action in the bulk.

This distinction between the brane and the bulk fields looks rather artificial and it may not really persist if one considers heavy branes of finite size rather than the light and infinitely thin branes considered in string theory. It is tempting therefore to speculate that there may be some sort of relation between the the Born Infeld actions of governing the vector fields and the action for gravity. (cf.Born-Infeld Gravity).

In fact it has been known for some time that there are certain actions for gravity of determinantal form resembling the Born Infeld action. In fact these actions pre-date Born-Infeld theory and were introduced by Eddington and their properties were elaborated by Einstein and later Schrödinger in their search for a unified theory of gravity and electro-magnetism in which they tried to combine the gauge connection A_μ and the Levi-Civita connection given by the Christoffel symbol $\{{}_\mu{}^\alpha{}_\nu\}$ of the spacetime metric $g_{\mu\nu}$ into a general affine connection $\Gamma_\mu{}^\alpha{}_\sigma$.

Symmetric Affine Theories In purely affine formalism in which the affine connection $\Gamma_{\alpha}{}^{\beta}{}_{\gamma}$ is regarded as the primary concept and the metric as a secondary or derived notion. For example the General Unitary Theories or (to use his own acronym) GUT's considered by Schrödinger, following earlier work by Eddington and Einstein, assumed the existence of a Lagrange density $\mathcal{L}$ depending only in a local way on the Ricci tensor contraction

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta} \tag{27}$$

of the curvature tensor

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_{\mu}\Gamma_{\nu}{}^{\alpha}{}_{\beta} + \Gamma_{\mu}{}^{\alpha}{}_{\sigma}\Gamma_{\nu}{}^{\sigma}{}_{\beta} - (\mu \leftrightarrow \nu) \tag{28}$$

Perhaps the most appealing choice would be Eddington's simple and elegant suggestion, made long before Born-Infeld theory, and possibly inspiring it,

$$\mathcal{L} = \sqrt{\frac{\det R_{\mu\nu}}{\Lambda}}. \tag{29}$$

In general, the aim of unitary theories was two-fold

- To unify the gravitational and electromagnetic fields in a common geometrical structure, in this case that of an affine connection and

- To exhibit everyhere source free solutions of the classical equations of motion describing electrically charged particles, thus ending the need to postulate additional equations for the sources. Nowadays we would call these solutions "solitons".

In fact Born-Infeld theory was an attempt to construct a unitary theory of electro-magnetism in the second sense. However it failed because although it admits solutions with finite self-energy they still have delta-function sources and are not smooth. Since those solutions cannot be called solitons. A better name is "BIons".

In the simplest GUT's the connection was assumed to be symmetric or torsion free

$$\Gamma_\mu{}^\alpha{}_\beta = \Gamma_\beta{}^\alpha{}_\mu. \tag{30}$$

The first Bianchi identity then becomes

$$R^\alpha{}_{[\mu\nu\rho]} = 0, \tag{31}$$

which implies that

$$R^\alpha{}_{\alpha\mu\nu} = (R_{\mu\nu} - R_{\nu\mu}). \tag{32}$$

This means that the other contraction is not independent and provides some partial justification for the restriction that $\mathcal{L}$ depend solely on the Ricci tensor.

Thus if one defines

$$\mathcal{L}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial R_{\mu\nu}}, \tag{33}$$

and uses the Palatini lemma one finds that the field equations become

$$\mathcal{L}^{(\mu\nu)}{}_{;\alpha} + \frac{1}{3}\delta^{\mu}_{\alpha}\mathcal{L}^{[\nu\beta]}{}_{;\beta} + \frac{1}{3}\delta^{\nu}_{\alpha}\mathcal{L}^{[\mu\beta]}{}_{;\beta} = 0, \tag{34}$$

*

*where, as usual ; denotes covariant differentiation with respect to the connection $\Gamma_{\mu}{}^{\alpha}{}_{\nu}$ and round or square brackets denote symmetrization or anti-symmetrization with weight one.

The interpretion of these equations is facilited by defining

$$\mathcal{M}^{\mu\nu} = \mathcal{L}^{(\mu\nu)} \tag{35}$$

and

$$\mathcal{N}^{\mu\nu} = \mathcal{L}^{[\mu\nu]}. \tag{36}$$

One may then go on to define a symmetric contravariant tensor $M^{\mu\nu}$ and its inverse symmetric covariant tensor $M_{\mu\nu}$ by "de-densitizing" $\mathcal{M}^{\mu\nu}$ in the usual way just as if it were the spacetime metric. In fact as we shall see shortly a multiple of $M_{\mu\nu}$ *is* turns out to be precisely the spacetime metric. One may then use $M_{\mu\nu}$ to de-densitize and raise and lower the indices of other tensor or vector densities such as $\mathcal{N}^{\mu\nu}$ in the usual way. We shall adhere to the usual convention that such tensors are given the same Latin letter as their Calligraphic antecedents.

Thus we define a conserved vector density by

$$\mathcal{O}^{\mu} = \mathcal{N}^{\mu\nu}{}_{;\nu} = \partial_{\nu}\mathcal{N}^{\mu\nu}\,, \tag{37}$$

whence

$$\partial_{\mu}\mathcal{O}^{\mu} = 0\,. \tag{38}$$

The equation of motion (34) becomes

$$\mathcal{M}^{\mu\nu}{}_{;\alpha} + \frac{1}{3}\mathcal{O}^{\mu}\delta^{\nu}_{\alpha} + \frac{1}{3}\mathcal{O}^{\nu}\delta^{\mu}_{\alpha} = 0\,. \tag{39}$$

This may be solved to give

$$\Gamma_{\mu}{}^{\alpha}{}_{\nu} = \{_{\mu}{}^{\alpha}{}_{\nu}\} - \frac{1}{2}M_{\mu\nu}O^{\alpha} + \frac{1}{6}\delta^{\alpha}_{\nu}O_{\mu} + \frac{1}{6}\delta^{\alpha}_{\mu}O_{\nu}, \tag{40}$$

where $\{_{\mu}{}^{\alpha}{}_{\nu}\}$ are the Christoffel symbols constructed from $M_{\mu\nu}$ in the usual fashion.

*

If we break the Ricci tensor into its symmetric $S_{\mu\nu}$ and anti-symmetric $F_{\mu\nu}$ parts one then has:

$$S_{\mu\nu} = R^M_{\mu\nu} - \frac{1}{3} O_\mu O_\nu, \tag{41}$$

where $R^M_{\mu\nu}$ is the Ricci tensor of the metric $M_{\mu\nu}$ and

$$F_{\mu\nu} = \frac{1}{6}(\partial_\mu O_\nu - \partial_\nu O_\mu). \tag{42}$$

In fact

$$\Gamma_\mu{}^\alpha{}_\alpha = \partial_\mu(\ln\sqrt{M}) + \frac{1}{3} O_\mu. \tag{43}$$

*Although the connection $\Gamma_\mu{}^\alpha{}_\nu$ is not a Weyl connection it does have the property, which is important for causality and horizons, that null geodesics of the metric $M_{\mu\nu}$ are also auto-parallel curves.

Evidently from a geometric point of view, $F_{\mu\nu}$ is proportional to the curvature of that part of the affine connection associated to parallel propagation of volumes. If it vanishes the connection would be "equi-affine" with structural group $SL(4, \mathbf{R})$ rather than the full $GL(4, \mathbf{R})$.

Physically and historically it was originally tempting to try to identify $F_{\mu\nu}$ as the electromagnetic field strength comprising $\mathbf{E}$ and $\mathbf{B}$ and because

$$\mathcal{N}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} \tag{44}$$

it then seemed reasonable to associate $\mathcal{G}^{\mu\nu} = -2\mathcal{N}^{\nu\mu}$ with its conjugate comprizing $\mathbf{D}$ and $\mathbf{H}$. Clearly the vector potential A_μ should be given by

$$A_\mu = \frac{1}{6} O_\mu. \tag{45}$$

However (37) then shows that electromagnetic gauge invariance is broken and what one has arrived at is a non-linear version of Proca theory coupled to gravity.

As Schrödinger realized,there is a remarakable and precise one to one equivalence between the class of symmetric affine unitary theories and and the class of non-linear electrodynamic theories with a mass term, including Proca-Born-Infeld, coupled to ordinary Einstein gravity.

To see this remarkable equivalence recall, that the symetric part of the Ricci tensor $S_{\mu\nu}$ satisfies

$$\mathcal{M}^{\mu\nu} = \frac{\partial \mathcal{L}(S_{\mu\nu}, F_{\mu\nu})}{S_{\mu\nu}}. \tag{46}$$

Modifying an original idea of Einstein's, Shrödinger performed a partial Legendre transform by defining:

$$\overline{\mathcal{L}}(\mathcal{M}^{\mu\nu}, F_{\mu\nu}) = \mathcal{L} - \mathcal{M}^{\mu\nu} S_{\mu\nu}. \tag{47}$$

Thus

$$S_{\mu\nu} = -\frac{\partial \overline{\mathcal{L}}}{\partial \mathcal{M}_{\mu\nu}}. \tag{48}$$

and

$$\mathcal{G}^{\mu\nu} = -2\frac{\partial \overline{\mathcal{L}}}{\partial F_{\mu\nu}} \tag{49}$$

Then one finds that (41) becomes

$$R^M_{\mu\nu} = -\frac{\partial \overline{\mathcal{L}}}{\partial \mathcal{M}_{\mu\nu}} + 6 A_\mu A_\nu. \tag{50}$$

and (37) becomes

$$\partial_\mu \mathcal{G}^{\mu\nu} = -12 \mathcal{M}^{\mu\nu} A_\nu. \tag{51}$$

These are of course, for a general Lagrangian, exactly the Einstein-nonlinear-Proca equations and may be obtained by varying the Lagrangian

$$\mathcal{M}^{\mu\nu} R^M_{\mu\nu} + \overline{\mathcal{L}} - 6 \mathcal{M}^{\mu\nu} A_\mu A^\nu. \tag{52}$$

with respect to $\mathcal{M}^{\mu\nu}$ and A_μ. Thus indeed one may regard $M_{\mu\nu}$ as the metric in units in which

$$16\pi G = 1. \tag{53}$$

Consider the linear case

$$\overline{\mathcal{L}} = -\frac{1}{4e^2} F_{\mu\nu} F_{\alpha\beta} M^{\mu\alpha} M^{\beta\nu} \sqrt{-M}. \tag{54}$$

It is easily seen that that the mass m_V of the vector boson is given in physical units by

$$m_V^2 = \frac{3e^2}{4\pi G}. \tag{55}$$

It is quite striking that the Planck mass enters formula (55) in the same way as the usual Higgs expectation value enters the mass in the standard Higgs mechanism. Put in another more provocative way we see here a purely gravitional version of the abelian Higgs mechanism.

Unfortunately it is rather hard to carry out explicitly the partial Legendre transformation (47, 48) to get the Lagrangian $\mathcal{L}$ in terms of $S_{\mu\nu}$ and $F_{\mu\nu}$. It would be even harder if one chose for $\overline{\mathcal{L}}$ the Born-Infeld Lagrangian.

Starting with the Lagrangian $\mathcal{L}$ and calculating the Lagrangian $\overline{\mathcal{L}}$ is no easier. Thus while it is easy to see that Eddington's Lagrangian (29) gives the equations:

$$R_{\mu\nu} = \Lambda L_{\mu\nu} \tag{56}$$

where $L_{\mu\nu}$ is the inverse of of $M^{\mu\nu} - \frac{1}{2}G^{\mu\nu}$ it is difficult to see precisely what version of non-linear electrodyamics this correponds to.

Summary

- A breakdown of gauge invariance with a tachyonic vector can give ries to an effective positive cosmological constant

- Such broken gauge invariance arises naturally from symmetric-affine theories of gravity

- There are intriguing hints of possible connections with branes and Born-Infeld gravity

- For applications to the late time universe , there remains a problem of fine tuning.