Multiply wound Polyakov loops at strong coupling

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Sean Hartnoll[1] Prem Kumar[2]

[1]Department of Applied Mathematics and Theoretical Physics
University of Cambridge

[2]Department of Physics
University of Wales Swansea

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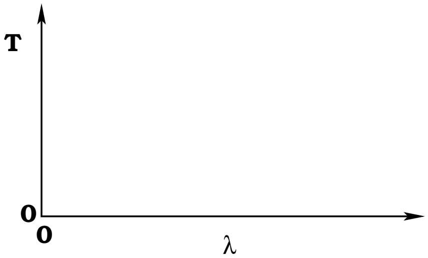
Outline

- Phase structure of $\mathcal{N} = 4$ SYM theory?
- Computing multiply wound Polyakov loops at strong coupling.
- D3 and D5 branes in Schwarzschild-AdS.
- Non analyticity of N^{-1} expansion and eigenvalue distributions.
- Summary.

Phase structure of $\mathcal{N} = 4$ SYM theory on $S^3 \times S^1$

Hawking-Page (1983), Witten (1998), Sundborg (1999), Aharony *et al.* (2003) ...

- Parameters: 't Hooft coupling λ and temperature T.



- Low temperature phase? Nonrenormalisation theorems for correlators. (Brigante-Festuccia-Liu, 2005)
- High temperature phase? Threshold branch cuts in correlators at small coupling coupling vs. quasinormal poles at strong coupling. (Hartnoll-Kumar 2005)

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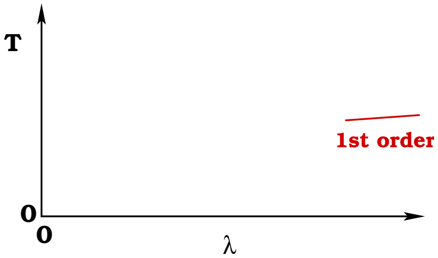

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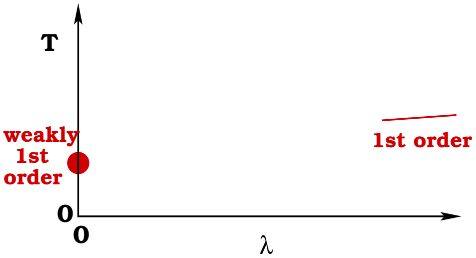

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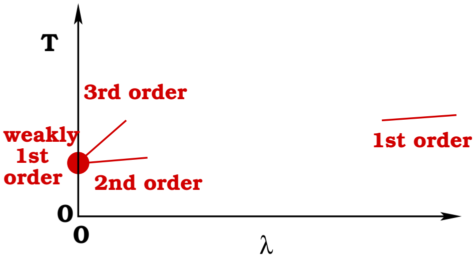

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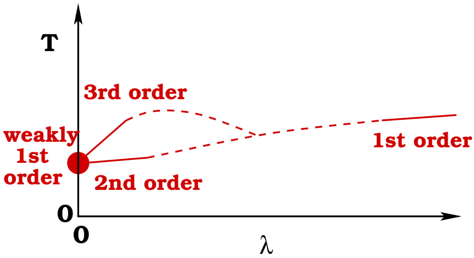

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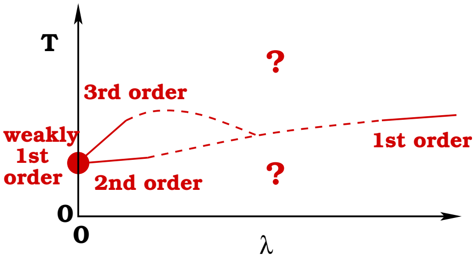

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- The order parameter for these transitions is the large N eigenvalue distribution of the Polyakov loop

$$U = P\, e^{i \oint A_0 \, d\tau} \,.$$

- The eigenvalue distribution can be computed from $\langle \mathrm{tr}\, U^k \rangle$

$$\rho(\theta) = 1 + 2 \sum_{k=1}^{\infty} \frac{1}{N} \langle \mathrm{tr}\, U^k \rangle \, \cos k\theta \,.$$

- 'Deconfinement': separates uniform vs. non uniform.
- 'Gross-Witten': separates gapped vs. non gapped.

What is the eigenvalue distribution at strong coupling?

Multiply wound loops from fundamental strings?

Gross-Ooguri (1998)

- AdS/CFT dictionary at strong coupling:

$$\frac{1}{N}\langle \mathrm{tr}\, U^k \rangle = e^{-S_{\text{k-winding}}} \,,$$

where U is now the (non-unitary) Polyakov-Maldacena loop

$$U = P\, e^{i \oint [A_0 - i\Phi_I \theta^I(\tau)] d\tau} \,,$$

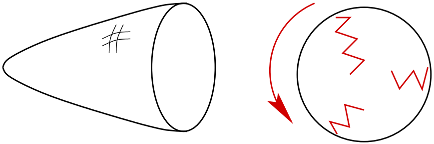
and $S_{\text{k-winding}}$ is the action of a string instanton in the bulk that winds around the thermal circle k times at infinity.

- The bulk is the Euclidean AdS-Schwarzschild black hole

$$ds^2 = R^2 \left[f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 + d\Omega_5^2 \right] \,,$$

with the horizon at $r = r_+$.

- The required string instanton wraps the cigar of the black hole. The multiple winding is implemented by $k-1$ branch cuts running from the interior of the worldsheet to infinity.



- This suggests the naïve result: $\frac{1}{N}\langle \mathrm{tr}\, U^k \rangle = e^{-kS_1}$.
- Problems:
  - The worldsheet has conical singularities.
  - There is a moduli space of embeddings, need to integrate over the location of the branch points.
- Both of these problems present serious technical difficulties. The naïve result, which would lead to a delta function eigenvalue distribution, is generally incorrect.

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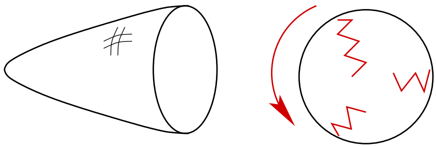

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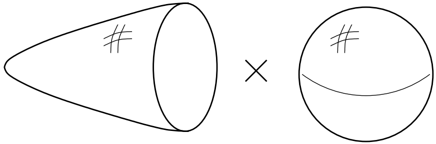
Multiply wound loops from D3 branes

Drukker-Fiol (2005)

- Considered a k wound circular spatial Wilson loop in $\mathbb{R}^4$.
- Used a single probe D3 brane carrying k units of string charge induced from a worldvolume gauge field

$$k = -i \frac{\delta S_{D3}}{\delta F_{tr}} \, .$$

- Think of as k strings blown up to a D brane via the Myers effect. Geometry is $AdS_2 \times S^2 \subset AdS_5$.



- No conical singularities. No moduli space. Fully reliable probe brane embedding!

- Druker-Fiol solved the probe D3 brane equations of motion and evaluated the action on the solution to obtain

$$\frac{1}{N}\langle \mathrm{tr}\, U^k \rangle = e^{2N\left[\kappa\sqrt{1+\kappa^2}+\sinh^{-1}\kappa\right]},$$

where $\kappa = \frac{k\sqrt{\lambda}}{4N}$. Would require a partial resummation of higher genus corrections in the string picture.

- This circular loop is supersymmetric, so can compare with a field theory computation using the Drukker-Gross Hermitian matrix model

$$\frac{1}{N}\langle \mathrm{tr}\, U^k \rangle = \frac{1}{Z}\int \mathcal{D}M \frac{1}{N}\mathrm{tr}\, e^{kM} e^{-\frac{2N}{\lambda}\mathrm{tr}\, M^2}.$$

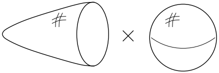
- They evaluated this matrix integral and found exact agreement with the above D brane expression.

(No) D3 branes in the black hole background

- Adapt Drukker-Fiol to the case of multiply wound Polyakov loops. The D3 brane action is

$$S = T_{D3} \int d\tau d^3\sigma e^{-\Phi} \sqrt{\det\left({}^\star g + 2\pi\alpha' F\right)} - i g_s T_{D3} \int {}^\star C_4 \,,$$

- Worldvolume: cigar of black hole $\times$ $S^2 \subset S^3$. Solve for size of S^2, $\alpha(r)$, and for electric field, $2\pi F_{tr}(r) = i\lambda^{1/2} F(r)$.

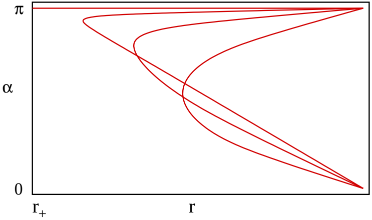


- Equations of motion

$$\kappa = \frac{r^2 \sin^2 \alpha F}{\sqrt{1 + r^2 f \left(\frac{d\alpha}{dr}\right)^2 - F^2}} \,,$$

$$r^2 \sin^2 \alpha \left[r^2 \sin 2\alpha \frac{F}{\kappa} + 4r \right] = \frac{d}{dr} \left(r^2 f \frac{\kappa}{F} \frac{d\alpha}{dr} \right) \,.$$

- Need $\alpha \to \pi$ as $r \to \infty$. There is a one parameter family of finite action boundary conditions at infinity. Integrate these in from infinity, what happens?



- The only solution that reaches the horizon is the 'collapsed' solution: $\alpha = \pi$. The S^2 does not blow up: $S_{D3} = kS_{F1}$.
- This takes us outside the validity of the D3 brane action. End up back in the multiply wound string picture.

D3 branes do not capture corrections to $\rho(z) = \delta(z - z_0)$.

Multiply wound loops from D5 branes

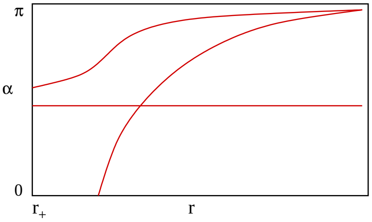
Callan *et. al* (1998), Hartnoll-Kumar (2006), Yamaguchi (2006)

- There are also D5 brane configurations compatible with the symmetries of multiply wound Polyakov or Wilson loops.
- Instead of $S^2 \subset S^3$ blown up, now have $S^4 \subset S^5$. Can have induced string charge k as before. With $k = N$, this setup has been used as a description of baryon vertices.
- Recall D5 brane action

$$S = T_{D5} \int d\tau d^5\sigma e^{-\Phi} \sqrt{\det\left({}^{\star}g + 2\pi\alpha' F\right)} - i g_s T_{D5} \int 2\pi\alpha' F \wedge {}^{\star}C_4 \,,$$

- Now need to solve equations of motion for $\alpha(r)$, the size of the $S^4 \subset S^5$.
- The parameter $\kappa = k\sqrt{\lambda}/4N$ no longer appears. Instead, the equations depend on $\kappa' = k/N$. D5 branes capture a different set of corrections.

- Integrate in from $r \to \infty$. This time there are solutions that do not go back out to infinity. At least three types of solution:



- The solution that reaches $\alpha = 0$ wraps the S^5 and so must have $k = N$ to be regular. This is the baryon vertex.
- The constant solutions are possible at discrete angles

$$\pi(\kappa' - 1) = \sin\alpha_0 \cos\alpha_0 - \alpha_0 \,.$$

There is a unique α_0 for each value of $\kappa' = k/N \in [0, 1]$. (This expression is familiar from studies of confining strings.)

- The constant solutions may be treated analytically. The action evaluated on the solution is given by

$$S = -\frac{N\sqrt{\lambda}\beta r_+}{3\pi^2} \sin^3 \alpha_0 \,.$$

- Another interesting regime that simplifies is the limit $k/N \to 0$. In this limit the function $\alpha(r)$ scales as $\alpha = k^{1/3}\tilde{\alpha}$. The leading order equation of motion is

$$\frac{d}{dr}\left[f(r)\frac{d\tilde{\alpha}}{dr}\right] = 4\tilde{\alpha}^4\left(\tilde{\alpha}^3 - 1\right) \,. \qquad (1)$$

  The action evaluated on solutions to this equation is

$$S = kS_{F1} + A(r_+)\frac{k^{5/3}}{N^{2/3}} + \cdots \,, \qquad (2)$$

  where $A(r_+)$ can be computed numerically.

- The non analyticity here in N^{-1}, the string coupling, is rather suggestive. Corrections scaling as $N^{-2/3}$ arise, for instance, in the vicinity of a Gross-Witten phase transition, where the N^{-1} expansion breaks down. (Periwal and Shevitz, 1990)

  However, this begs the question...

- What do the D5 brane solutions compute? Probably not $\mathrm{tr}\,U^k$. Other possibilities are $(\mathrm{tr}\,U)^k$ and $\mathrm{tr}_{R_k} U$, where R_k is a representation of $SU(N)$ with N-ality k. These are related to $\mathrm{tr}\,U^k$ through the Frobenius formula. For instance

$$\mathrm{tr}\,U^2 = \mathrm{tr}_{\square\square}\, U - \mathrm{tr}_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\, U\,. \quad (3)$$

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- Yamaguchi argues that D5 branes correspond to Wilson lines in the k^{th} antisymmetric representation. Near horizon limit of N D3s, 1 D5 and k F1s. In this system the F1s are fermionic.
- The non constant D5 solutions are natural candidates for antisymmetric representations as their tension tends asymptotically to that of k F1s. From our solutions one could thus compute $\mathrm{tr}_{A_k} U$ numerically.
- The constant solutions may provide a direct connection to the eigenvalue distribution. Yamaguchi found $AdS_2 \times S^4$ duals to circular spatial Wilson loops. Their action is basically identical to the constant solutions described above. The Drukker-Gross matrix model shows that the $N^{-2/3}$ non analyticity stems from an underlying gapped Wigner eigenvalue distribution.
- Work in progress builds on this to suggest that the Polyakov loop eigenvalue distribution at strong coupling is gapped with a temperature dependent width.

Summary

- Computing the eigenvalue distribution of the Polyakov loop at strong coupling is necessary for understanding the phase structure of $\mathcal{N} = 4$ SYM theory.
- Information about this distribution may be accessed by using probe D5 branes, carrying k units of string charge, in the dual black hole background.
- Interesting features:
  - Solutions with a constant size $S^4 \subset S^5$.
  - Non analyticity in N^{-1} expansion.
- We expect to present arguments in the near future indicating that the eigenvalue distribution at strong coupling is related to a gapped Wigner distribution with a temperature dependent width.