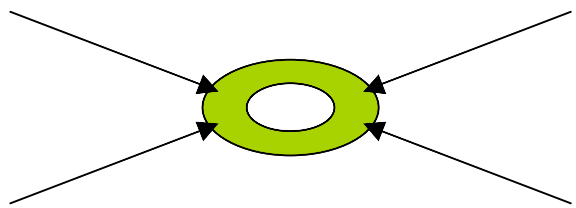
Quantum structure of Black Holes

Samir D. Mathur

The Ohio State University

Work done in collaboration with:

S. Giusto, O. Lunin, A. Saxena, Y Srivastava

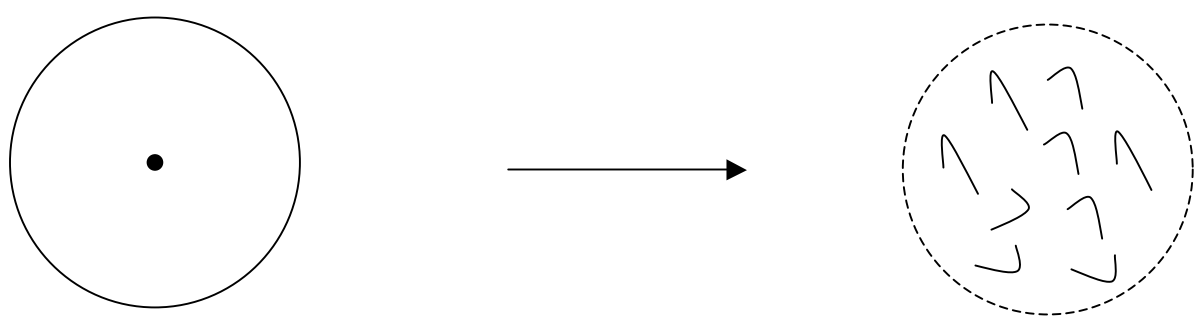


Graviton scattering: Quantum effects important at scale

$$l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} \sim 10^{-33} \text{ cm}$$

When we bind N particles together

$$l_p \longrightarrow N^{\square}\, l_p \quad ??$$



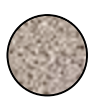
‘Fuzzball’

??

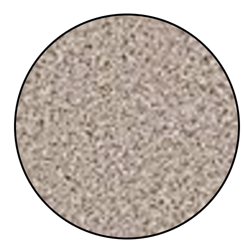
Bound states of two or more kinds of mutually BPS objects have a large entropy

$$
\begin{aligned}
S &= 2\sqrt{2}\pi\sqrt{n_1 n_2} \\
S &= 2\pi\sqrt{n_1 n_2 n_3} \\
S &= 2\pi\sqrt{n_1 n_2 n_3 n_4}
\end{aligned}
$$

What is the structure of these microstates?



g small



g large

??

Plan of the talk

1. 2-charge systems: constructing microstates

2. 3-charge systems

   (a) Constructing special microstates
   (b) Base-Fiber structure of geometries

3. Quantum corrections: some comments

4. A microstate for 3-charge ring

5. Conclusions

Notation:

Type IIA string theory: **gravitons, NS1, NS5**

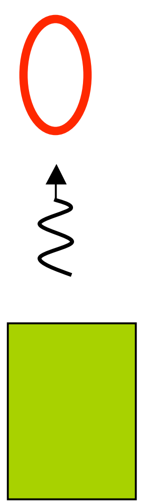
Compactify: $M_{9,1} \rightarrow M_{4,1} \times S^1 \times T^4$

Radius of S^1 **is** R_y , **Volume of** T^4 **is** $(2\pi)^4 V$

NS1 wrapped on S^1

Momentum modes P along S^1

NS5 branes wrapped on $S^1 \times T^4$



Dualities permute NS1, P, NS5 in all possible ways

$$NS1\,NS5\,P\,(IIB) \xrightarrow{T_5} P\,NS5\,NS1\,(IIA)$$

$$\xrightarrow{T_6} P\,NS5\,NS1\,(IIB)$$

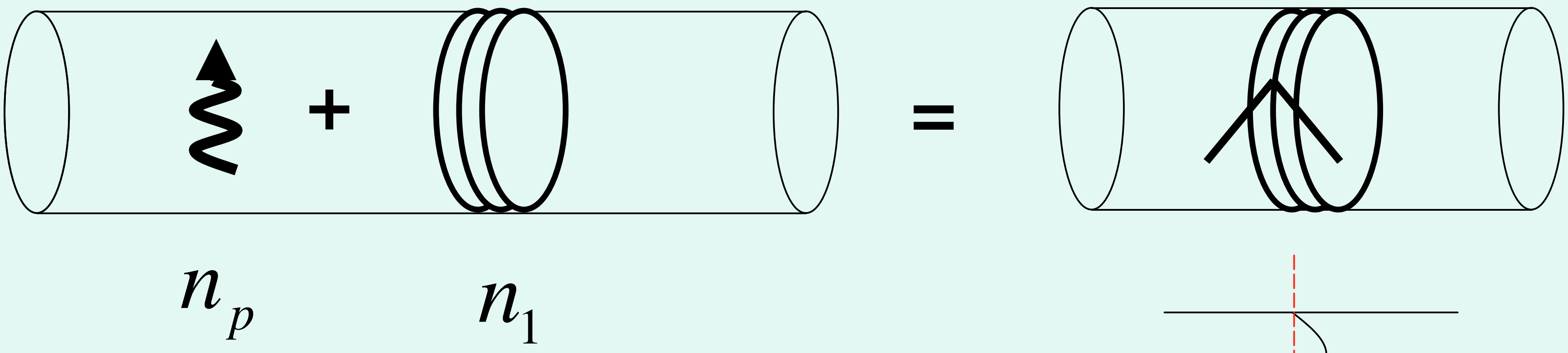
$$\xrightarrow{S} P\,D5\,D1\,(IIB)$$

$$\xrightarrow{T_{6789}} P\,D1\,D5\,(IIB)$$

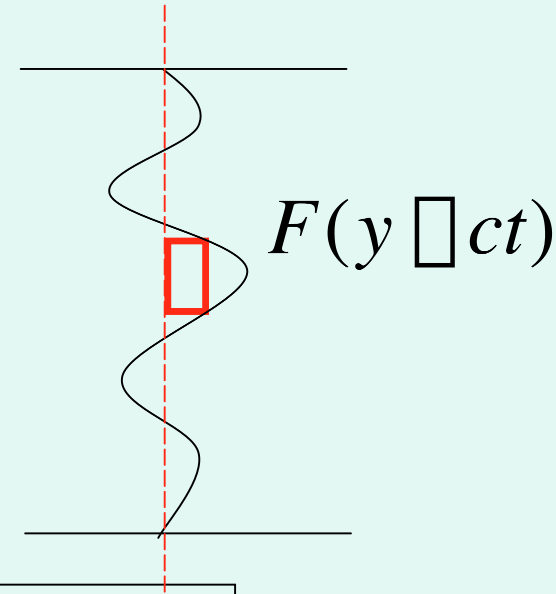
$$\xrightarrow{S} P\,NS1\,NS5\,(IIB)$$

$$NS1\,NS5\,P \xrightarrow{S} D1\,D5\,P$$

Thus we can map D1-D5 to NS1-P which is just an elementary string with winding and momentum charges



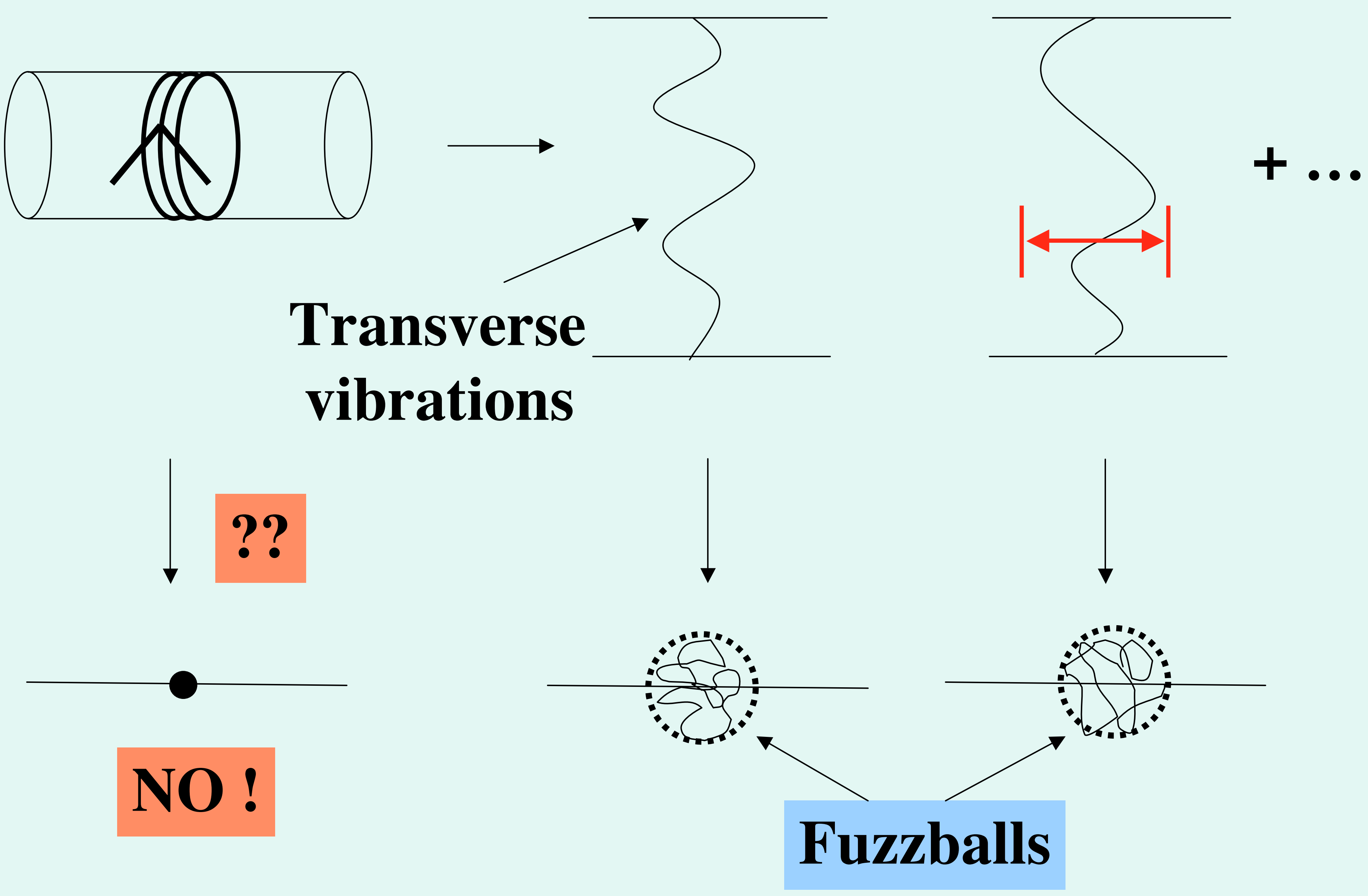
Traveling waves



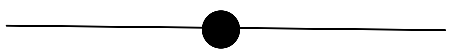
Many ways to partition the momentum among different harmonics

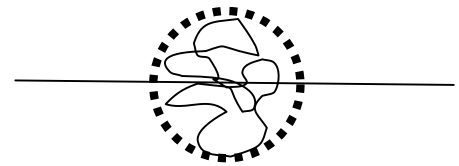
→ $$e^{2\pi\sqrt{2}\sqrt{n_1 n_p}}$$ **states**

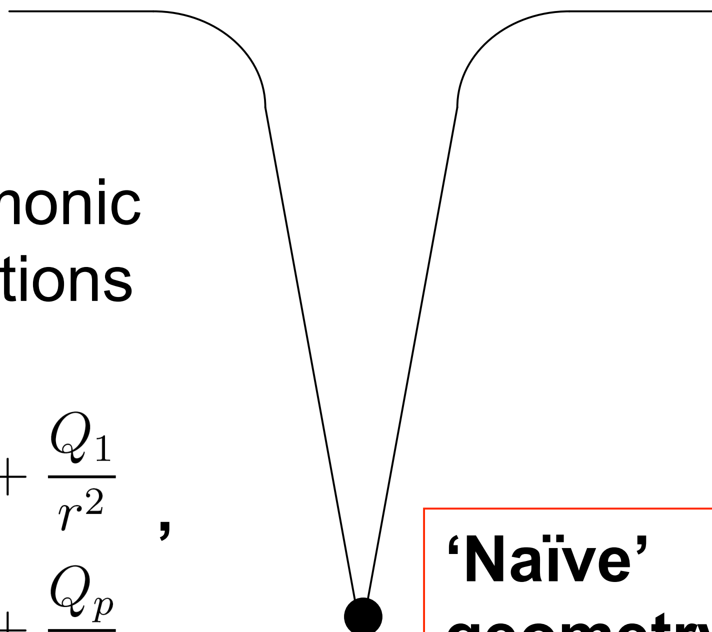
‘Size’ of the bound state



`Capped' geometries created by `fuzzballs'



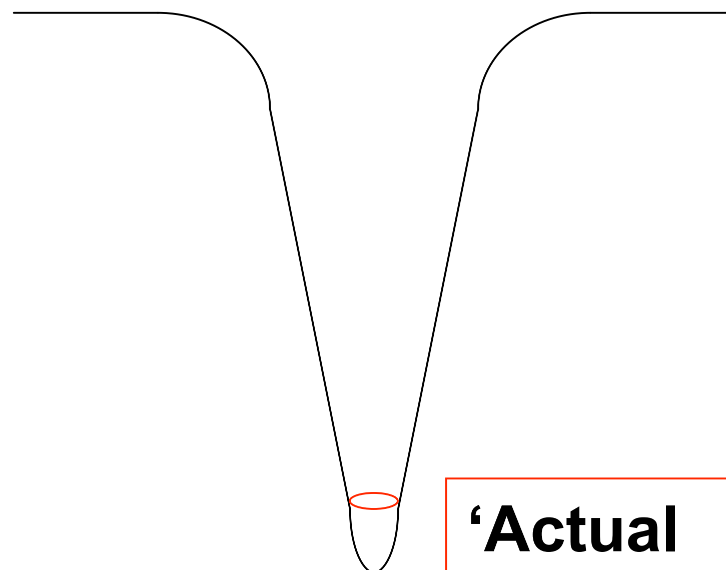




harmonic
functions

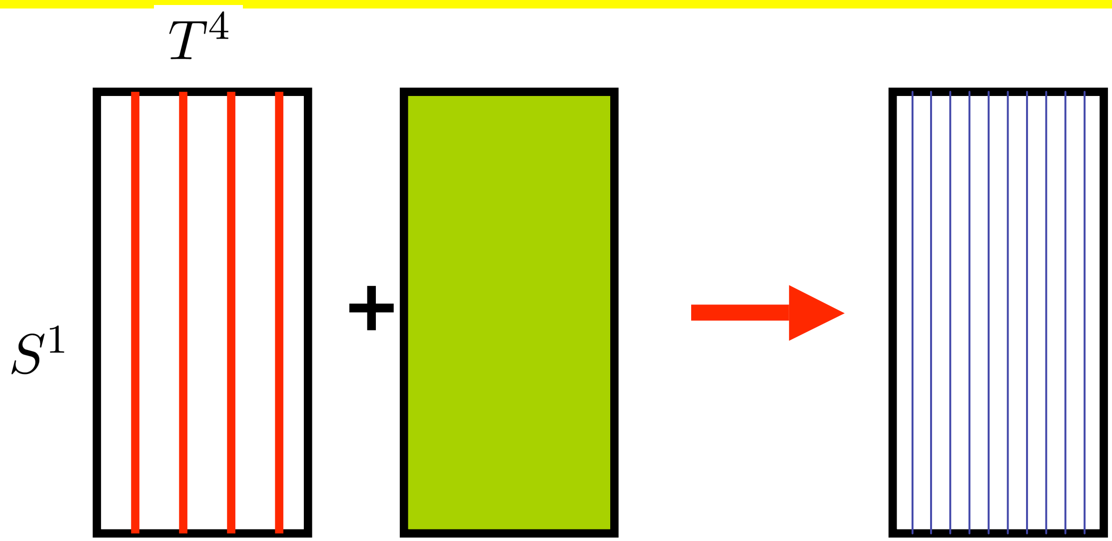
$$1+\frac{Q_1}{r^2}\ ,\qquad 1+\frac{Q_p}{r^2}$$

'Naïve' geometry



'Actual geometries'

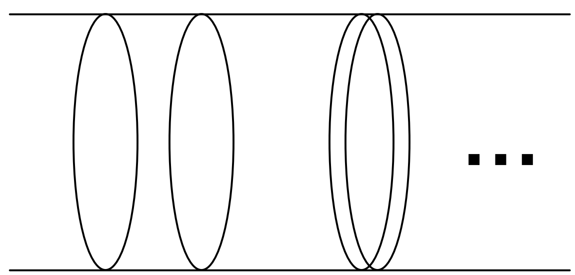
Bound state of NS1 -NS5 (or D1-D5)



NS1 n_1

NS5 n_5

Effective string:
Winding number
$n_1 n_5$

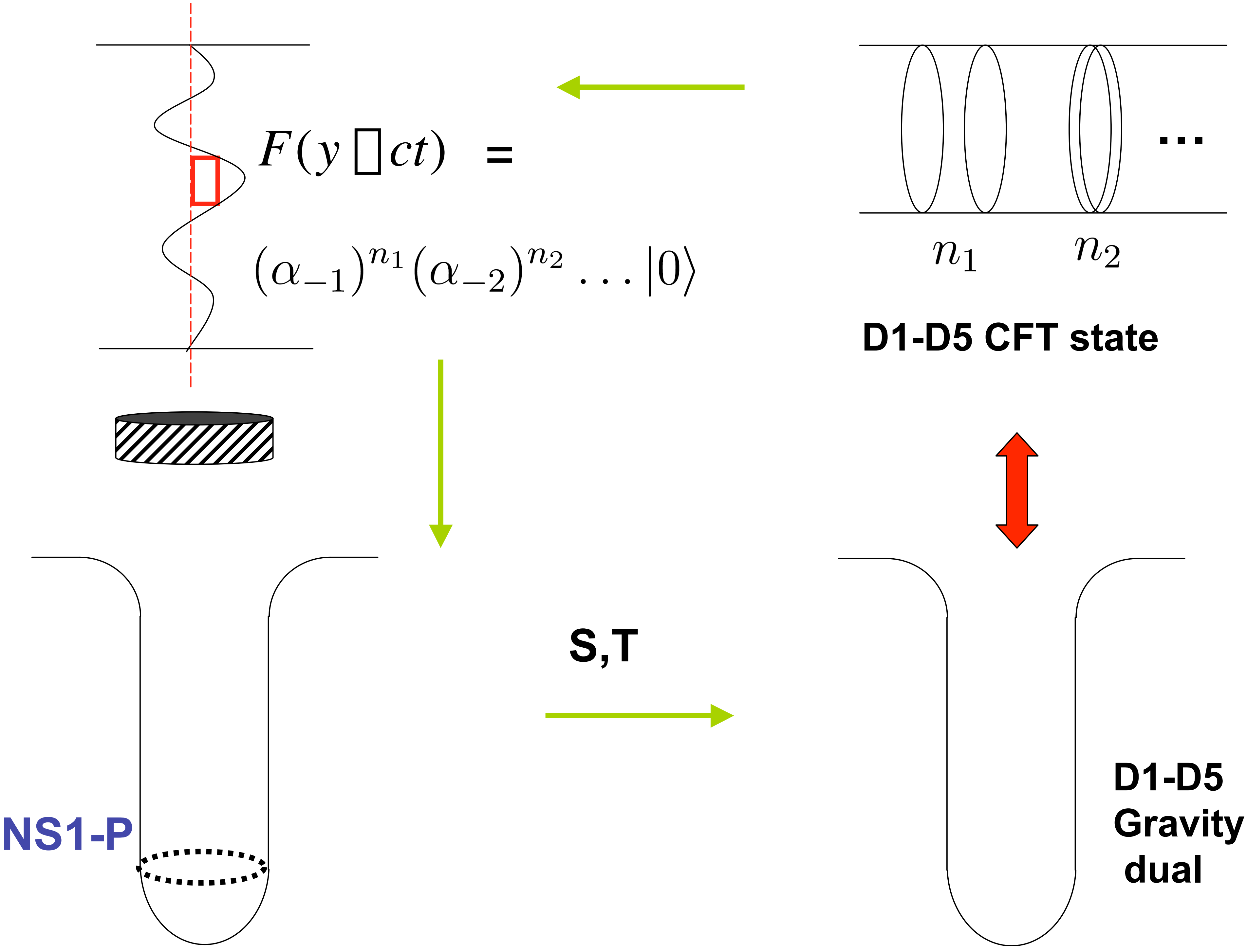


Dynamics:
left, right vibrations

Partitions of $n_1 n_5$ →

$$e^{2\pi\sqrt{2}\sqrt{n_1 n_5}}$$

states



$F(y \square ct)$ =
$(\alpha_{-1})^{n_1}(\alpha_{-2})^{n_2}\ldots|0\rangle$
n_1
n_2
...
D1-D5 CFT state
S,T
NS1-P
D1-D5
Gravity
dual

$$ds^2 = \sqrt{\frac{H}{1+K}}[-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2] + \sqrt{\frac{1+K}{H}} dx_i dx_i + \sqrt{H(1+K)} dz_a dz_a$$

$$H^{-1} = 1 + \frac{Q}{L_T} \int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2},$$

$$K = \frac{Q}{L_T} \int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2},$$

$$A_i = -\frac{Q}{L_T} \int_0^{L_T} \frac{dv\ \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}$$

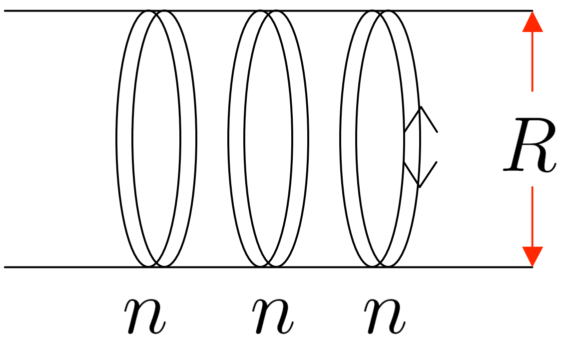
$$dB = - *_4 dA$$

First such metric:
Balasubramanian+
De Boer+ Keski-Vakkuri
+ Ross; Maldacena+Maoz

General metrics: Lunin+SDM

Also,
'Supergravity supertubes'
Emparan+Mateos+Townsend

D1-D5 CFT state

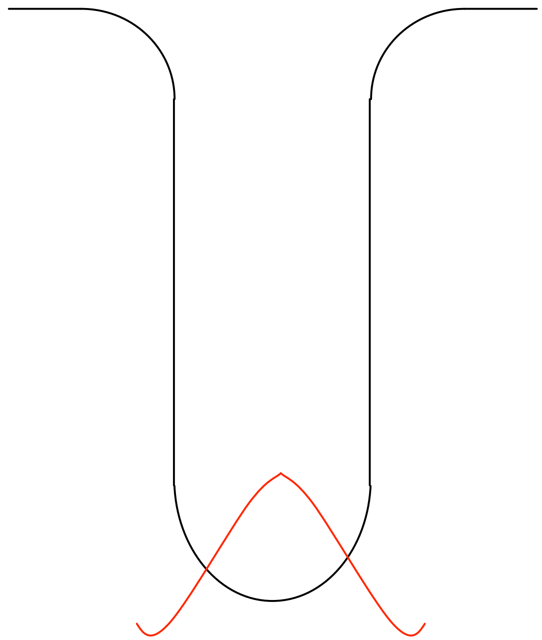


$$\Delta E = \frac{1}{nR} + \frac{1}{nR} = \frac{2}{nR}$$

Longer 'component strings' → lower energy

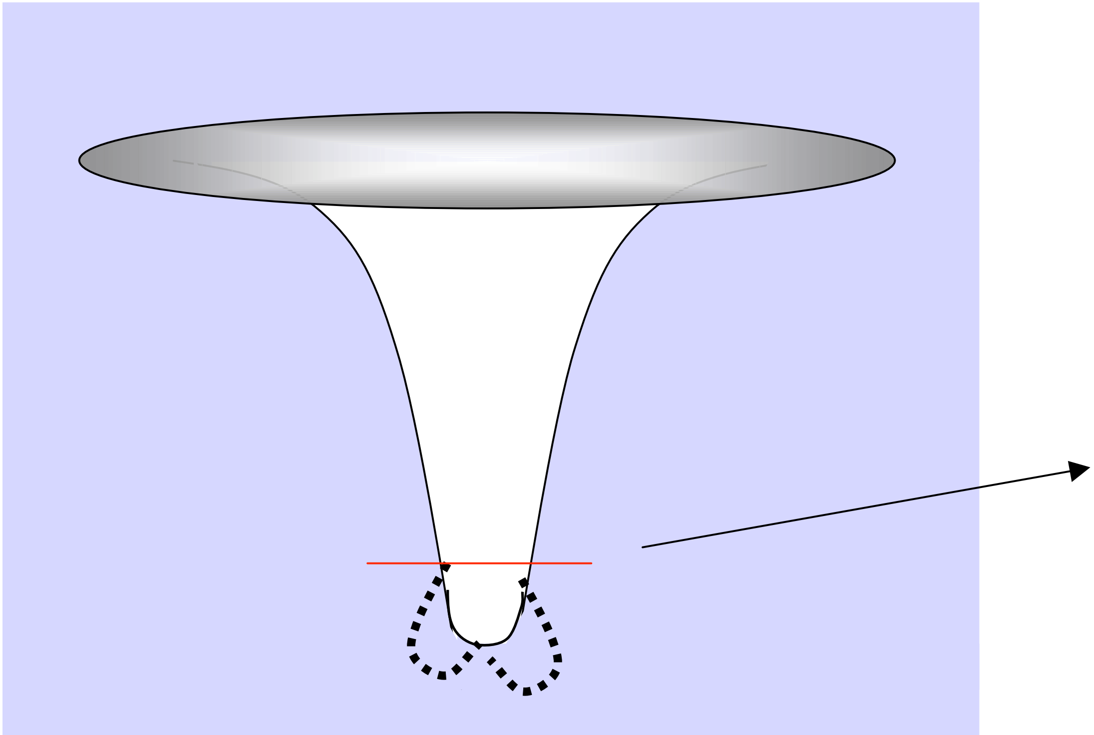
↔

D1-D5 Sugra solution



$$\Delta E = \frac{2}{nR}$$

Deeper throat, more redshift, lower energy

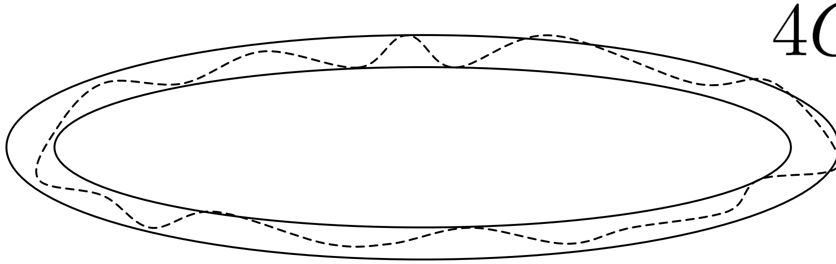


$$\frac{A}{4G} \sim \sqrt{n_1 n_5} \sim S$$

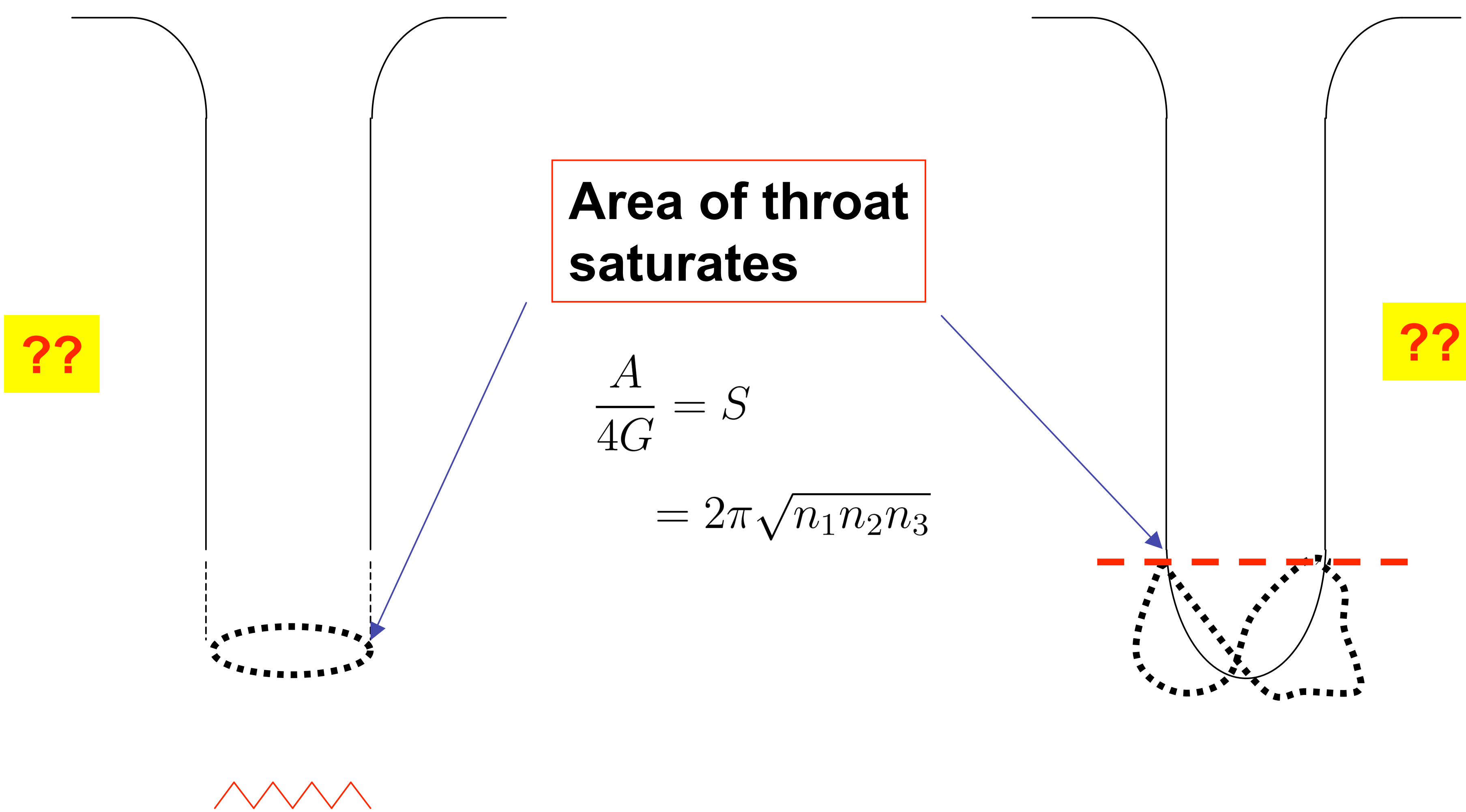
The 'size' of the typical fuzzball is such that the area of its surface yields a Bekenstein type relation

$$\frac{A}{4G} \sim \sqrt{n_1 n_5 - J} \sim S$$

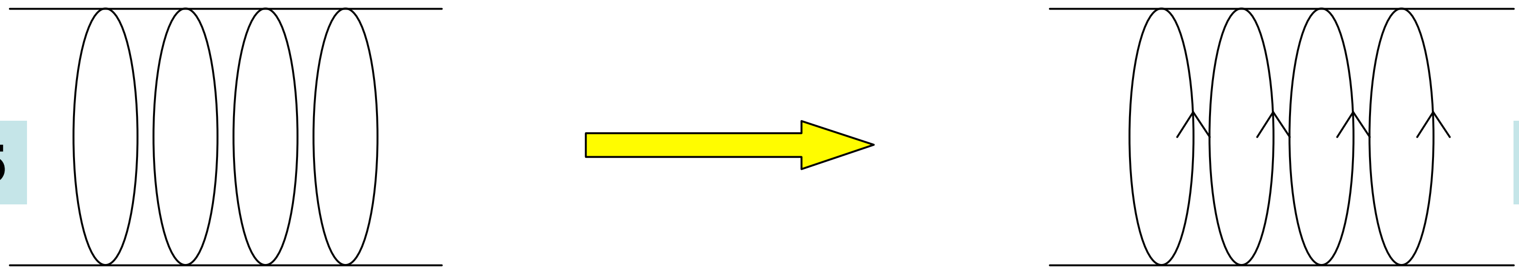
Highly rotating 2-charge



3-charge holes: NS1-NS5-P (or D1-D5-P)



D1-D5



D1-D5-P

Spectral flow on left movers :

R → NS → R (n times)

$$|n\rangle^{total} = (J^{-,total}_{-(2n-2)})^{n_1 n_5} (J^{-,total}_{-(2n-4)})^{n_1 n_5} \dots (J^{-,total}_{-2})^{n_1 n_5} \; |1\rangle^{total}$$

Right movers unchanged

$$h - \bar{h} = n(n+1) n_1 n_5$$

P charge

Spectral flow in AdS is a coordinate transformation

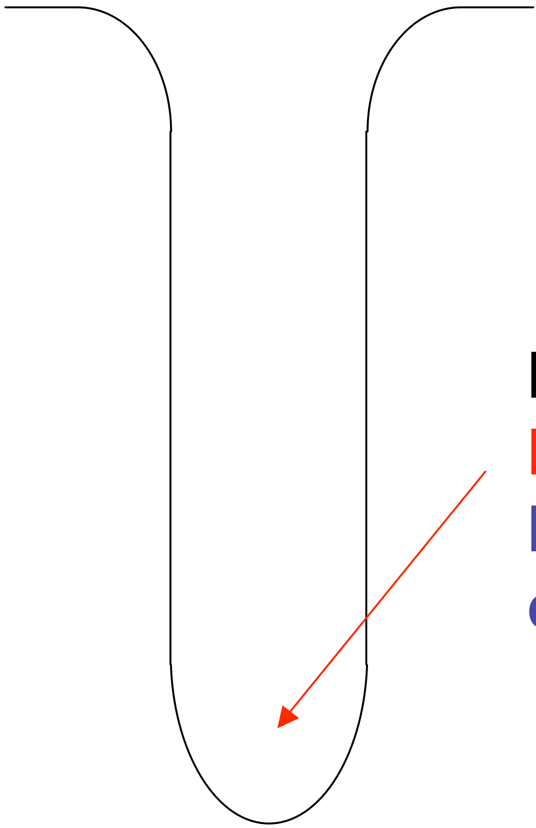
Balasubramanian+De Boer+ Keski-Vakkuri+ Ross '00; Maldacena+Maoz '00
Cvetic-Youm '95

$$
\begin{aligned}
ds^2 &= -\frac{1}{h}(dt^2 - dy^2) + \frac{Q_p}{hf}(dt - dy)^2 + hf\left(\frac{dr_N^2}{r_N^2 + a^2\eta} + d\theta^2\right) \\
&+ h\left(r_N^2 - na^2\eta + \frac{(2n+1)a^2\eta Q_1 Q_5 \cos^2\theta}{h^2 f^2}\right)\cos^2\theta d\psi^2 \\
&+ h\left(r_N^2 + (n+1)a^2\eta - \frac{(2n+1)a^2\eta Q_1 Q_5 \sin^2\theta}{h^2 f^2}\right)\sin^2\theta d\phi^2 \\
&+ \frac{a^2\eta^2 Q_p}{hf}\left(\cos^2\theta d\psi + \sin^2\theta d\phi\right)^2 \\
&+ \frac{2a\sqrt{Q_1 Q_5}}{hf}\left[n\cos^2\theta d\psi - (n+1)\sin^2\theta d\phi\right](dt - dy) \\
&- \frac{2a\eta\sqrt{Q_1 Q_5}}{hf}\left[\cos^2\theta d\psi + \sin^2\theta d\phi\right]dy + \sqrt{\frac{H_1}{H_5}}\sum_{i=1}^{4} dz_i^2
\end{aligned}
$$

$$
\begin{aligned}
f &= r_N^2 - a^2\eta\, n\sin^2\theta + a^2\eta\,(n+1)\cos^2\theta \\
h &= \sqrt{H_1 H_5},\ \ H_1 = 1 + \frac{Q_1}{f},\ \ H_5 = 1 + \frac{Q_5}{f}
\end{aligned}
$$

$$
\eta \equiv \frac{Q_1 Q_5}{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}
$$

D1-D5-P extremal geometry



No horizon
No singularity
No closed timelike curves

But not a very generic state:
axially symmetric,
large angular momentum …

Can we make all 3-charge microstate geometries?

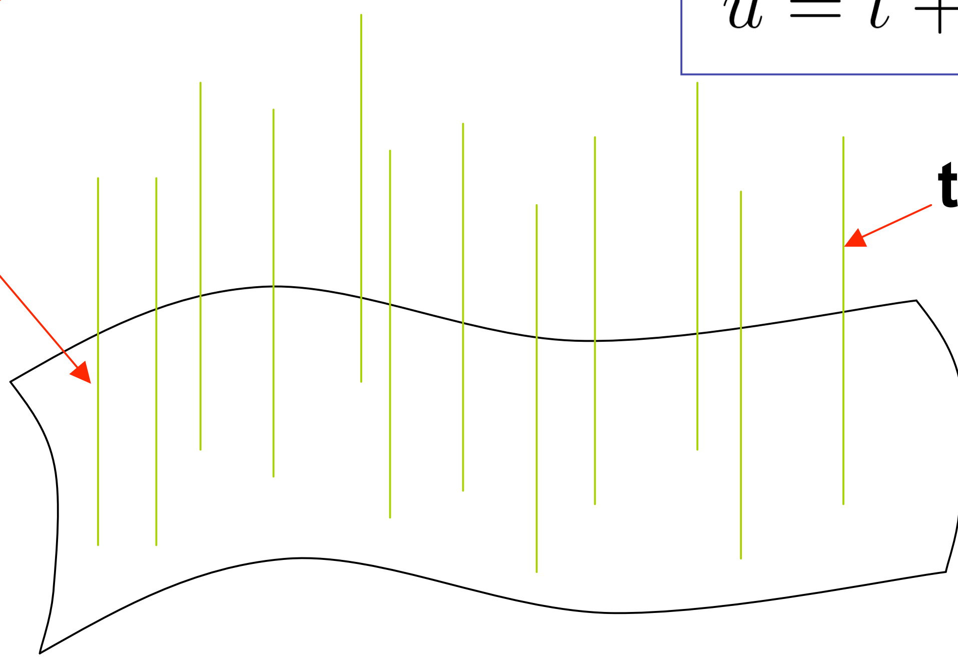
All D1-D5-P BPS supergravity solutions can be written as

$$ds^2 = -H^{-1}(dv + \sqrt{2}\beta)\Big(du + \sqrt{2}\omega + \frac{F}{2}(dv + \sqrt{2}\beta)\Big)$$

$$+H h_{mn} dx^m dx^n$$

$$u = t + y\,, \quad v = t - y$$

Hyperkahler base



t, y (S^1) fiber

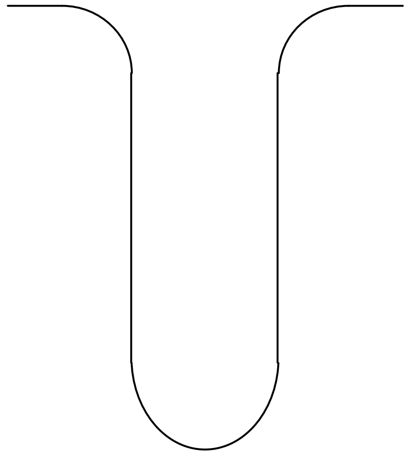
Coefficient functions are `harmonic functions with sources'

$$\mathcal{G}^{+} = H^{-1}\left(\frac{d\omega + \star d\omega}{2} + \frac{F}{2}\, d\beta\right)$$

$$\begin{aligned} d\beta &= \star d\beta \\ d\mathcal{G}^{+} &= 0 \\ d \star d\, H + d\beta\, \mathcal{G}^{+} &= 0 \\ d \star d\, F + \left(\mathcal{G}^{+}\right)^{2} &= 0 \end{aligned}$$

Gauntlett+Gutowski+Hull+ Pakis+ Reall 2002,
Gutowski+ Martelli+ Reall 2003,
Bena+Warner 2004

$$|n\rangle^{total} = (J^{-,total}_{-(2n-2)})^{n_1 n_5} (J^{-,total}_{-(2n-4)})^{n_1 n_5} \dots (J^{-,total}_{-2})^{n_1 n_5} \; |1\rangle^{total}$$



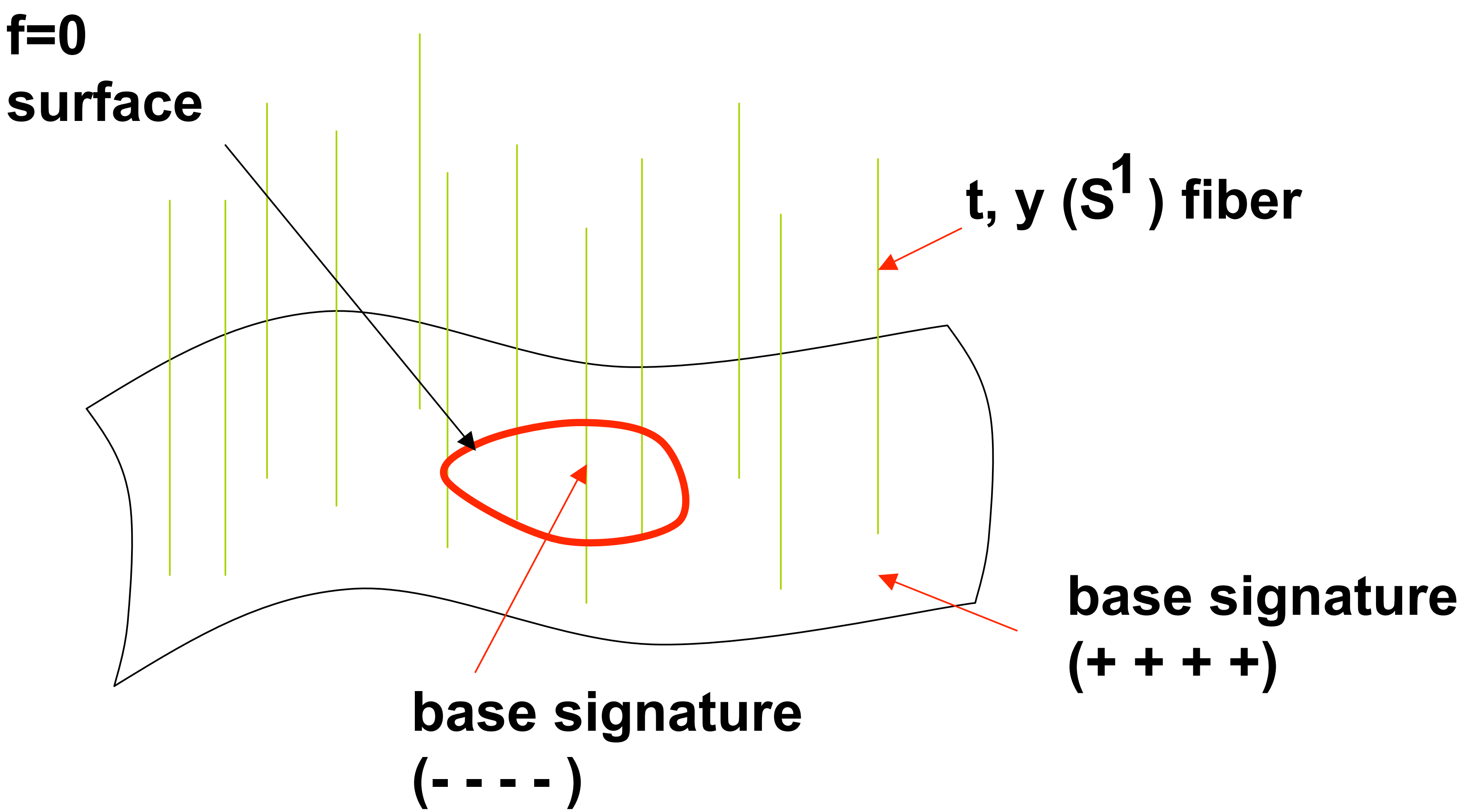
Write as base + fiber

$$h_{mn} dx^m dx^n = H_2^{-1} (d\tau + \chi \, d\tilde{\phi})^2 + H_2 \, ds_3^2$$

$$H_2 = \frac{n+1}{|\vec{x}|} - \frac{n}{|\vec{x} - \vec{c}|}$$

Base is Gibbons-Hawking, a special subset of hyperkahler

But …



Base is `pseudo-hyperkahler'

Full metric is regular , signature (-, +, +, +, +, +)

All Gibbons-Hawking type pseudo-hyperkahler base solutions constructed:

Bena+Warner '05, Berglund+Gimon+Levy '05

→

Get regular 3-charge solutions, with many properties suggesting that they could be black hole microstates (size, angular momentum of maximally rotating BMPV reproduced)

4-charge smooth microstates … (Bena+ Kraus '05)

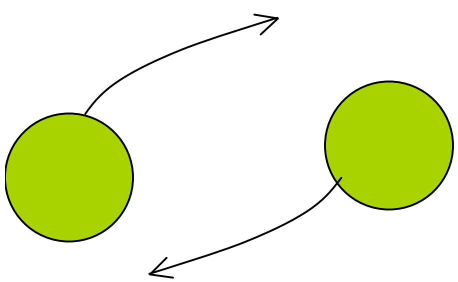
Is there a way to make sure a 3-charge geometry is a microstate?

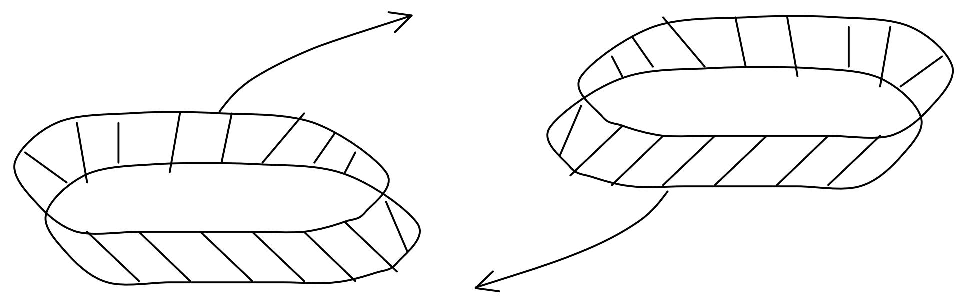
The difficulty:

Classically, the superposition of several 3-charge BPS solutions is a BPS solution …

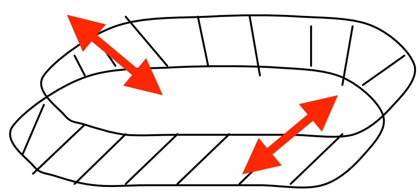
But this is not a black hole microstate *because it is not a bound state of the charges*

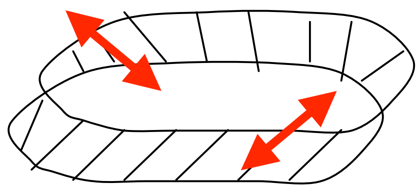
2-charge: Unbound states have a zero mode (the drift mode)





But a single bound state has only \`quasi-oscillations'





Even though there is a continuous family of classical configurations
With the same mass, charge, angular momentum….

The low energy dynamics is not a slow drift along a 'moduli space'

Conjecture: Same holds for 3-charge ….

i.e. If a geometry has a 'drift' zero mode then It is unbound, otherwise bound.

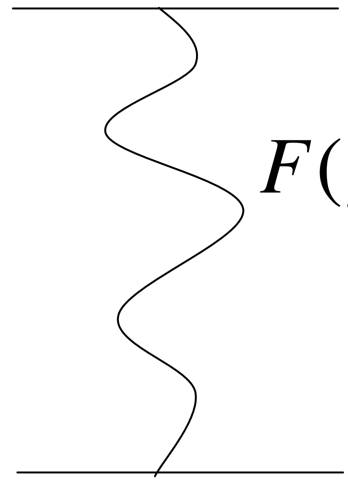
All BPS geometries known, select bound ones by this method … get all microstates ??

Quantum corrections in 2-charge geometries: some comments

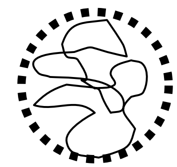
NS1-P state: $(\alpha_{-k_1})^{n_1}(\alpha_{-k_2})^{n_2}\ldots|0\rangle$

Fix total energy

Few modes k, large n : ***Coherent state***

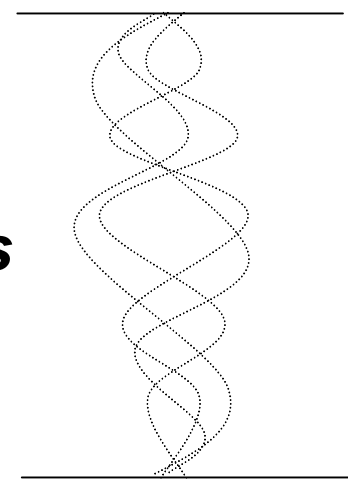


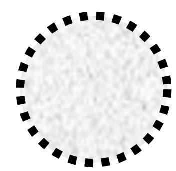
$F(y \,\square\, ct)$



Many k, All n ~ 1

Quantum energy eigenstate for Harmonic oscillators of each fourier mode





'Fuzzball'

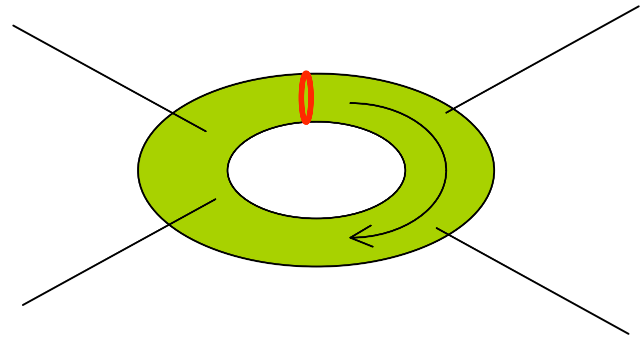
Size for generic state estimated from classical geometries

$$M_{9,1} \rightarrow M_{4,1} \times K3 \times S^1$$

D1

← D5 →

Winding mode of NS1 around S[1]



$$S_5 = \frac{2\pi^4 R_0 V_0}{G_{10}} \int dx^5 \sqrt{-g_{(E)}} \Big[R_{(E)} + \frac{c_2}{6} \frac{g^2 \alpha'^4}{V_0 R_0^2} \left(\frac{V}{V_0} e^{-2\phi} \right)^{-1/3} \left(\frac{R_0}{R} \right)^{4/3} R^{(E)}_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}_{(E)} \Big]$$

Radius of S^1

Cardoso, de Wit, Mohaupt '00,
Dabholkar '04

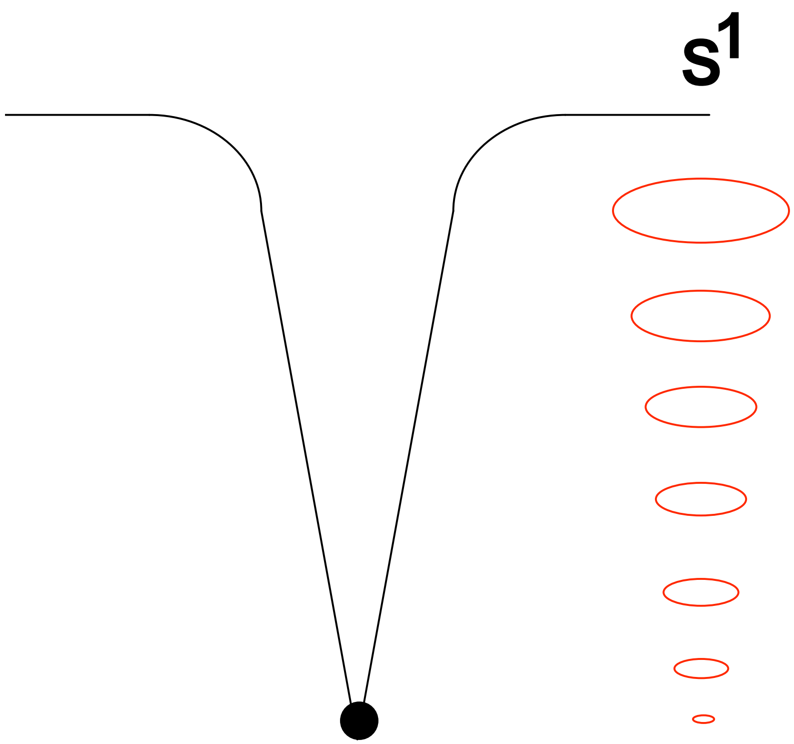
Naïve geometry:

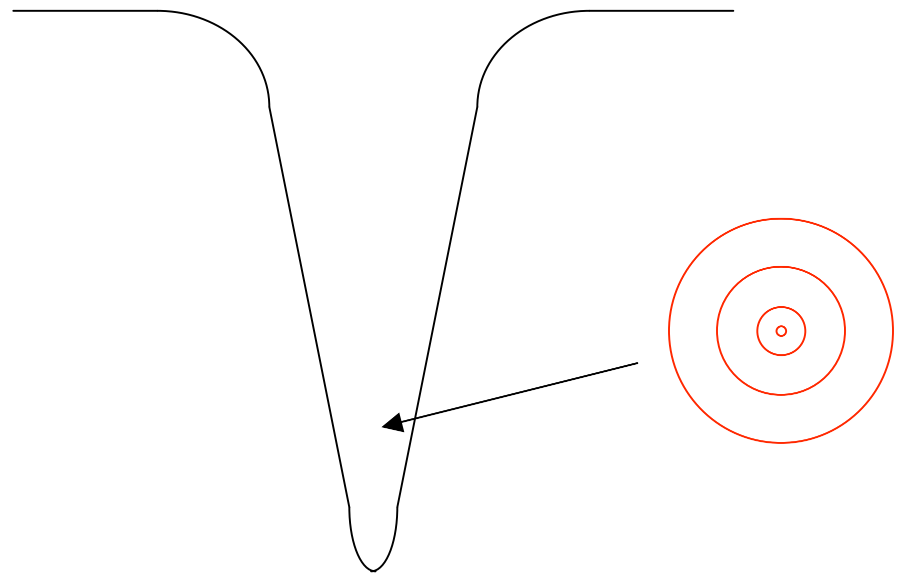
S^1 shrinks to zero size, correction can *diverge*

Actual geometry

S^1 is nontrivially fibered over the with the angular S^3

Shrinks to zero as the angular circle in a plane, like in the KK monopole

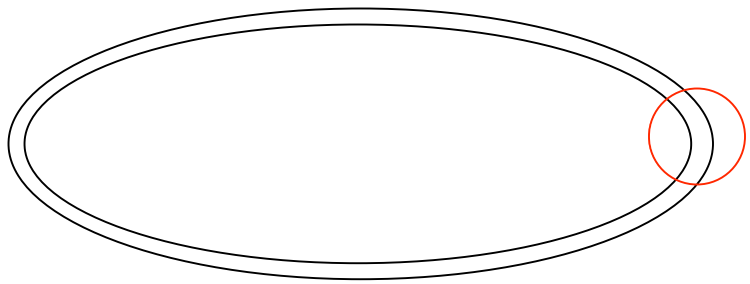


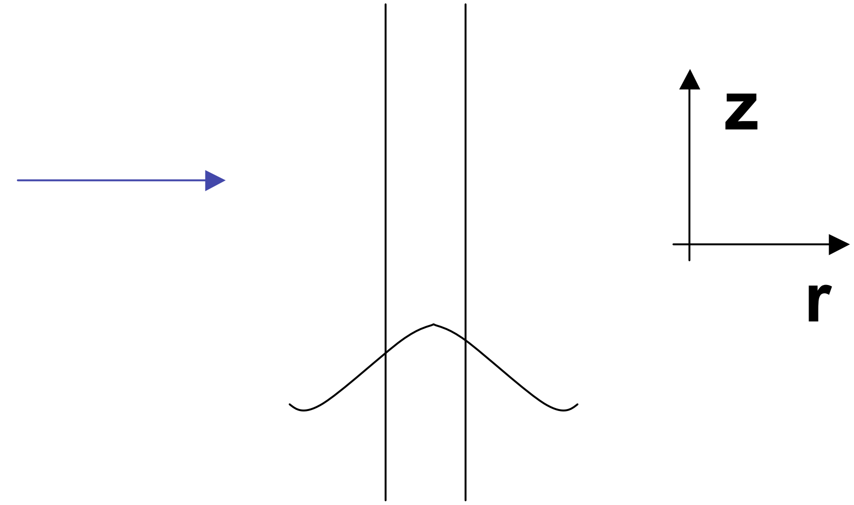


⇒ ***Corrections bounded***

A simple microstate for the 3-charge black *ring*

A microstate for the 3-charge black ring





Smooth D1-D5 geometry

Add one unit of P

CFT state $|\psi\rangle = J^{-}_{-1}|\psi\rangle_R$

Wavefunction

$$w = e^{-i(t+y)-ikz}\,\tilde{w}(r,\theta,\phi)$$

$$B^{(2)}_{MN} = e^{-i(t+y)-ikz}\,\tilde{B}^{(2)}_{MN}(r,\theta,\phi)$$

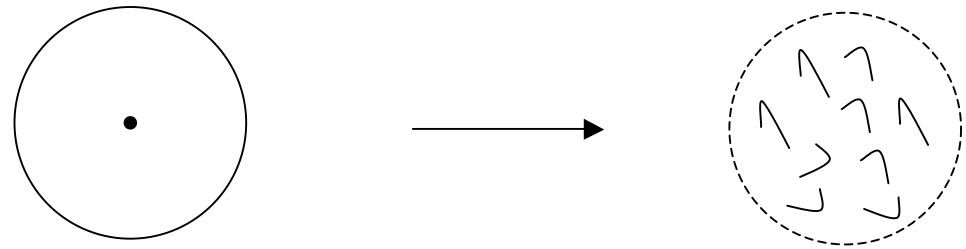
$$w = e^{-\frac{1}{2Q}(t+y)}\, e^{i(\phi-kz)}\, \cos\frac{\theta}{2}\, e^{-kr}\, \frac{r^{1/2}}{Q+r}$$

$$\begin{aligned}
B^{(2)} = {} & e^{-\frac{1}{2Q}(t+y)}\, e^{i(\phi-kz)}\, e^{-kr}\, r^{1/2} \Big\{ -\frac{1}{2Q}\, \cos\frac{\theta}{2}\, dt\wedge dz \\
& + \frac{r}{2(Q+r)^2}\, \cos\frac{\theta}{2}\, [dy - Q(1+\cos\theta)d\phi]\wedge\left[dt - \frac{2Q+r}{Q}\, dz\right] \\
& + ik\, \cos\frac{\theta}{2}\, dr\wedge dz + \frac{1}{2}\sin\frac{\theta}{2}\, dr\wedge[d\theta - i\sin\theta d\phi] \\
& \qquad - \frac{i}{2}\, r\, \cos\frac{\theta}{2}\sin\theta\, d\theta\wedge d\phi \Big\}
\end{aligned}$$

Conclusions

$l_p \longrightarrow N^{\square}\, l_p$

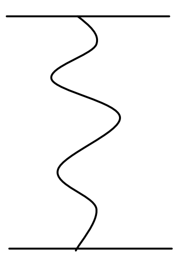
??

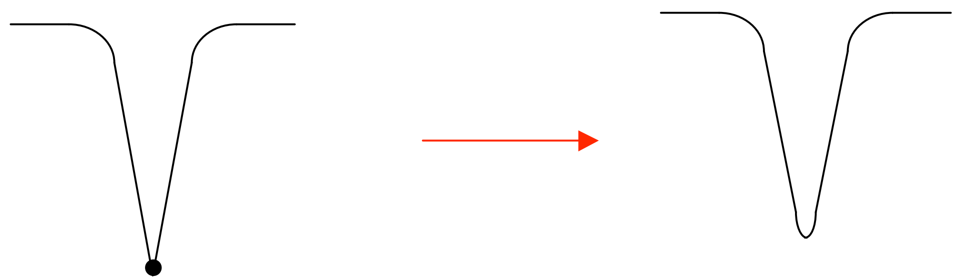


‘Matter description’: Fractionation generates low tension floppy objects that stretch upto horizon scale

$$T \to T/(n_1 n_2 n_3)$$

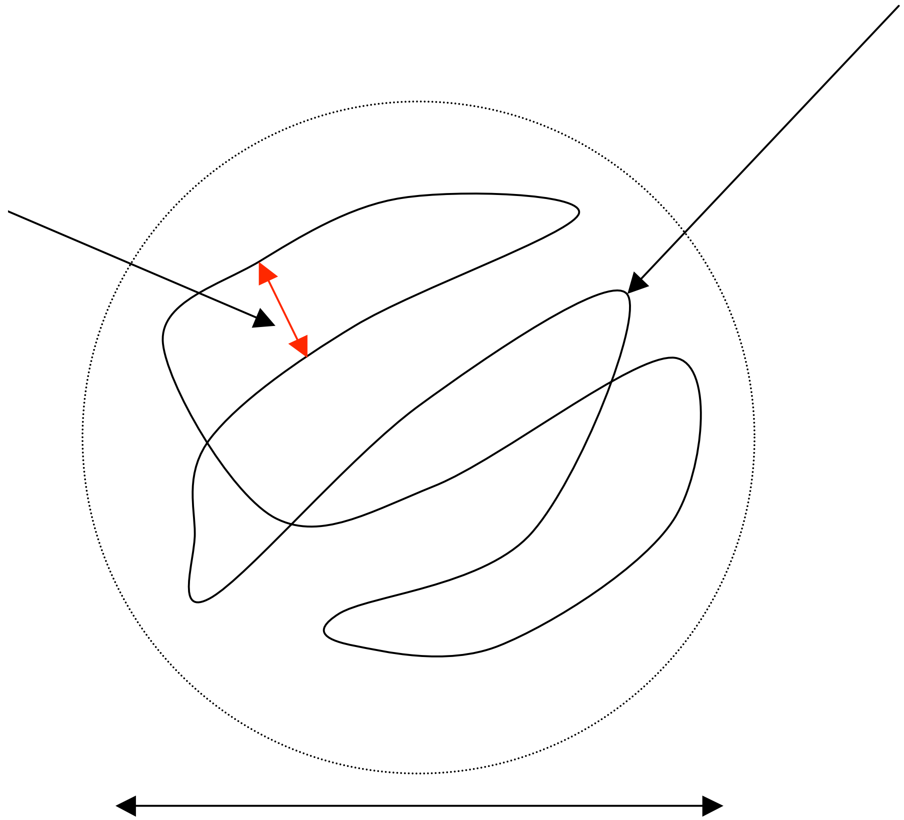
Black holes: D1-D5 CFT states all break spherical symmetry, compact directions twist over and merge with noncompact ones to ‘cap off’ throat before a horizon forms





Cosmology: Mass inside horizon radius is of same order as that needed to make a black hole …. Quantum effects may correlate across horizon distances **…??**

Charges $n_1 \sim n_5 \sim n$



Suggests that curvature length scale of the generic state is $\sim n^{\frac{1}{4}}\ l_p$