Pure Spinor Noncritical Strings in Various Dimensions

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Motivations

- Polyakov's idea of holography: noncritical backgrounds with RR flux dual to four dimensional gauge theories.
- Perturbative $\mathcal{N} = 4$ SYM is integrable. On the string side, look for integrability of pure spinor $AdS_5 \times S^5$ sigma model with RR flux at large α'. Integrability of noncritical $AdS_p \times S^q$ with RR flux?
- How do the pure spinors work in lower dimensions?
- Witten and Nekrasov suggested to apply the Cech cohomology to the pure spinor space.

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σ–model in D dimensions

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g}\, [G_{MN}(X) + B_{MN}(X)]\, \partial X^M \bar{\partial} X^N + \ldots + \alpha' r^{(2)} \Phi(X)$$

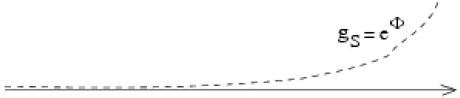
Conformal invariance requires β functions to vanish

$$\beta^{\Phi} = \frac{D-10}{\alpha'} + 4(\nabla\Phi)^2 - 4\nabla^2\Phi - R + \frac{1}{12} H_{(3)}^2 = 0$$

if $D \neq 10$ a solution is: flat metric + linear dilaton $\Phi = \frac{Q}{2}\varphi$

- cosm. const. $\sim (D-10)/\alpha'$ $\Rightarrow$ no consistent sugra approx.
- varying string coupling $g_S(\Phi) = e^{\frac{Q}{2}\varphi}$

$\varphi = -\infty$
$g_S \to 0$



$\varphi = +\infty$
g_S blows up

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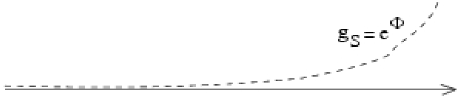

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Type II on $\mathbb{R}^d \times \mathbb{R}_\varphi \times U(1)_x$ [Seiberg,Kutasov '90]

Two ways to regulate the strong coupling region: *sine Liouville* or the *cigar*. But don't worry, we stay in the perturbative region around $\varphi = -\infty$.

$$T = -\frac{1}{2}(\partial x^\mu)^2 - \frac{1}{2}(\partial x)^2 - \frac{1}{2}(\partial\varphi)^2 + \frac{Q}{2}\partial^2\varphi + ferm. + ghosts$$

requiring $c_{tot} = 0$ fixes the slope $Q^2 = 4 - d/2$.

- w.s. $N = 2$ *superconformal algebra* $\{T, G^\pm, J\}$
- spacetime supercharges Q_α restricted to spin fields of $\mathbb{R}^d$
  1. only d–dimensional susy
  $$\{Q_\alpha, Q_\beta\} = \gamma^\mu_{\alpha\beta} P_\mu, \quad \mu = 1, \ldots, d$$
  2. Q_α depends on $x \Rightarrow$ GSO projects x on a circle with $R = 2/Q$

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Type IIB on $\mathbb{R}^d \times \mathbb{R}_\varphi \times U(1)_x$

- the full $d+2$ dim. spectrum contains: usual NSNS gravity, 'non tachyonic' NSNS tachyon, odd RR.
- no $d+2$ multiplets: but it fits into *offshell supermultiplets of susy in $SO(d)$ Lorentz*: sugra + tachyon. Different (n, w) RR modes fit into different $SO(d)$ multiplets.

$d = 0$: $\mathcal{N} = 2$ susy in $d = 0$ with $U(1)_R$
sugra $\emptyset$
tachyon $2 \oplus 2$

$d = 4$: $\mathcal{N} = 2$ susy on $SO(4)_{lorentz} \times U(1)_R$
sugra $32 \oplus 32$
tachyon $8 \oplus 8$ (vector multiplet)

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Spectrum
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Critical Superstring [Berkovits,. . .]

$$S = \int d^2z \left(\frac{1}{2} \partial X^\mu \bar{\partial} X_\mu + p_\alpha \bar{\partial} \theta^\alpha + w_\alpha \bar{\partial} \lambda^\alpha + \alpha' r^{(2)} \Omega(\lambda) \right)$$

(X^μ, θ^α) are $\mathcal{N} = 1$ $D = 10$ superspace coord.
$\lambda^\alpha \gamma^\mu_{\alpha\beta} \lambda^\beta = 0$ defines the pure spinor space

$$c_m = 10 - 32$$
$$c_{gh} = 22$$
$$c_{tot} = 0$$

- $\Omega(\lambda)$ is the *top form* on pure spinor space [Witten; Nekrasov '05]
- By using the matter variables we construct the current algebra $(d_\alpha, \Pi^\mu, \partial\theta^\alpha)$, realizing $\mathcal{N} = 1$ $D = 10$ spacetime susy

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Cohomology

The susy current algebra reads

$$d_\alpha(z)d_\beta(0) \sim -\frac{\gamma^\mu_{\alpha\beta}\Pi_\mu}{z}$$

The BRST charge $Q_B = \oint \lambda^\alpha d_\alpha$ is nilpotent on $\lambda\gamma^\mu\lambda = 0$.

- Physical states are in the cohomology at $\#_{gh} = 1$, *massless* states are *zero weight* vertex ops $\mathcal{U} = \lambda^\alpha A_\alpha(X, \theta)$

$$Q_B\mathcal{U} = 0 \quad \Rightarrow \quad (\gamma^{\mu\nu\rho\sigma\tau})^{\alpha\beta} D_\alpha A_\beta = 0$$
$$\delta\mathcal{U} = Q_B\Lambda \quad \Rightarrow \quad \delta A_\alpha = D_\alpha\Lambda$$

which are the $8 \oplus 8$ states of the onshell 10D SYM.

- The *closed string* spectrum can be computed as usual by

$$\text{closed} = \text{open} \otimes \widetilde{\text{open}}$$

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The Cech Operator

The *full* BRST charge contains the Cech operator

$$Q_B = \oint \lambda^\alpha d_\alpha - \delta_{Cech}$$

The pure spinor space $\{\lambda^\alpha : \lambda\gamma^\mu\lambda = 0\} = \prod_{\alpha=1}^{16} P_{(\alpha)}$ *locally on a patch* is described by 11 $\beta\gamma$ systems, by solving the constraint

1. the cohomology of $\oint \lambda^\alpha d_\alpha$ on a patch $P_{(\alpha)}$ gives $16 \oplus 16$ states $\Phi_{(\alpha)}$: twice as many!
2. impose the additional requirement that $\Phi_{(\alpha)}$ and $\Phi_{(\beta)}$ glue on the intersection $P_{(\alpha)} \cap P_{(\beta)}$, i.e. they are in the Cech cohomology

$$\delta_{Cech}\Phi = \Phi_{(\alpha)} - \Phi_{(\beta)} = 0$$

and kill half of the states leaving $8 \oplus 8$. [Grassi,LM,Oz]

With this Q_B the b antighost satisfies $\{Q_B, b\} = T$.

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$(x^1, \dots, x^4, \varphi, x)$ and (p_{iA}, θ_i^A), $A = 1, \dots, 4;\ i = 1, 2$ $\quad c_m = 6 - 16$

$\epsilon^{ij} \lambda_i^A \gamma_{AB}^M \lambda_j^B = 0$, pure spinors $\quad c_{gh} = 10$

$c_{tot} = 0$

Spacetime symmetries are realized as w.s. *current algebras*

- ignoring dilaton term, $(d_{iA}, \partial\theta_i^A, \Pi^M)$ form an $SO(6)$ susy algebra with 8 supercharges
- due to dilaton term, we only have 4 supercharges that close on susy along the flat $(x^1, \dots, x^4)$

$$(d_{1A}, \partial\theta_1^A, \Pi^\mu)$$

in two copies related by a $\mathbb{Z}_2$ symmetry $1 \leftrightarrow 2$.

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Spectrum

- The *BRST charge* $Q_B = \oint \lambda^A_i d_{iA} - \delta_{Cech}$ is *nilpotent* on $\lambda\Gamma^M\lambda = 0$.
- Open Q_B cohomology, mod out by $\mathbb{Z}_2$ symmetry, take $\text{closed} = \text{open} \otimes \widetilde{\text{open}}$.

Unlike $10D$, we have two vertex ops at $\#_{gh} = 1$ and weight zero

sugra

$$\mathcal{U} = \lambda^A A_A(\varphi - ix, \theta)$$

gives $4 \oplus 4$ states in one sector $\Rightarrow \mathcal{N} = 2$ $SO(4)$ offshell sugra.

tachyon

$$\mathcal{T} = e^{\frac{1}{Q}(\varphi + ix)} \lambda^A \partial\theta^B B_{AB}(\varphi - ix, \theta)$$

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The Map from RNS to Pure Spinors [Adam,Grassi,LM,Oz]

We can map the full RNS into the pure spinor theory, e.g. $d + 2 = 6$

1. Bosonize the RNS fields $\psi^\dagger \psi = \partial H$, $cb = \partial \chi$, $\gamma\beta = \partial\phi + \xi\eta$, $\ldots$
2. The pure spinor w.s. theory has twice as many θ's $\Rightarrow$ double the superspace

   4 physical q_A's
   4 unphysical $\tilde{q}_A$'s

3. Pick up four q's and define

$$\eta = q_+ e^{\tilde{\phi}+\tilde{\kappa}}, \qquad b = q_+ e^{(\tilde{\kappa}-\tilde{\phi})/2}$$

   which maps the RNS onto one patch $P_{(+)}$ of the pure spinors

$$\lambda^+ = e^{\tilde{\phi}+\tilde{\kappa}}, \quad w_+ = \partial\kappa e^{-\tilde{\phi}-\tilde{\kappa}}$$

4. Add four BRST quartets (λ^A, w_A) and $(\tilde{p}_A, \tilde{\theta}^A)$ to reconstruct the full pure spinor theory.

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4. Add four BRST quartets (λ^A, w_A) and $(\tilde{p}_A, \tilde{\theta}^A)$ to reconstruct the full pure spinor theory.

The Map from RNS to Pure Spinors [Adam,Grassi,LM,Oz]

We can map the full RNS into the pure spinor theory, e.g. $d+2=6$

1. Bosonize the RNS fields $\psi^\dagger\psi = \partial H$, $cb = \partial\chi$, $\gamma\beta = \partial\phi + \xi\eta, \ldots$
2. The pure spinor w.s. theory has twice as many θ's $\Rightarrow$ double the superspace

   4 physical q_A's
   4 unphysical $\tilde{q}_A$'s

3. Pick up four q's and define

$$\eta = q_+ e^{\tilde{\phi}+\tilde{\kappa}}, \qquad b = q_+ e^{(\tilde{\kappa}-\tilde{\phi})/2}$$

   which maps the RNS onto one patch $P_{(+)}$ of the pure spinors

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Features of the map

- Unlike pure spinor CY compactifications, the linear dilaton has a precise w.s. RNS description to match.
- For all d, the RNS stress tensor is mapped into the *pure spinor stress tensor*

$$T = -\frac{1}{2}(\partial x^M)^2 - d_A\partial\theta^A + w_A\partial\lambda^A + \frac{Q}{2}\partial^2(\varphi - ix) - \frac{1}{2}\partial^2 \log \Omega(\lambda)$$

  including the dependence on the top form $\Omega(\lambda)$
  $\Rightarrow$ the map knows about anomalies on pure spinor space.
- The same map in flat $10D$ reproduces the $\Omega(\lambda)$ term introduced by [Witten; Nekrasov '05], confirming that one must remove the singular point at the origin $\lambda^\alpha = 0\ \forall\alpha$.
- The zero modes saturation rule is reproduced by the map

$$\langle c\partial c\partial^2 ce^{-2\phi}\rangle = 1 \leftrightarrow \langle \lambda^3\theta^{d/2+1}e^{Q(\varphi - ix)}\rangle = 1$$

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Why
RNS
Pure Spinors
Future

Conclusions and Things to Do

- Use the properties of Cech cohomology (and the antighost b) to simplify multiloop superstring computations with pure spinors.
- Clarify the ten dimensional pure spinor superstring compactified on CY_n.

Pure spinors make sense in lower dimensions, at least as noncritical superstrings.

- Consider $AdS_p \times S^q$ with RR flux:
  - Gauge/string duality, e.g. Klebanov–Maldacena $AdS_5 \times S^1$ as a dual of SQCD.
  - Integrability of w.s. σ models at large α'.