Light-Cone Structure of $\mathcal{N} = 4$ Yang-Mills

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Motivations

UV Divergences of $D = 11$ Sugra

8th order $SO(9)$ Casimir Deficit Curtright

Massless Particles with Spin > 2

$N = 8$ Light-Cone Analysis

$N = 4$ Light-Cone Analysis

$\mathcal{N} = 4$ Yang-Mills in Light Cone Superspace

Constrained Chiral Superfield

(Gauge group adjoint index suppressed)

Brink et al, 1982

$$\varphi\,(y) \;=\; \frac{1}{\partial^+}\,A\,(y) \;+\; \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}_{mn}\,(y)$$

$$+\,\frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y)$$

$$+\,\frac{i}{\partial^+}\,\theta^m\,\bar{\chi}_m(y) \;+\; \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y)$$

Grassmann variables: $\theta^m \;, \bar{\theta}_m \;, m = 1, \ldots, 4$

Chiral Coordinates: $y = (\, x,\, \bar{x},\, x^+,\, x^- - \frac{i\,\theta^m\,\bar{\theta}_m}{\sqrt{2}}\,)$

Chiral Derivatives:

$$d^{\,m} = -\frac{\partial}{\partial\,\bar{\theta}_m} - \frac{i}{\sqrt{2}}\,\theta^m\,\partial^+\,; \qquad \bar{d}_{\,n} \;=\; \frac{\partial}{\partial\,\theta^n} + \frac{i}{\sqrt{2}}\,\bar{\theta}_n\,\partial^+$$

$$\{\, d^m \,,\; \bar{d}_n \,\} \;=\; -i\,\sqrt{2}\,\delta^m{}_n\,\partial^+$$

Chiral Constraints:

$$d_m\,\varphi \;=\; 0\;; \qquad \bar{d}_m\,\overline{\varphi} \;=\; 0$$

$$\bar{d}^m\,\bar{d}_n\,\varphi = \frac{1}{2}\,\epsilon_{mnpq}\,d^p\,d^q\,\bar{\varphi}\,; \qquad d^m\,d^n\,\bar{\varphi} = \frac{1}{2}\,\epsilon^{mnpq}\,\bar{d}_p\,\bar{d}_q\,\varphi$$

SuperPoincaré Algebra

Kinematical Generators at $x^+ = 0$

$$p^+ \;=\; -\,i\,\partial^+\;,\quad p\;=\;-\,i\,\partial\;,\quad \bar p\;=\;-\,i\,\bar\partial$$

$$j^+ \;=\; i\,x\,\partial^+\;,\qquad \bar j^+\;=\;i\,\bar x\,\partial^+$$
$$j^{+-}\;=\;i\,x^-\,\partial^+ - \frac{i}{2}\,(\,\theta^p\bar\partial_p + \bar\theta_p\,\partial^p\,) + i$$
$$j\;=\;x\,\bar\partial - \bar x\,\partial + \frac{1}{2}\,(\,\theta^p\,\bar\partial_p\;-\;\bar\theta_p\,\partial^p\,)\;-1$$

$$q^{\,m}_{\,+} = -\partial^m\;+\;\frac{i}{\sqrt 2}\,\theta^m\,\partial^+\;;\qquad \bar q_{\,+\,n} = \quad\bar\partial_n\;-\;\frac{i}{\sqrt 2}\,\bar\theta_n\,\partial^+$$

$$\{\,q^{\,m}_{\,+}\,,\,\bar q_{\,+\,n}\,\}\;=\;i\,\sqrt 2\,\delta^m{}_n\,\partial^+$$

(anticommute with chiral derivatives)

Dynamical Generators at $x^+ = 0$

$$p^- \;=\; -\,i\frac{\partial\bar\partial}{\partial^+}$$

$$j^- \;=\; i\,x\,\frac{\partial\bar\partial}{\partial^+} \;-\; i\,x^-\,\partial \;+\; i\left(\theta^p\bar\partial_p - \lambda - 1\right)\frac{\partial}{\partial^+}$$

$$\bar j^- \;=\; i\,\bar x\,\frac{\partial\bar\partial}{\partial^+} \;-\; i\,x^-\,\bar\partial \;+\; i\left(\bar\theta_p\partial^p + \lambda - 1\right)\frac{\bar\partial}{\partial^+}$$

$$q^m_- \;=\; \frac{\bar\partial}{\partial^+}\,q^{\,m}_{\,+}\;, \qquad \bar q_{\,-\,m} \;=\; \frac{\partial}{\partial^+}\,\bar q_{\,+\,m}$$

(boosted kinematical supersymmetries)

$$\{\,q^m_-\,,\,\bar q_{\,-\,n}\,\} \;=\; \sqrt{2}\,\delta^{\,m}{}_{n}\,p^-$$

SuperConformal Algebra

Kinematical Generators ($x^+ = 0$)

$$K^+ \;=\; 2i\,x\,\bar{x}\,\partial^+$$

$$D \;=\; i\left(x^-\partial^+ - \,x\bar{\partial}\, - \,\bar{x}\partial - \frac{1}{2}\theta\frac{\partial}{\partial\theta} \,-\, \frac{1}{2}\bar{\theta}\frac{\partial}{\partial\bar{\theta}}\right)$$

$$K \quad = \quad 2ix\left(x^-\partial^+ - x\bar{\partial} - \theta\frac{\partial}{\partial\,\theta} + 1\right)$$

$$\overline{K} \quad = \quad 2i\bar{x}\left(x^-\partial^+ - \bar{x}\partial - \bar{\theta}\frac{\partial}{\partial\bar{\theta}} - 1\right)$$

$$s^m_+ \;=\; i\sqrt{2}\,\bar{x}\,q^m_+ \qquad \bar{s}_{+\,n} \;=\; -\,i\sqrt{2}\,x\,\bar{q}_{+\,n}$$

Dynamical Generators ($x^+ = 0$)

$$K^- \;=\; 2\,i\left(x^-\,\partial^+ \,-\, \bar{x}\,\partial \,-\, \bar{\theta}\frac{\partial}{\partial\,\bar{\theta}}\right)$$

$$\times\left(x^-\,\partial^+ \,-\, x\,\bar{\partial} \,-\, \theta\frac{\partial}{\partial\,\theta} \,+2\right)\frac{1}{\partial^+}$$

$$s_-^m \;=\; i\,\sqrt{2}\left(x^-\,\partial^+ \,-\, x\,\bar{\partial} \,-\, \theta\,\frac{\partial}{\partial\,\theta} \,+2\right)\frac{1}{\partial^+}\,q_+^m$$

$$\bar{s}_{-\,n} \;=\; -\,i\,\sqrt{2}\left(x^-\,\partial^+ \,-\, \bar{x}\,\partial \,-\, \bar{\theta}\,\frac{\partial}{\partial\,\bar{\theta}}\right)\frac{1}{\partial^+}\,\bar{q}_{+\,n}$$

$$\{ q_+^m \, , \, \bar{s}_{-\,n} \} \; = -\delta^m_n(i\,D + ij^{+-} - j\,) \; + \; 2\,J^m{}_n$$

$SU(4)$ Generators appear

$$J^m{}_n \; = \; q_+^m \, \bar{q}_{+\,n} \frac{1}{\partial^+}$$

$$PSU(2,2\,|\,4)$$

$$PSU(2,2\,|\,4) \supset \; SO(4,2) \, \times \, SU(4) \; \sim \; SO(4,2) \, \times \, SO(6)$$

Free Theory: φ spans a **linear** representation

$$\delta_j \, \varphi \; \sim \; j \, \varphi \; , \qquad \delta_{p^-} \, \varphi \; \sim \; p^- \, \varphi \; , \quad etc...$$

Interacting $\mathcal{N} = 4$ Yang-Mills

Non-Linear $PSU(2,2\,|\,4)$:

Unchanged Kinematical Generators

$$\delta_{\mathrm{Kin}}\,\varphi \;=\; \delta^{\mathrm{free}}_{\mathrm{Kin}}\,\varphi$$

Non-Linear Dynamical Generators

$$\delta_{\mathrm{Dyn}}\,\varphi \;=\; \delta^{\mathrm{free}}_{\mathrm{Dyn}}\,\varphi \;+\; \delta^{g}_{\mathrm{Dyn}}\,\varphi \;+\; \delta^{g^2}_{\mathrm{Dyn}}\,\varphi \;+\; \cdots$$

Solve Commutation Relations

order by order in g

$$[\,\delta_{\rm Kin}\,,\delta_{\rm Kin}\,]\,\varphi \;=\; \delta_{\rm Kin}\,\varphi$$

$$[\,\delta_{\rm Kin}\,,\delta_{\rm Dyn}\,]\,\varphi \;=\; \delta_{\rm Kin}\,\varphi + \;\delta_{\rm Dyn}\,\varphi$$

$$[\,\delta_{\rm Dyn}\,,\delta_{\rm Dyn}\,]\,\varphi \;=\; \delta_{\rm Kin}\,\varphi \;+\; \delta_{\rm Dyn}\,\varphi$$

All SuperConformal Dynamics

fixed by

Supersymmetry!

$$\{\delta_{q_-}, \delta_{\bar{q}_-}\}\varphi \sim \delta_{p^-}\varphi$$

$$[\delta_{p^-}, \delta_K] \sim \delta_{j^-}\varphi$$

$$[\delta_{j^-}, \delta_K] \sim \delta_{K^-}\varphi$$

Exact Dynamical Supersymmetry

$$\delta_{\bar{q}_-}\,\varphi^a \;=\; \frac{\partial}{\partial^+}\,\bar{q}_{\,+\,m}\,\varphi^a - g\,f^{abc}\,\frac{1}{\partial^+}\left\{\,\frac{\partial}{\partial\theta}\,\varphi^b\,\partial^+\,\varphi^c\right\}$$

no order g^2 contributions!

UNIQUELY DETERMINED

from commutation relations AND chirality

Light-Cone Hamiltonian DERIVED

$$\delta^g_{p^-}\,\varphi^a \;=\; -i\frac{\partial\bar{\partial}}{\partial^+}\,\varphi^a - i\,g\,f^{abc}\,\frac{1}{\partial^+}\,(\,\bar{\partial}\,\phi^b\,\partial^+\,\phi^c) + \mathcal{O}(g^2) + \text{conj.}$$

$[\,\delta_{p^-}\,,\,\delta_{j^-}\,] \;=\; 0 \rightarrow$ Jacobi identities on f^{abc}

$\mathcal{N} = 4$ Light Cone Hamiltonian is a Quadratic Form!

Inner Product:

$$(\,\varphi\,,\,\varphi'\,)\;\equiv\;\frac{i}{\sqrt{2}}\int d^4\,x\,d^4\,\theta\,d^4\,\bar{\theta}\;\overline{\varphi}\,\frac{1}{\partial^+}\varphi'$$

$$H\;=\;(\,\mathcal{W}^a\,,\,\mathcal{W}^a\,)$$

$$\begin{aligned}\mathcal{W}^a &= \frac{\delta_{\bar{q}_-}\varphi^a}{\delta\,\omega} = \frac{\partial}{\partial^+}\bar{q}_+\varphi^a - g f^{abc}\frac{1}{\partial^+}\Big(\frac{\partial}{\partial\theta}\varphi^b\partial^+\varphi^c\Big)\\ &= \frac{1}{\partial^+}\mathcal{D}^{ab}\bar{q}_+\varphi^b\end{aligned}$$

$$\mathcal{D}^{ab}\;=\;\partial\,\delta^{ab} - g f^{abc}\partial^+\varphi^c$$

Vacuum Configurations $\leftrightarrow$ $\mathcal{W}^a = 0$

$$\begin{aligned}
D^{ab}\frac{1}{\partial^+}\bar{\chi}^b_m &= 0 \\
D^{ab}\bar{C}^b_{lm} &= -\frac{i}{\sqrt{2}}\,g\,f^{abc}\frac{1}{\partial^+}\bar{\chi}^b_l\bar{\chi}^c_m \\
\epsilon_{lmpq}D^{ab}\chi^{qb} &= -\,g\,f^{abc}\,\frac{1}{\partial^+}\bar{\chi}^b_l\partial^+\bar{C}^c_{mp} \,-\, 2\,g\,f^{abc}\,\bar{C}^b_{lm}\bar{\chi}^c_p \\
\epsilon_{lmpq}D^{ab}\partial^+\bar{A}^b &= 3\,g\,f^{abc}\,\partial^+\bar{C}^b_{mp}\bar{C}^c_{lq} \,-\, \frac{3}{\sqrt{2}}\,i\,g\,f^{abc}\,\epsilon_{lmpr}\chi^{rb}\bar{\chi}^c_q \\
&\quad + \frac{i}{\sqrt{2}}\,g\,f^{abc}\,\epsilon_{mpqr}\,\partial^+\chi^{rc}\frac{1}{\partial^+}\bar{\chi}^b_l
\end{aligned}$$

$$D^{ab} \equiv \partial\delta^{ab} - g f^{abc}A^c$$

Constraint

$$f^{abc}\left[\,\partial^+\bar{A}^b\frac{1}{\partial^+}\bar{\chi}^c_m + \bar{C}^b_{mn}\chi^{nc}\,\right] = 0$$

All Interactions mediated by (S)purion:

$$\begin{aligned}\Psi^a &\equiv gf^{abc}\frac{1}{\partial^+}(\frac{\partial}{\partial\theta^m}\varphi^b\partial^+\varphi^c) \\ &\sim gf^{abc}\epsilon_{mnpq}\theta^n\theta^p\theta^q\,\partial^+\overline{A}^b\,A^c \;+\;\cdots\end{aligned}$$

Spurion links gluons of *opposite* helicity

N-gluon Tree Amplitudes vanish if all gluon helicities are the same (or all but one)

QUANTUM CORRECTIONS

$$\delta_{\bar{q}_-}^{Classical}\,\varphi^a \;=\; \frac{\partial}{\partial^+}\,\bar{q}_+\,\varphi^a - g\,f^{abc}\,\frac{1}{\partial^+}\left\{\frac{\partial}{\partial\theta}\,\varphi^b\,\partial^+\,\varphi^c\right\}$$

$$H^{Classical} \;=\; \left(\delta_{\bar{q}_-}^{Classical}\,\varphi\,,\,\delta_{\bar{q}_-}^{Classical}\,\varphi\right)$$

$$\delta_{\bar{q}_-}^{Quantum}\,\varphi \;=\; \delta_{\bar{q}_-}^{Classical}\,\varphi \;+\; \sum_{L=1}^{\infty}\hbar^L\,\sum_{M=1}^{\infty}\mathcal{W}_L^{[M]}$$

$$\mathcal{W}^{[M]} \sim \varphi^M$$

HYPOTHESIS

$$H^{Quantum} \;=\; \left(\delta_{\bar{q}_-}^{Quantum}\,\varphi\;,\,\delta_{\bar{q}_-}^{Quantum}\,\varphi\right)\;?$$

$$\mathcal{W}_L^{[M]} \;\equiv\; \delta_{\bar{q}_-}^{[M,L]}\,\varphi$$

Dimensions, Helicity:

$$\delta_{\bar{q}_-}^{[M,L]}\,\varphi \sim \frac{(\hbar g^2)^L}{g}\,\frac{\partial}{\partial\theta}\,\left(\frac{\partial}{\partial^+}\right)^{2+A-B}\left(\frac{\bar{\partial}}{\partial^+}\right)^{A}\,(g\varphi)^{\frac{M+B}{2}}\,(g\overline{\varphi})^{\frac{M-B}{2}}$$

Chirality:

$$d_m\,\delta_{\bar{q}_-}^{[M,L]}\,\varphi \;=\; 0$$

Commutation relation with j^+ , $\bar{j}^+$:

$$[\,\delta_{j^+}\,,\,\delta_{\bar{q}_-}^{[M,L]}\,]\,\varphi \;=\; [\,\delta_{\bar{j}^+}\,,\,\delta_{\bar{q}_-}^{[M,L]}\,]\,\varphi \;=\; 0$$

Terms linear and quadratic in φ:

$$\mathcal{W}_L^{[1]\,a} \;=\; \sum_{L=1} a_L (\hbar g^2)^L \frac{\partial}{\partial^+}\,\bar{q}_+\,\varphi^a$$

$$\mathcal{W}_L^{[2]\,a} \;=\; \sum_{L=1}^{\infty} b_L (\hbar g^2)^L g\, f^{abc}\,\frac{1}{\partial^+}\left\{\frac{\partial}{\partial\theta}\,\varphi^b\,\partial^+\,\varphi^c\right\}$$

Same as Classical!

On-shell two and three point functions unaltered by loops

Green, Schwarz, Brink

Consistent with quadratic hypothesis

Superspace Diagrams in LC_2 Gauge

Lee and Milgram

Gluon contribution

$$p^2\delta^{ij}\left[(\frac{11}{3}-4z)\frac{1}{e_1}-\frac{70}{9}-4S_1(z)-4z\ln(z)\right]+\frac{2p_ip_j}{3(z-1)}$$

$$\frac{1}{e_1} = \frac{1}{\epsilon}+\ln(\pi p^2)+\gamma-i\pi \; ; \qquad z = \frac{2p^+p^-}{p^2}$$

$$S_1(z) = \ln(z)\,\zeta_1(z)-\zeta_2(z) \; ; \qquad \zeta_n(z) = \sum_{l=1}^{\infty}\frac{z^l}{l^n}$$

$N = 4$ Two-Point function

S.-S. Kim

$$-4\frac{g^2}{16\pi^2}\,p^2\delta^{ij}\,S_1(z)$$

Three-point LC_2 function (Work in Progress)

Determine $\mathcal{W}^{[3]}_{L=1}$

Extra Machinery needed:

$$T(z,\bar{z}) \;=\; \exp\left[\frac{\partial}{\partial^+}\bar{z} + \frac{\bar{\partial}}{\partial^+}z\right]$$

Commutation relation with Boosts j^-:

$$[\,\delta_{j^-}\,,\,\delta^{[3,1]}_{\bar{q}_-}\,]\,\varphi \;=\; [\,\delta_{\bar{j}^-}\,,\,\delta^{[3,1]}_{\bar{q}_-}\,]\,\varphi \;=\; 0$$

Chiral Constructs

$$(\overline{d}\,A \star B) \;\equiv\; (\overline{d}A)(\partial^+ B) - (\partial^+ A)(\overline{d}B)$$

$$(\overline{d}\,A \star \overline{B}) \;\equiv\; A\,\overline{B} - \overline{d}(A d\,\overline{B}) + \cdots$$

$$d\,(\overline{d}\,A \star B) \;=\; 0\,; \qquad d\,(\overline{d}\,A \star \overline{B}) \;=\; 0$$

Sample Term

$$\mathcal{W}_L^{[3]} \;=\; \frac{f^{abc}}{\partial^{+\,(l-m)}}\left(\frac{1}{\partial^{+\,m}}T^{-1}(z,\;\bar{z})\Psi^{b\,[k]}\partial^{+l}T(z,\;\bar{z})\varphi^c\right)_{z^p\bar{z}^{p+1}}$$

$$\Psi^{a\,[k]} \;=\; \frac{f^{abc}}{\partial^{+\,(2k+1)}}\left(\overline{d}\partial^{+k}\varphi^b\partial^{+(k+1)}\varphi^c\right)$$

Four-gluon one-loop amplitude

Green, Schwarz, Brink

On-Shell Algebra; IR divergences?

Compare finite parts?

Calculate in LC_2 gauge

À SUIVRE...