Half K3 Surfaces

or

K3, G2, E8, M, and all that

(work in progress)

Strings 2002

David R. Morrison, Duke University

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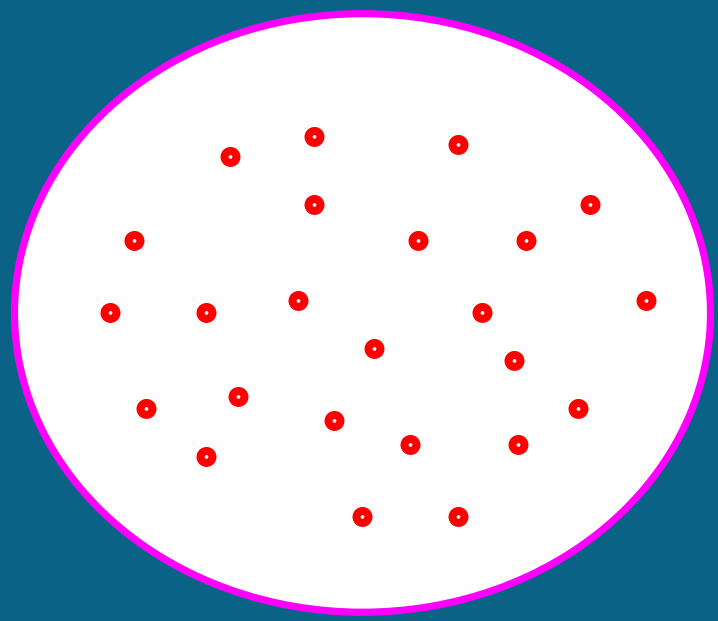
Thanks to: P. Aspinwall, M. Atiyah, E. Witten.

Duality in dimension eight



- F-theory in dimension 8: type IIB string compactified on S^2 with 24 D7-branes at points of S^2

- data: elliptically fibered K3 surface with 24 singular fibers (generically)

$$y^2 = x^3 + f_8(z)x + g_{12}(z)$$

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- singular fibers at $\Delta(z) = 4f(z)^3 + 27g(z)^2$

- Can take a limit[1] in which the S^2 stretches into a long tube with 12 of the D7-branes at one end and 12 at the other end.

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- j-invariant of the F-theory data is nearly constant in the middle of the tube

[1] Morrison–Vafa, `arXiv:hep-th/9603161`

- that constant value determines the T^2 metric for a heterotic dual description

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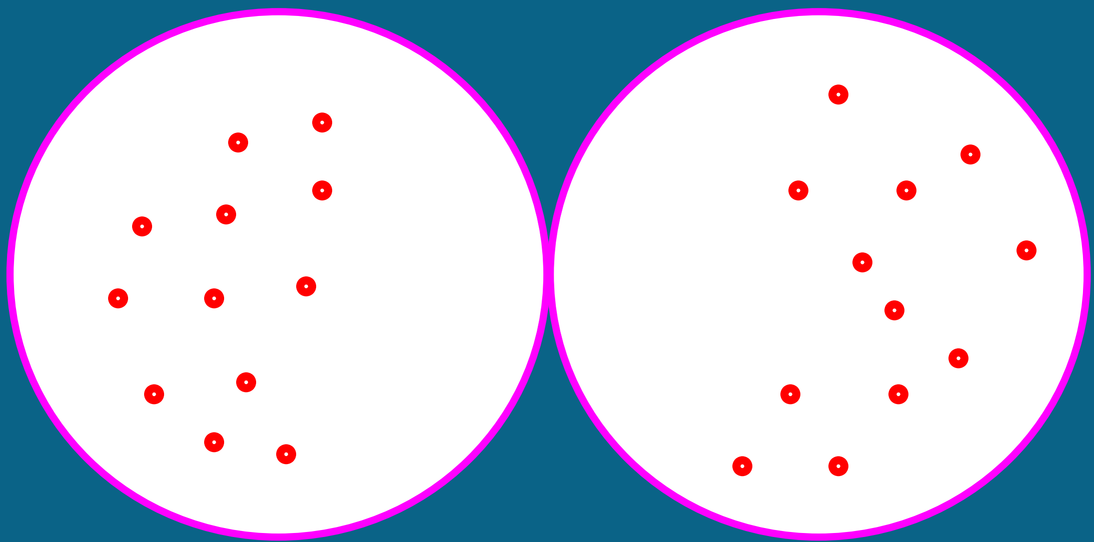
- each set of 12 points determines an E_8 gauge field configuration, e.g., via perturbations of the E_8 singularity

$$y^2 = x^3 + z^5$$

- More precise version:[2] by rescaling (e.g. $x \mapsto x/z^2$, $y \mapsto y/z^3$) get a limit in which the original S^2 splits into two S^2's meeting at a point



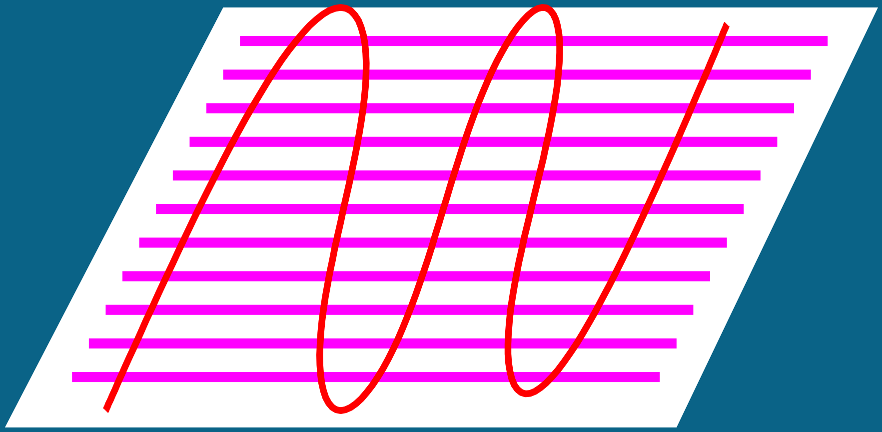
- Over each S^2 is a *rational elliptic surface with section.* (These are NOT the half K3 surfaces of the title.)

[2] Friedman–Morgan–Witten, `arXiv:hep-th/9701162`

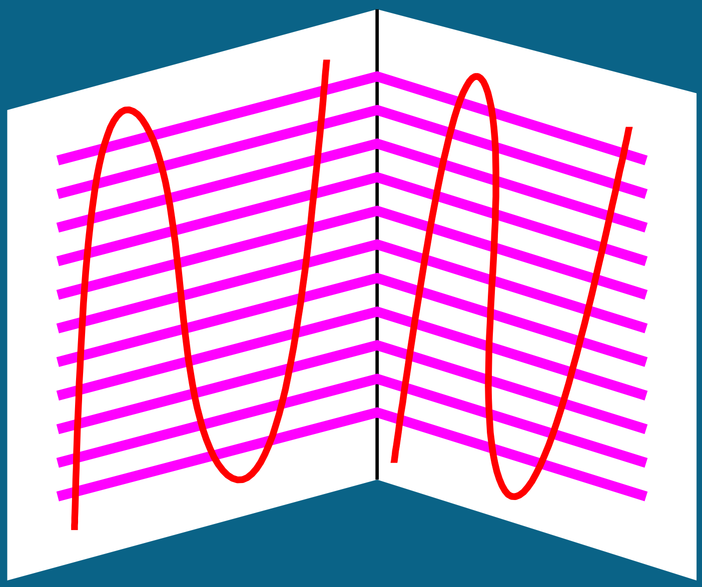
- Can blow down the section to give a del Pezzo surface of degree 1
- Looijenga showed that such surfaces are in one-to-one correspondance with E_8 gauge theory on a (fixed) 2-torus
- The degeneration corresponds to weak heterotic coupling

Heterotic string on elliptic fibrations

- If X has an elliptic fibration, the heterotic string on X should have an F-theory dual, obtained fibrewise
- Base of F-theory has a family of S^2's:

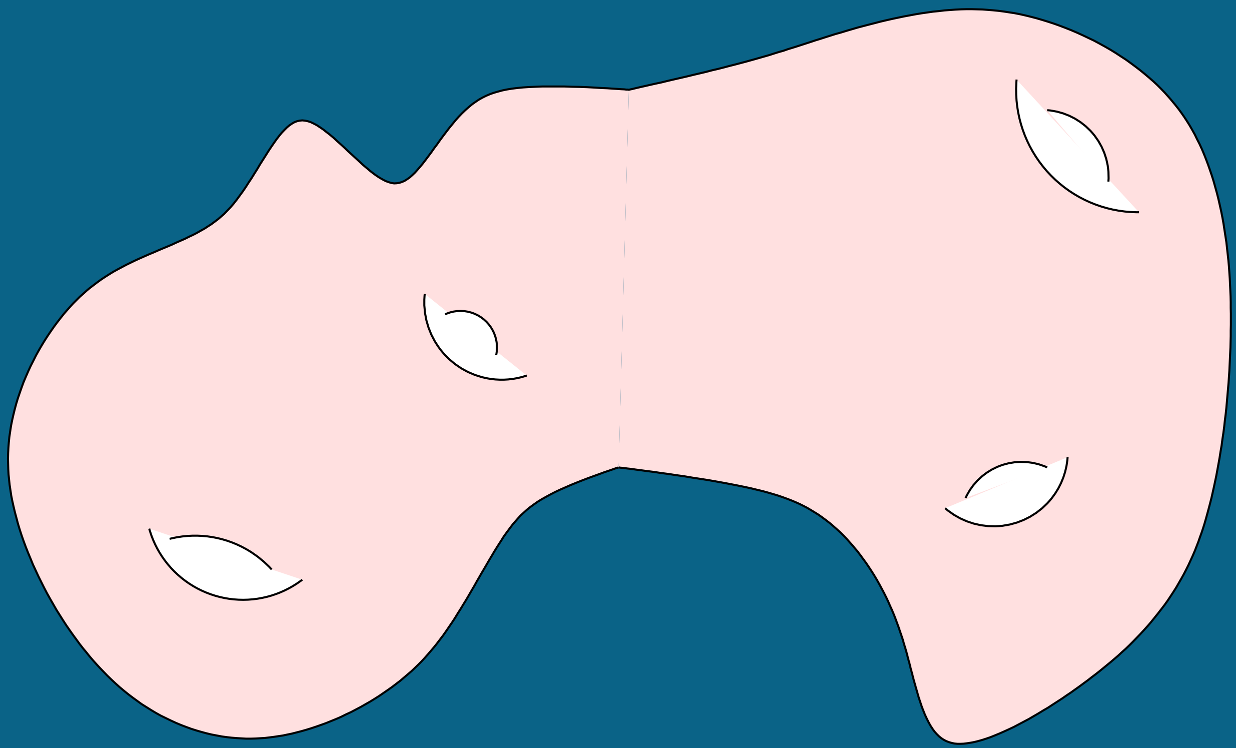


- weak heterotic coupling limit is dual to a degeneration



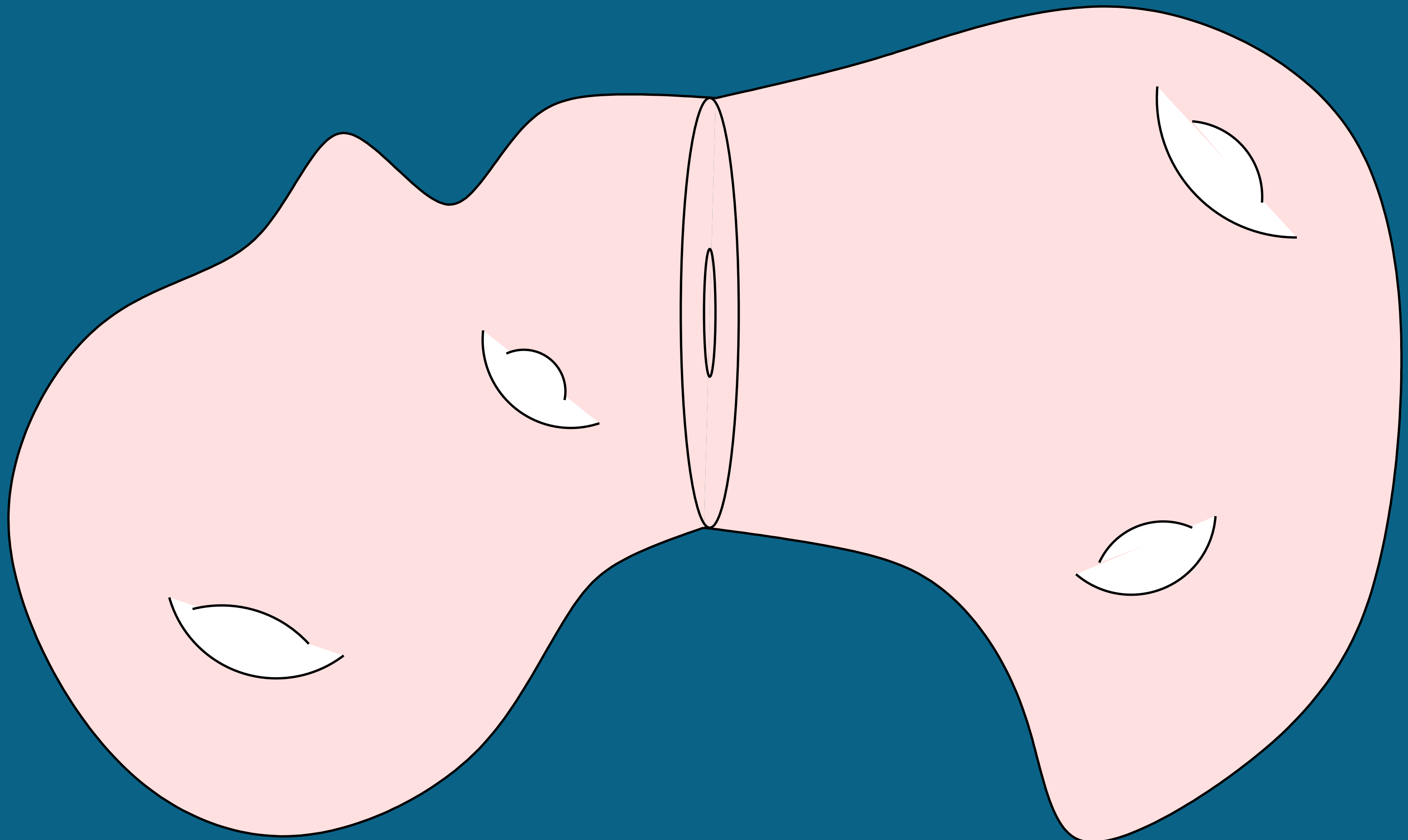
- precise correspondance between data; can be used to investigate many interesting phenomena on both sides
- X re-emerges as the intersection of the two elliptic fibrations in the limit

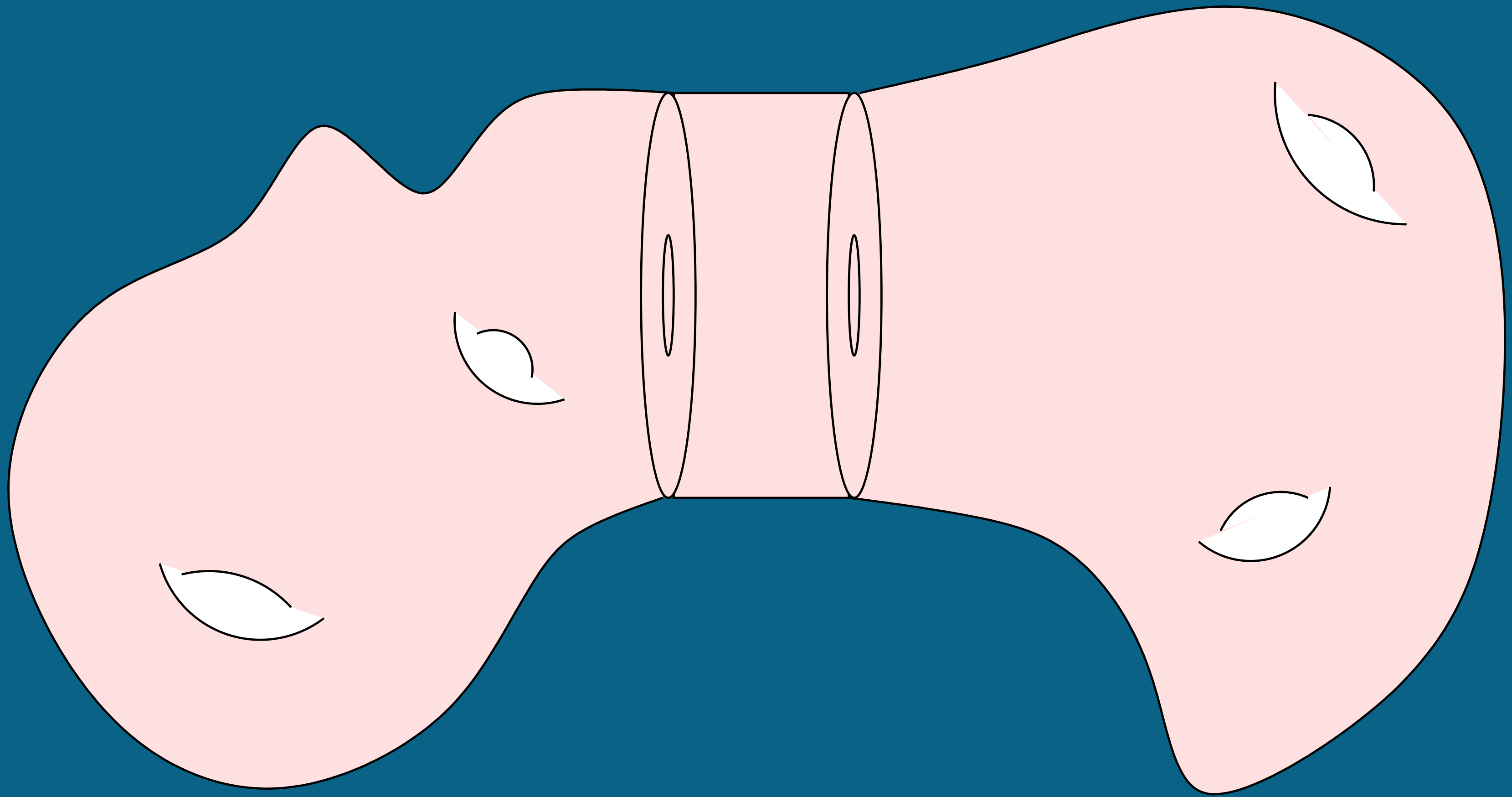
Duality in dimension seven

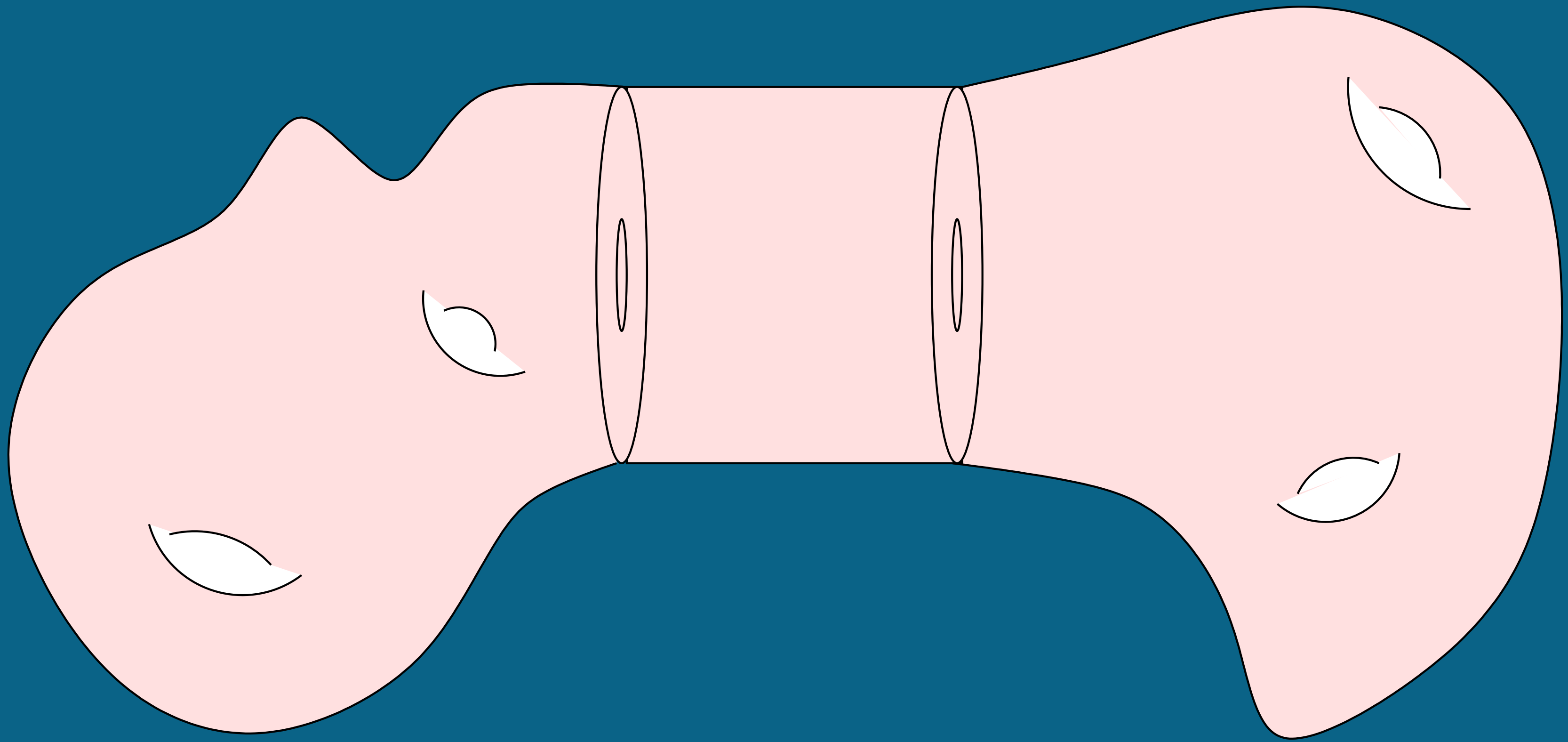


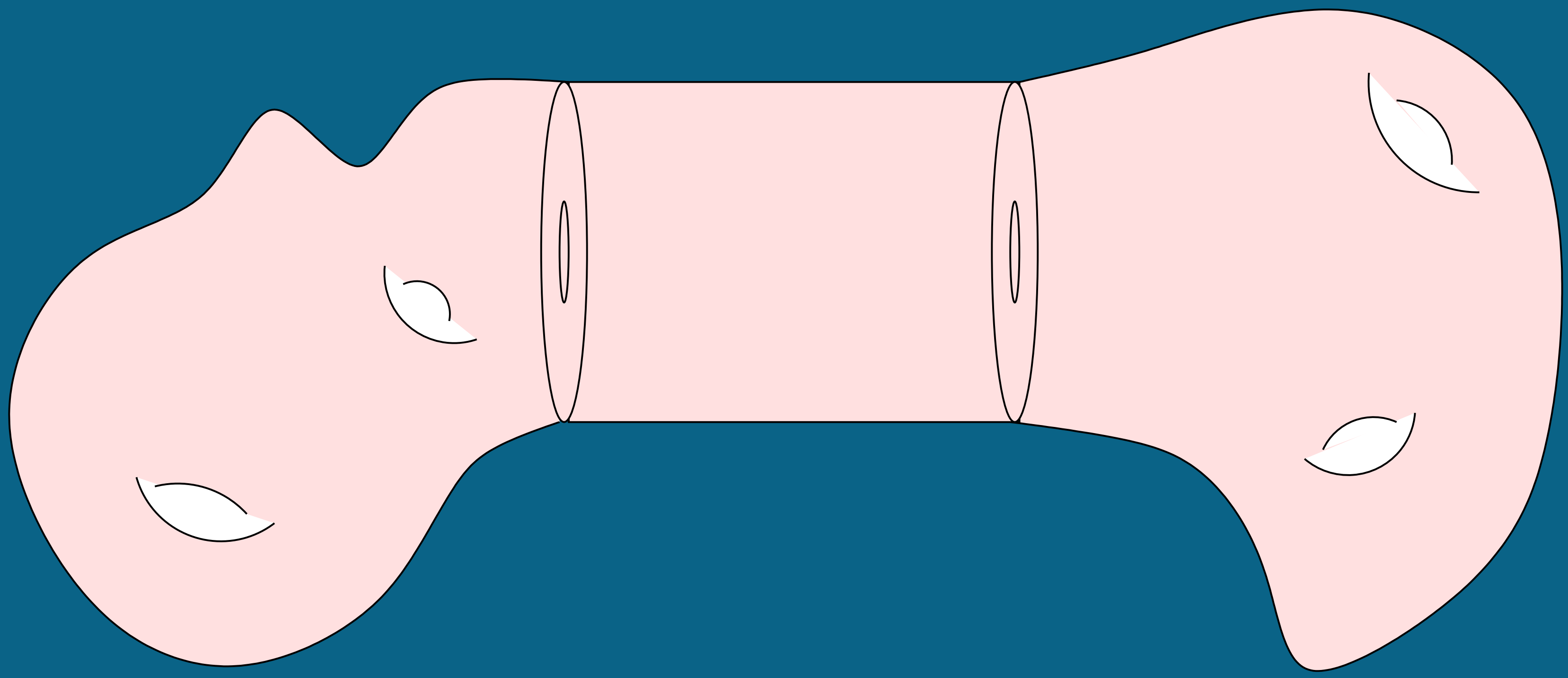
- We wish to find a similar picture in dimension 7, starting from M-theory on a K3 surface

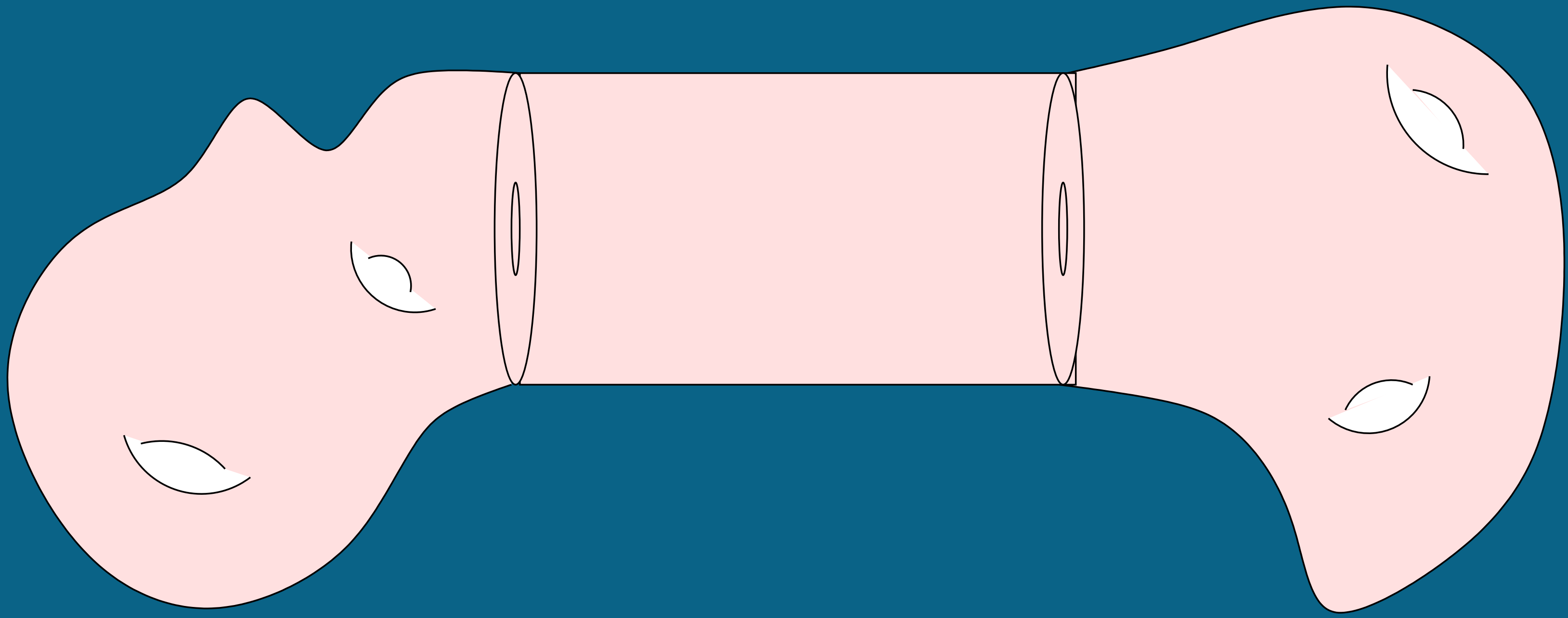
- There are limits in the space of metrics on $K3$, and presumably even in the space of Ricci-flat metrics, with the following behavior:

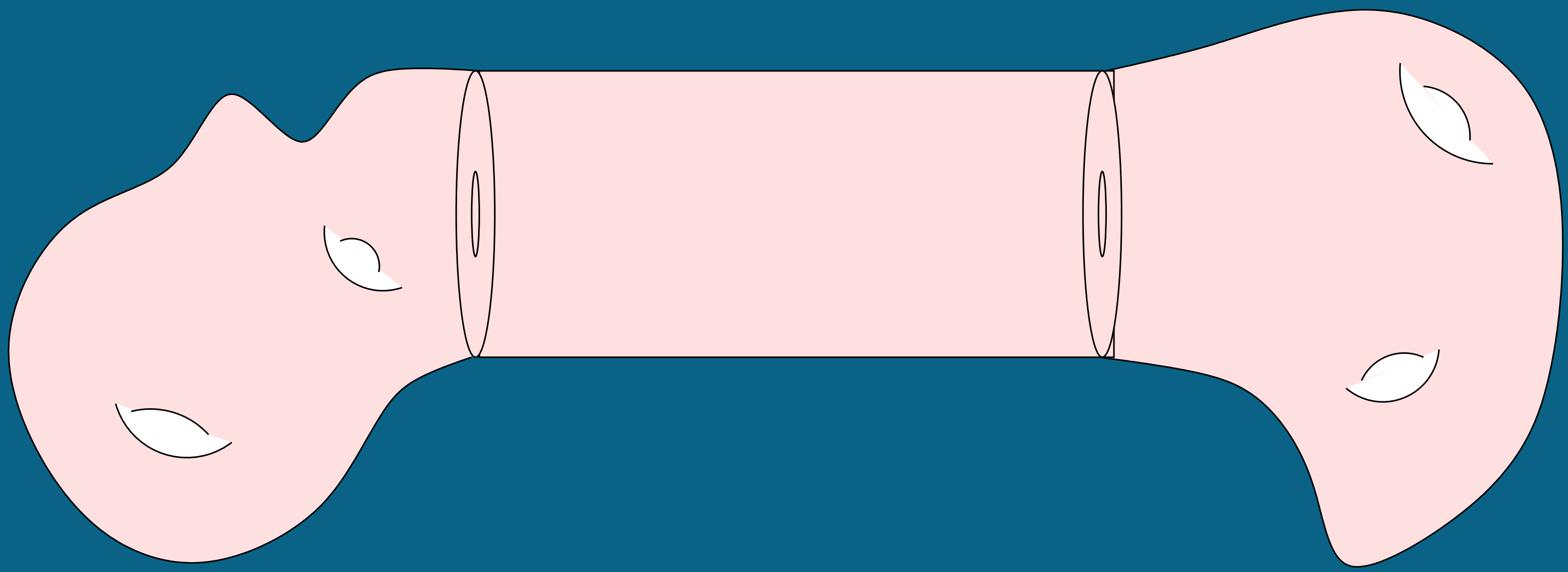


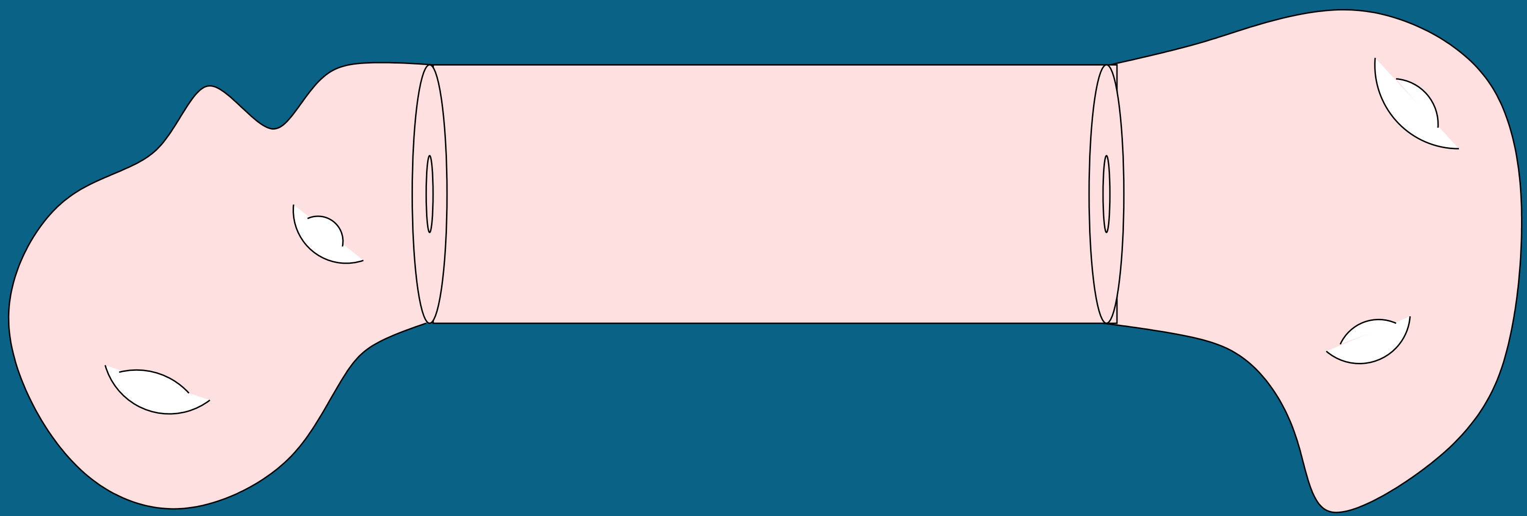


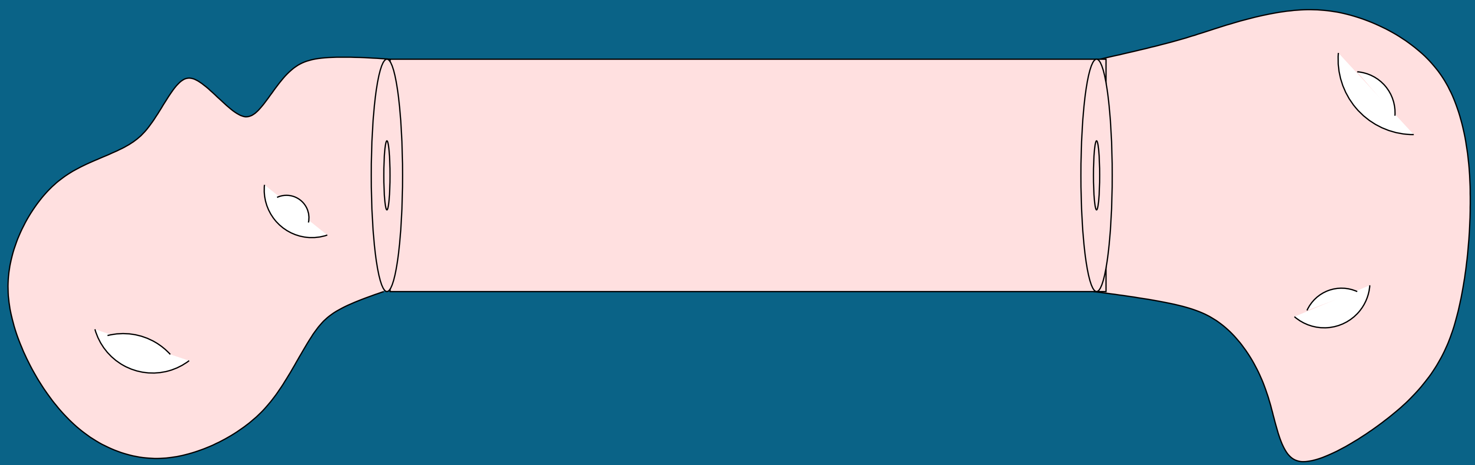


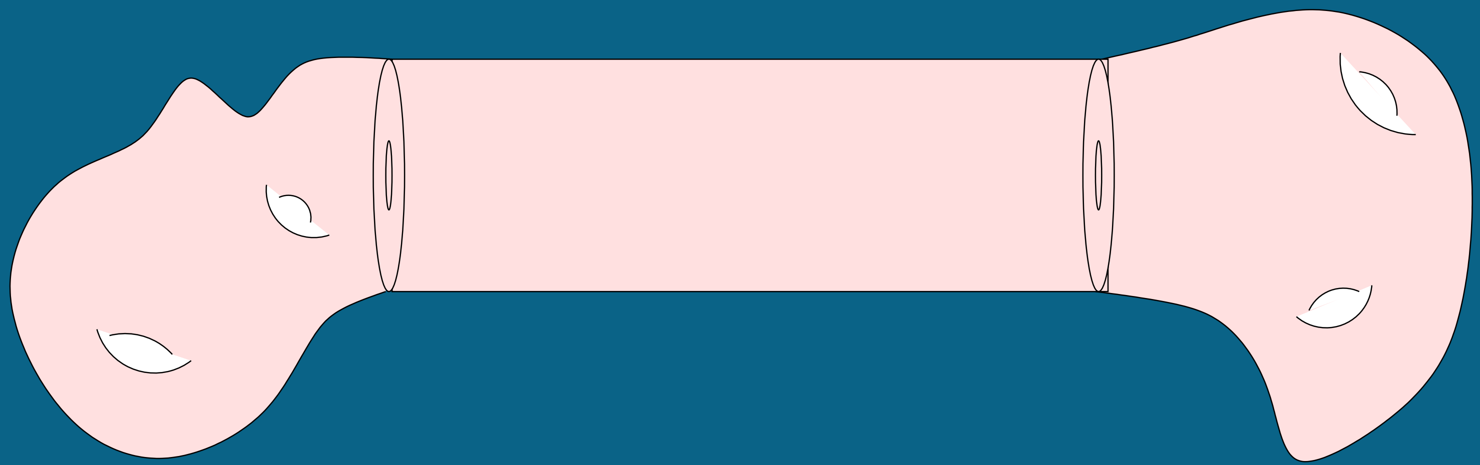












- A very long throat of the form $T^3 \times [0, R]$ has opened up in the middle of the $K3$ surface.

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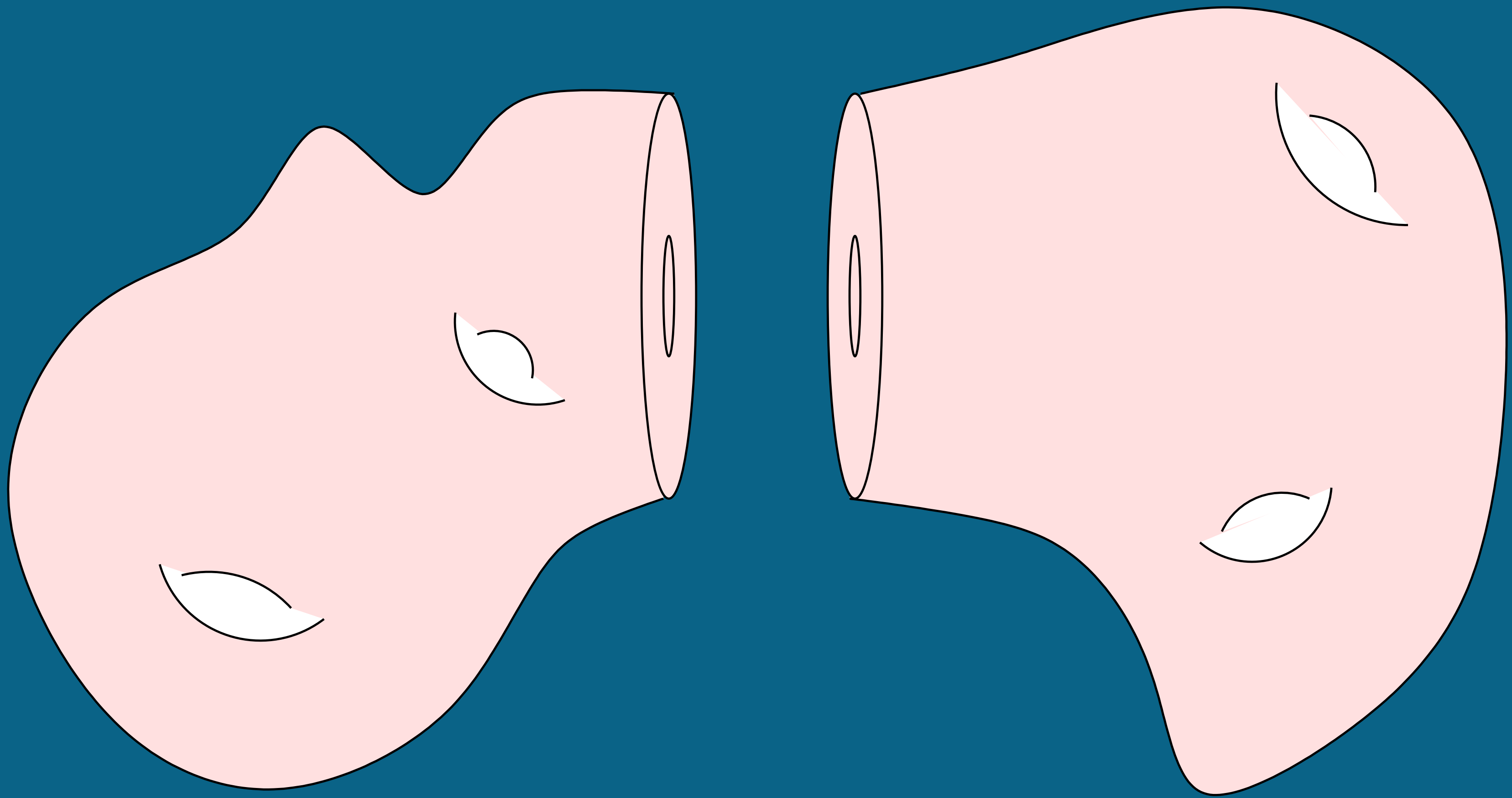
- An observer in the middle sees the two complicated ends recede; they are the **half K3 surfaces**

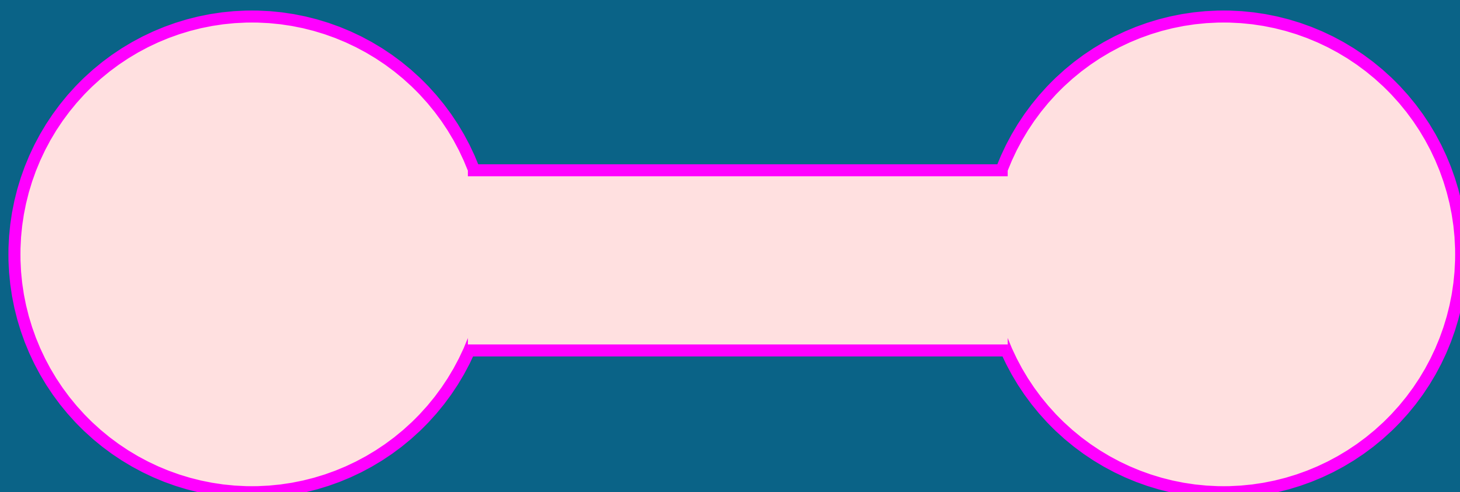
- we will see presently that each half K3 surface accounts for the data of an E_8 gauge field on T^3



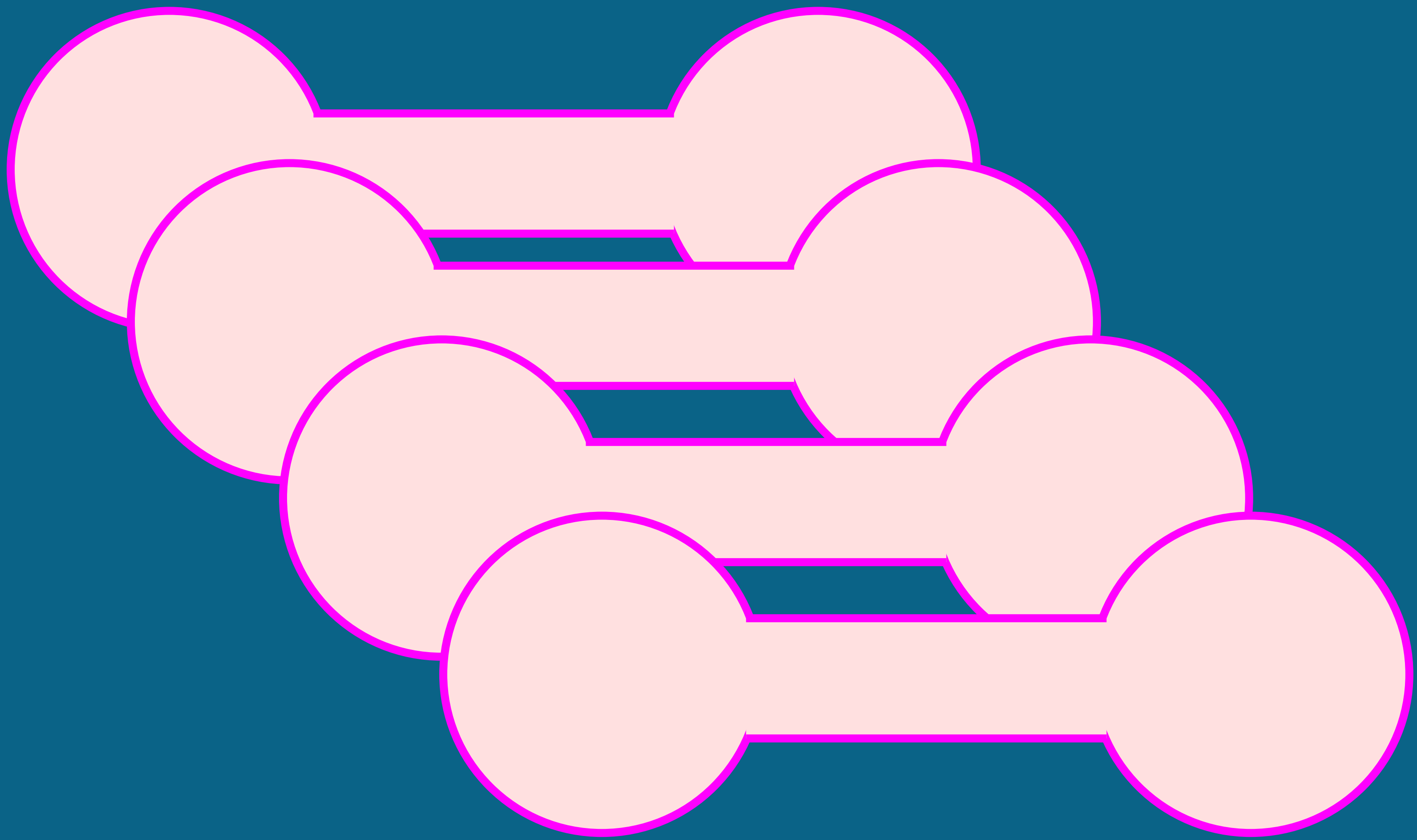
- Explicit realization of M/heterotic duality in dimension 7. Similar to Hořava–Witten but with a more geometric interpretation for the gauge fields at the ends.
- Again corresponds to weak heterotic coupling

Application: 4-dimensional duality

- As in the previous case, we can put this into families. To illustrate this, we simplify our stretched K3 to a cartoon:



- Now we can put a family of these together:



- The family of T^3's emerges as the common boundary

along which the families of half K3's meet.

- Can be applied to heterotic string on a Calabi–Yau 3-fold, with its supersymmetric T^3-fibration[3].
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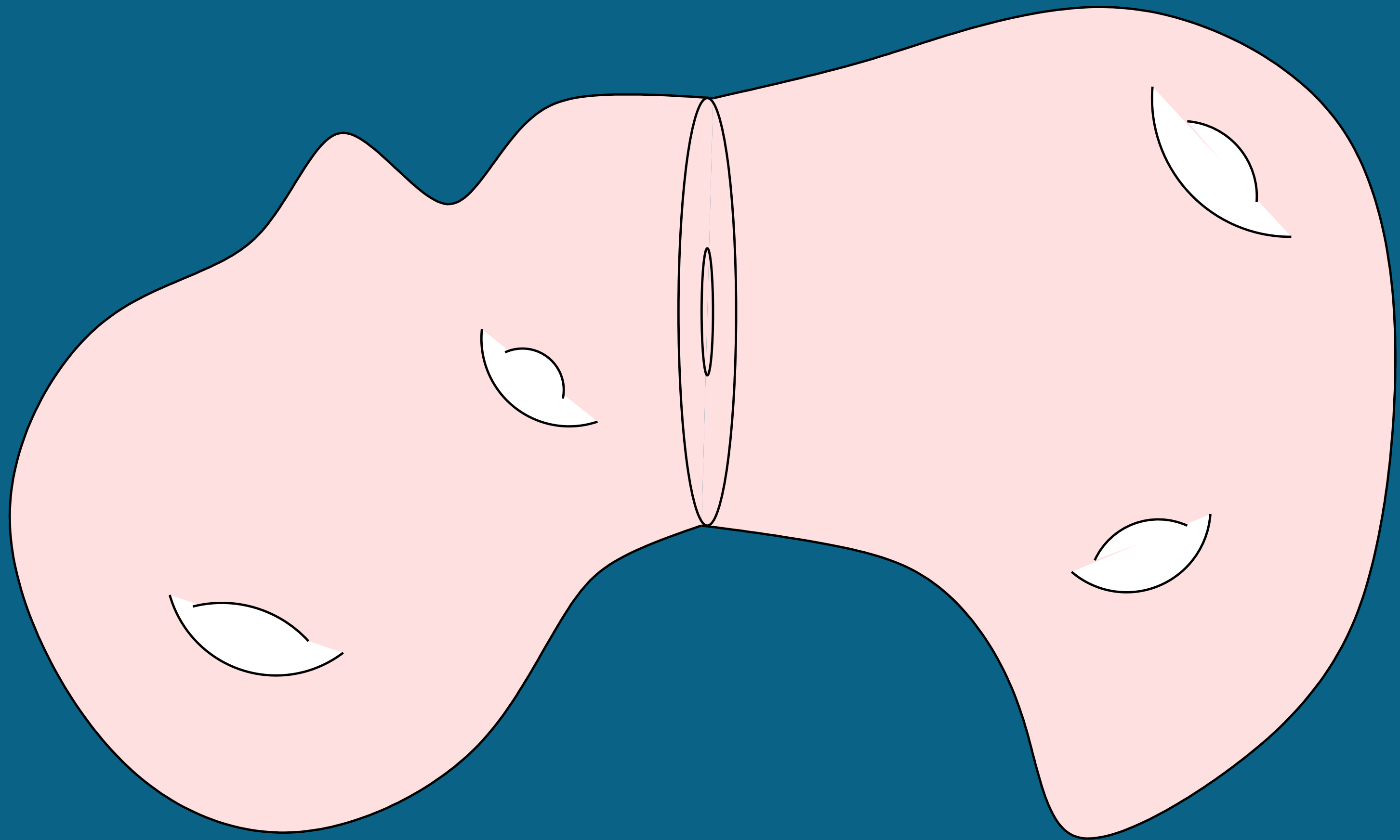
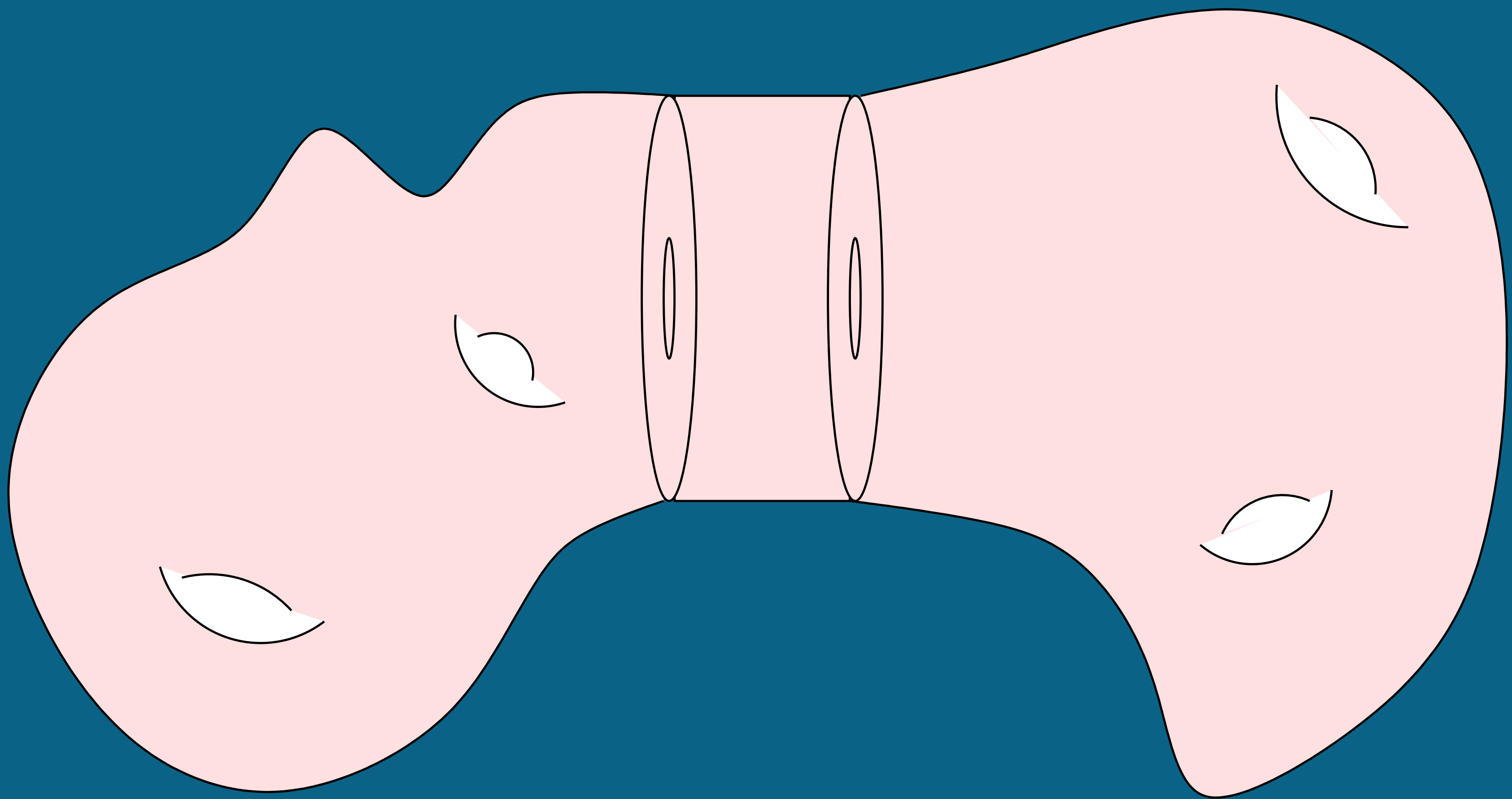
  - ⋆ Fibrewise duality gives M-theory on a family of $K3$ surfaces over S^3
  - ⋆ i.e., a fibered G_2 manifold
  - ⋆ weak heterotic coupling leads to a stretching limit
  - ⋆ the half-G_2's are 7-manifolds with boundary, whose common boundary is the Calabi–Yau 3-fold[4]

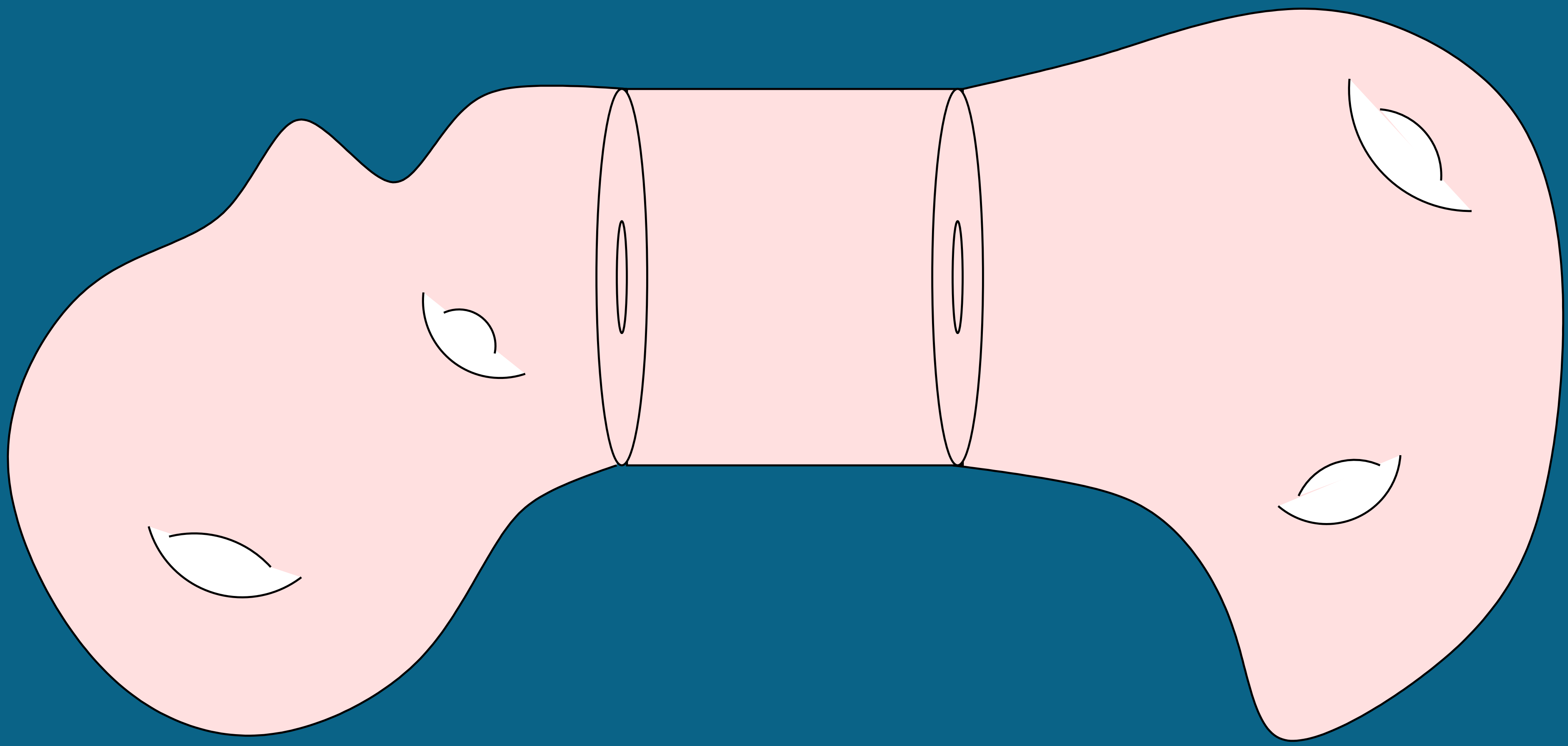
[3]Strominger–Yau–Zaslow
[4]cf. Donaldson–Thomas

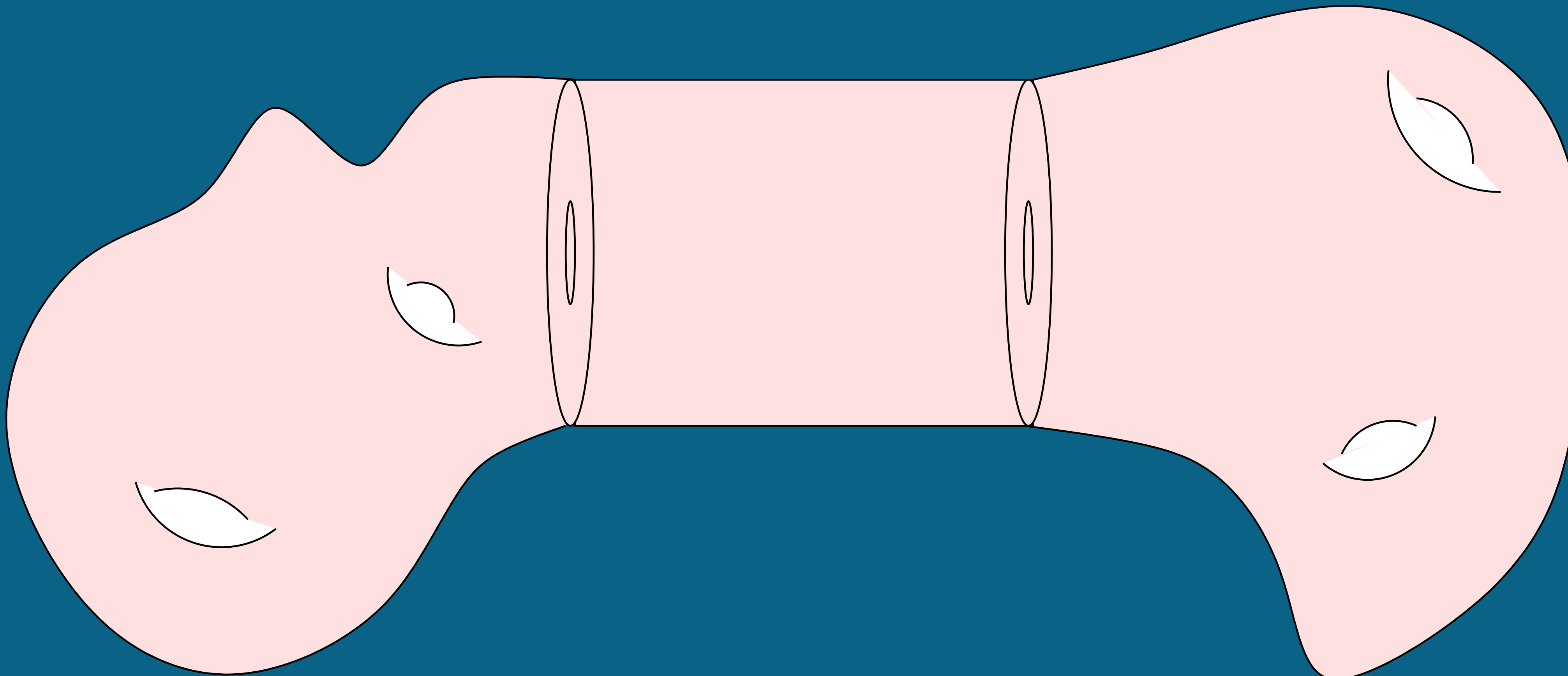
The half K3 limit

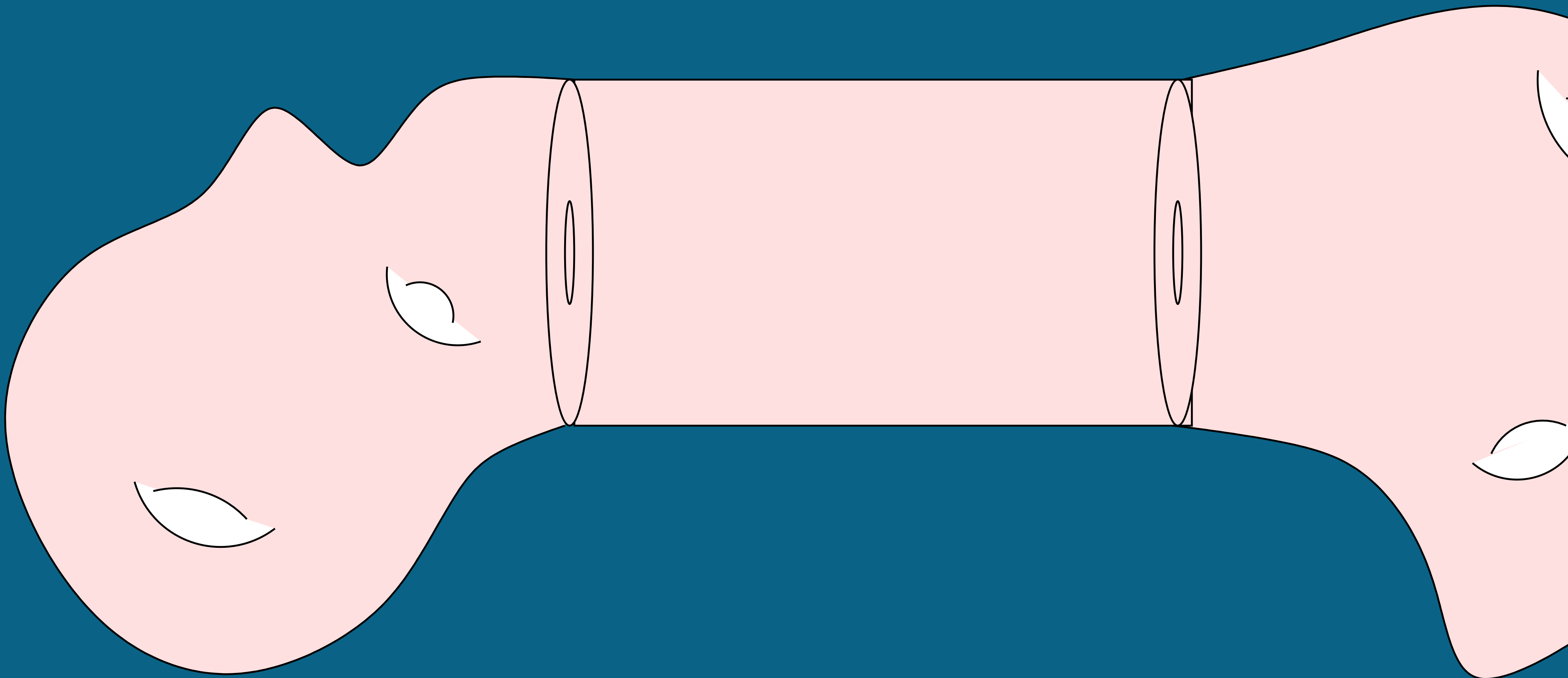
- An observer who stays well within one half of the K3 surface during scaling sees something different:

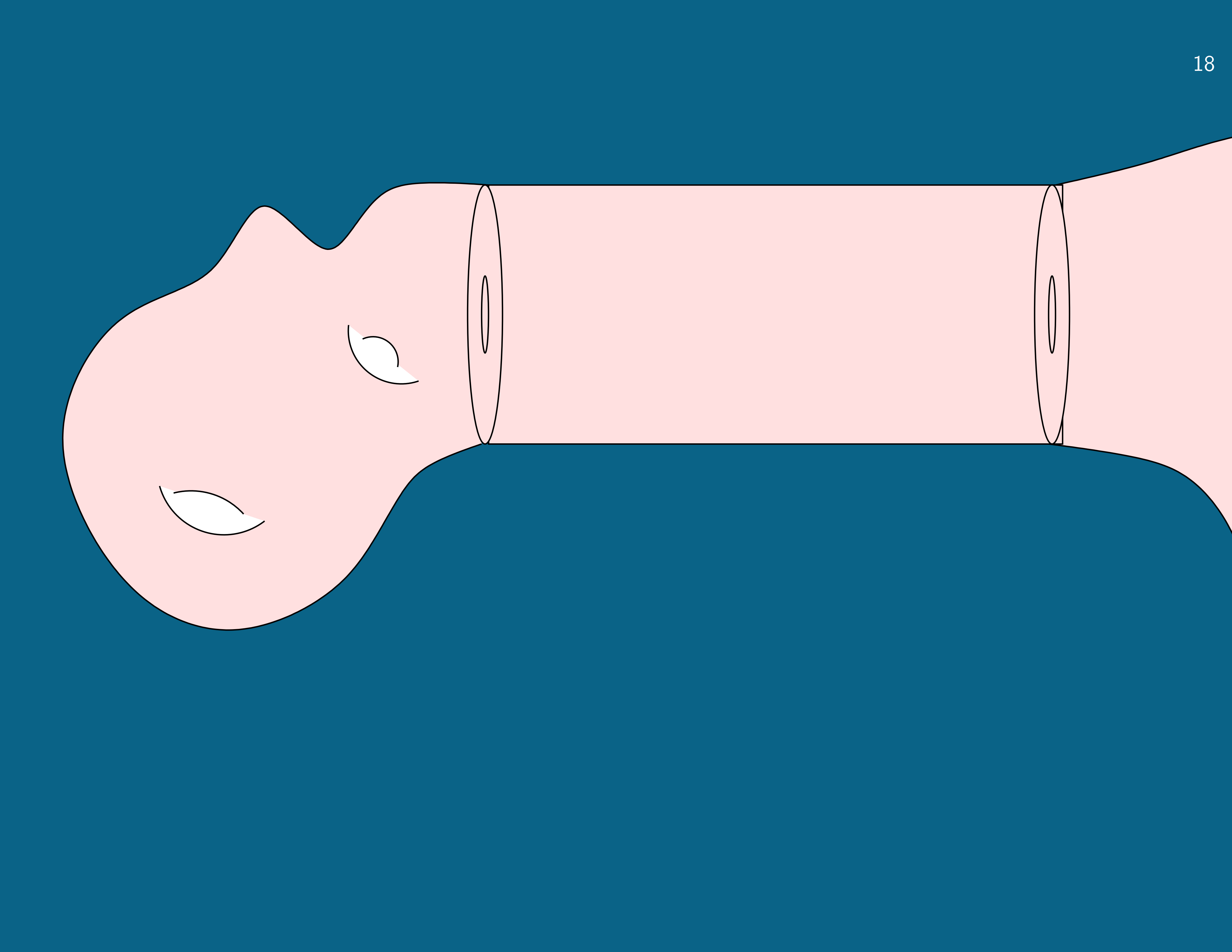


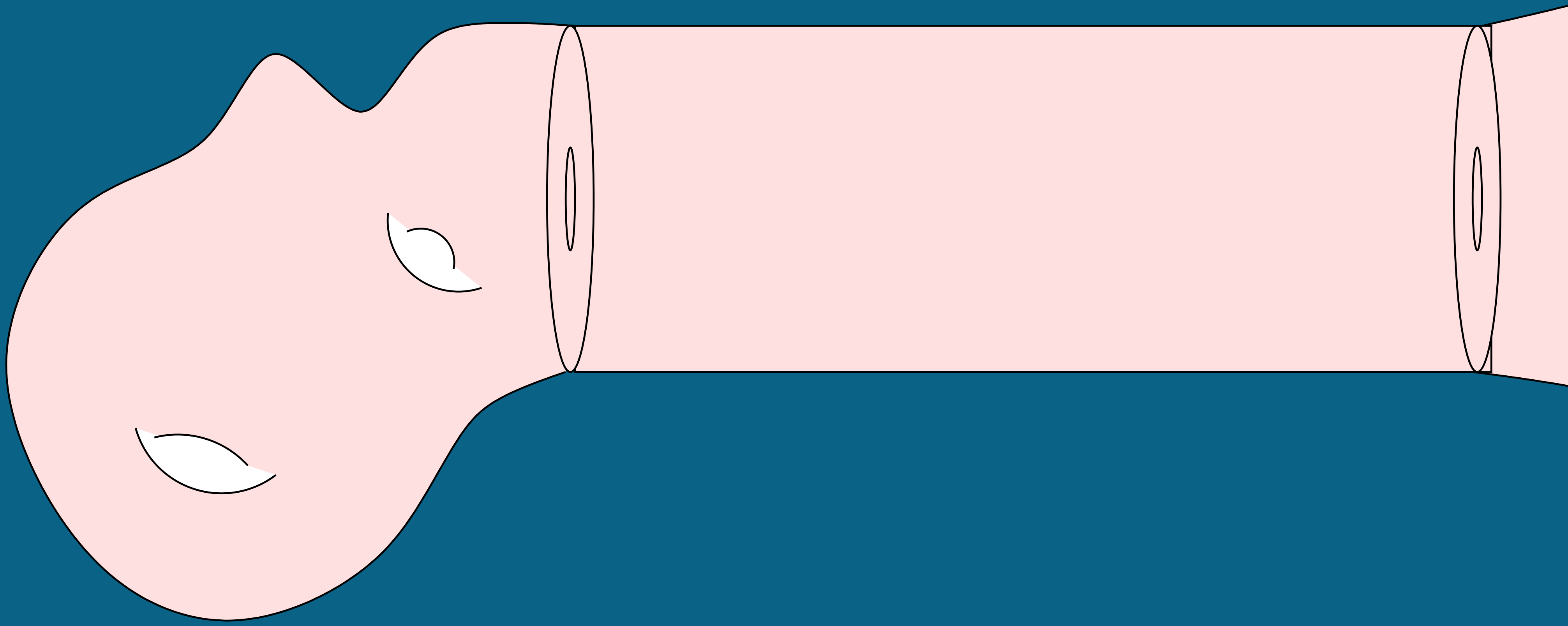


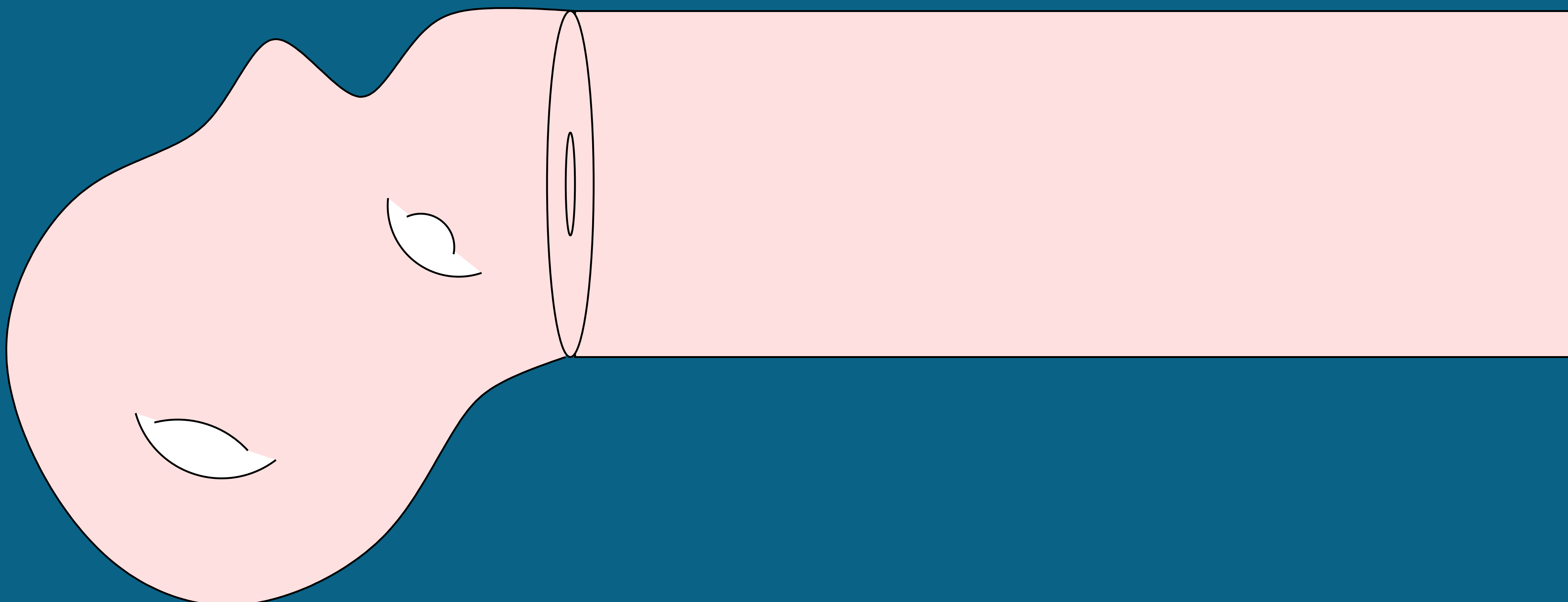


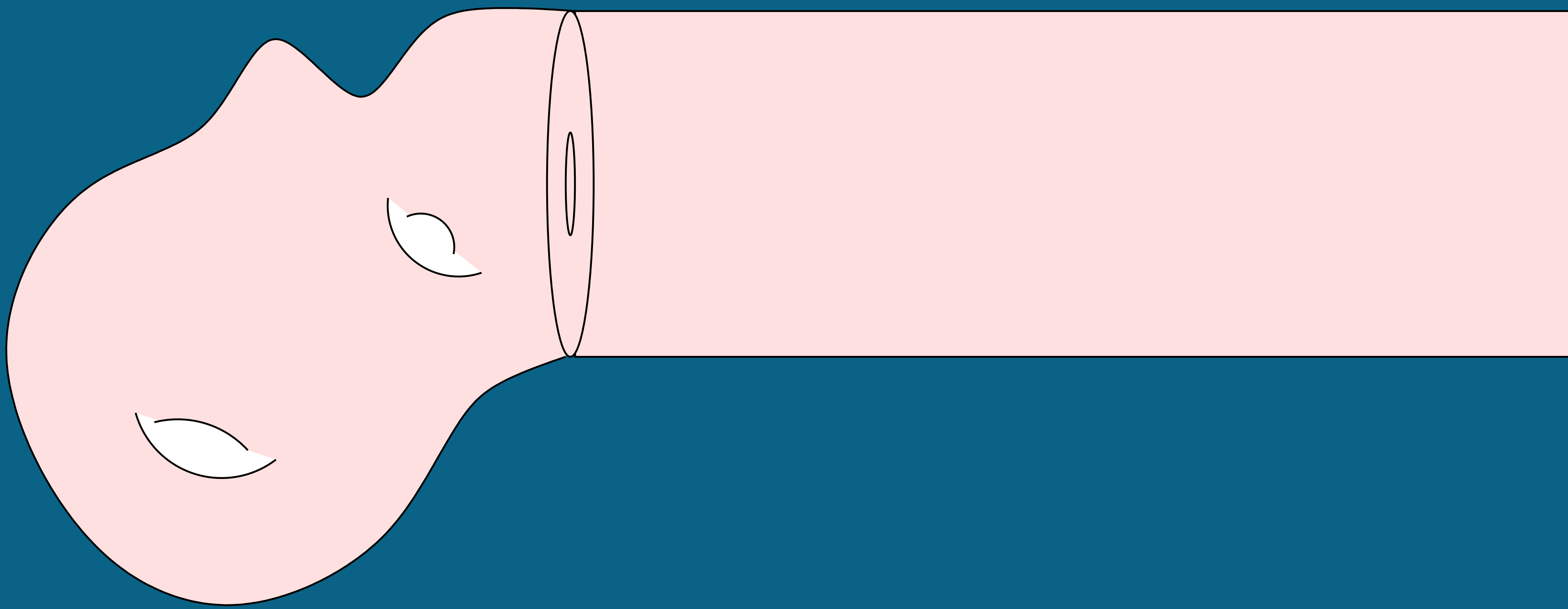












- From this perspective, an infinite throat has opened up, and the “other” half K3 surface has receded to inifinite distance.

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- The physics on the two halves does *not* decouple: this is heterotic weak coupling and the "other" half K3 surface has receded to inifinite distance.

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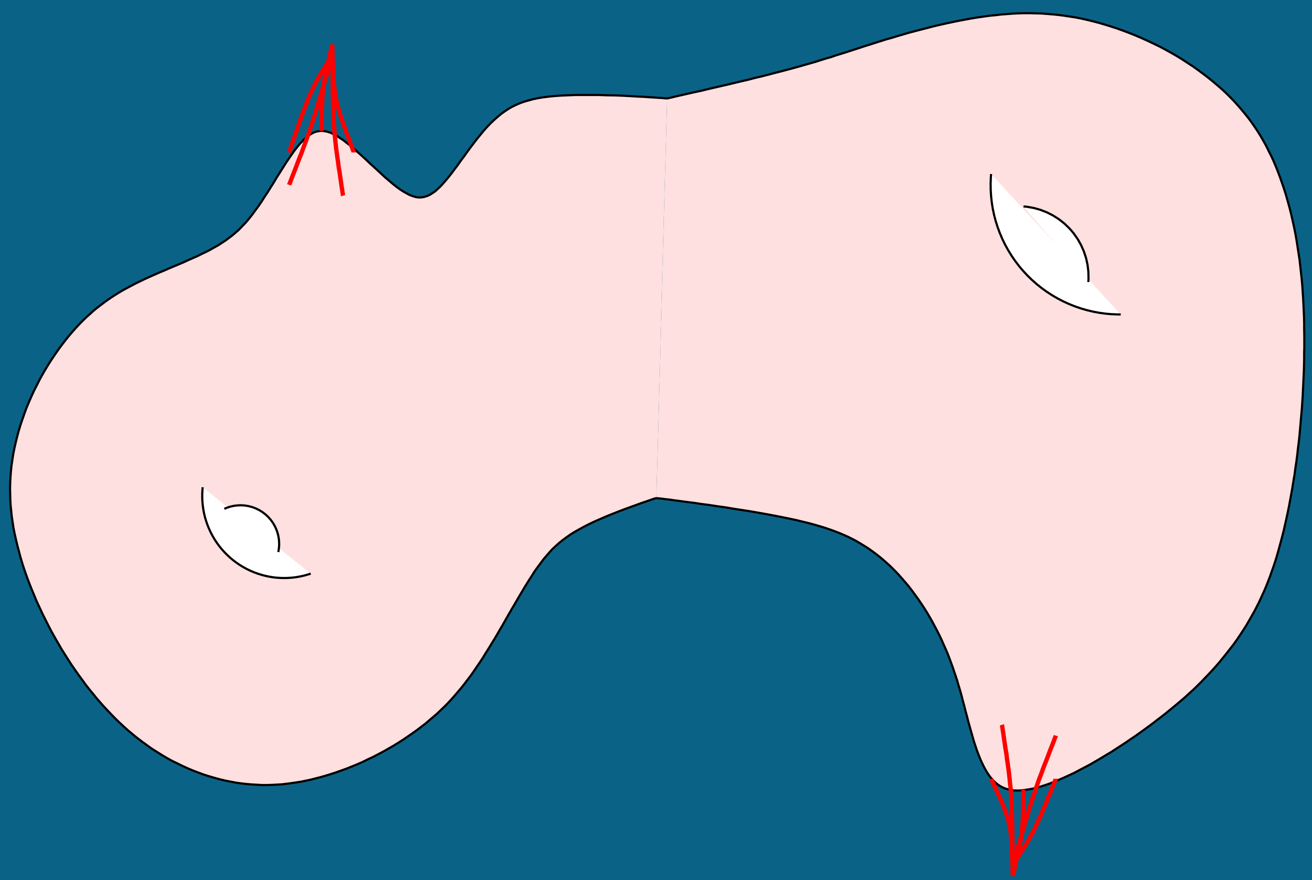
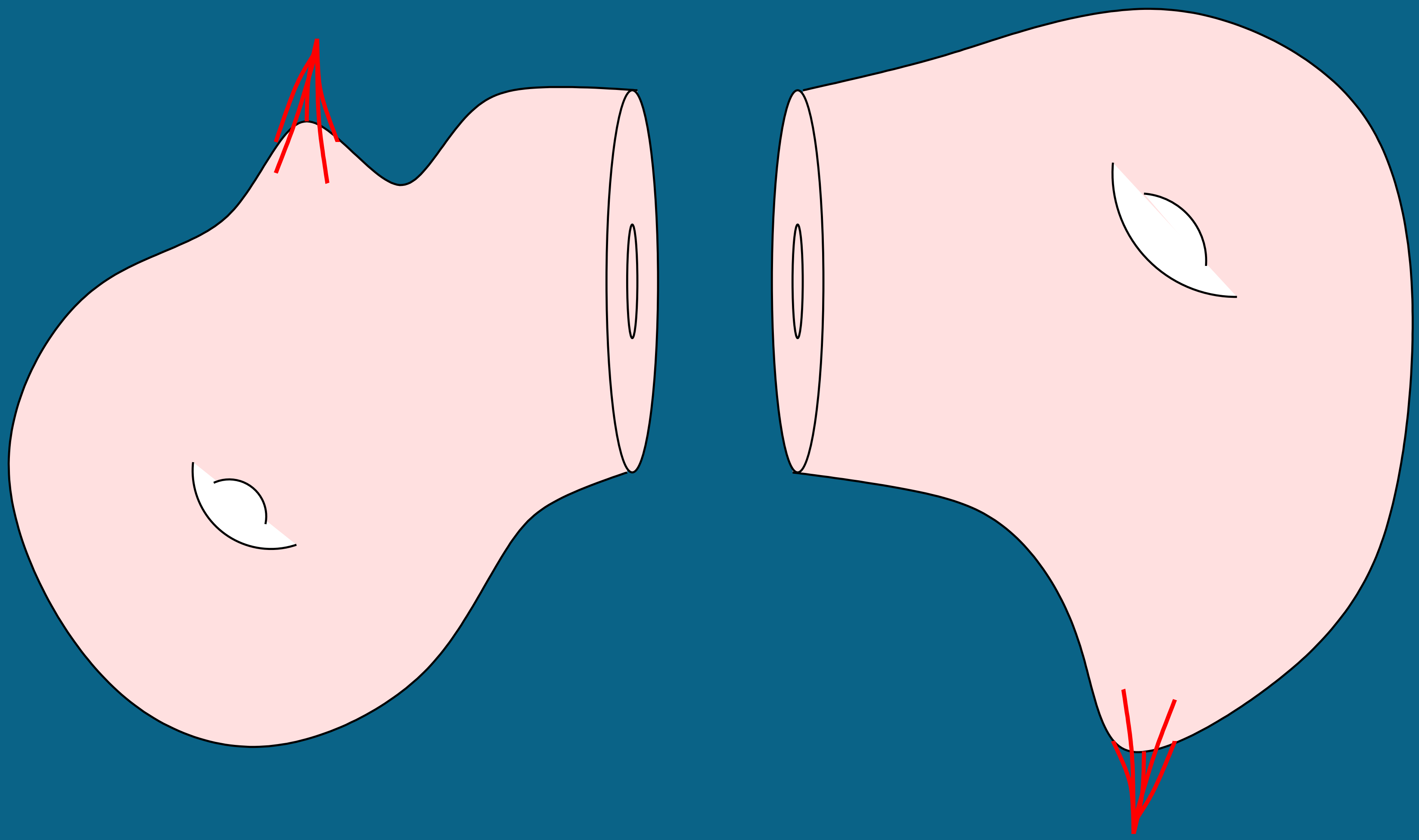
- The physics of the other half K3 surface must be encoded in the boundary of the manifold-with-boundary

Variant: K3 surfaces with frozen singularities

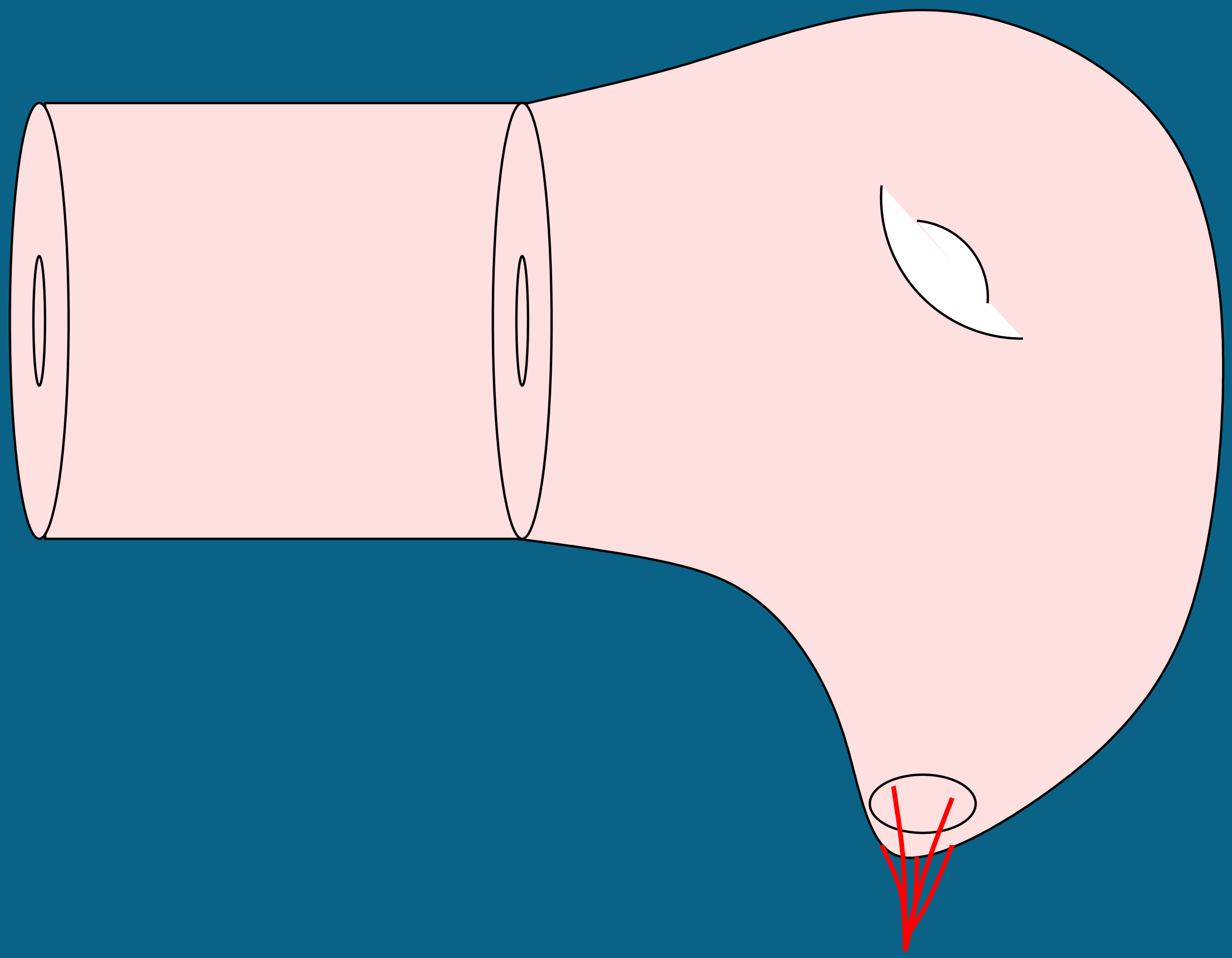


- There are M-theory compactifications on certain K3 surfaces with ADE singularities with 3-form flux at the singular points[5]
- Allowed values of 3-form flux: $k/N \pmod{\mathbb{Z}}$ where N is one of the multiplicities in the longest root of the corresponding root system
- At most 4 singularities, and the total 3-form flux vanishes
- These can similarly be cut in half along T^3, giving a half K3 with frozen singularities

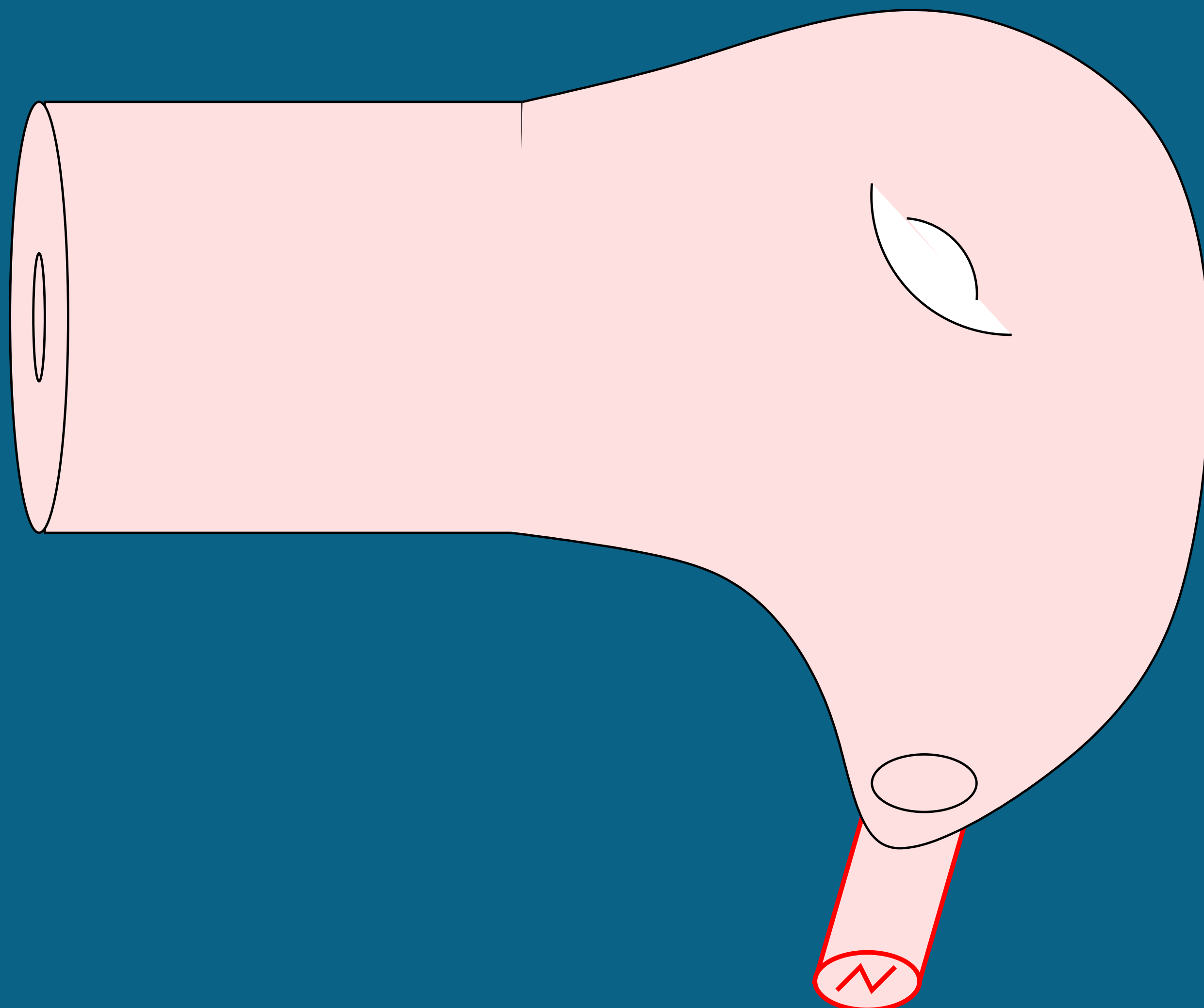
[5] Witten, de Boer et al. (Triples, fluxes, and strings), Atiyah–Witten



- We can extend T^3 to a long throat as before



- and also replace the singular point by an asymptotic tube whose boundary is S^3/Γ



M-theory on manifolds with boundary

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- Hořava–Witten: M-theory on a manifold with boundary must have an E_8 gauge field on the boundary, in addition to bulk fields
- We use coordinates with $x \geq 0$ near the boundary; the

metric is

$$ds^2 = \left(\frac{dx}{x}\right)^2 + d\tilde{s}^2$$

and the 3-form can be written

$$C = \frac{dx}{x} \wedge \alpha + \beta$$

- Hořava–Witten anomaly analysis for the boundary theory says:
  - ⋆ The limit of C on the boundary (caputured by β) equals $CS(A_3) - \frac{1}{2}CS(R_3)$ for the gauge connection A_3 and curvature R_3 on the boundary

- ⋆ α restricted to the boundary provides the B-field there
- ⋆ the familiar $\int C \wedge G \wedge G$ term, together with a new $\int C \wedge X_8$ term, gives rise to the Green–Schwarz mechanism on the boundary

- To describe our M-theory vacuum on the half K3 surface, we need a metric and 3-form field there, as well as an E_8 gauge field on the boundary T^3

- The E_8 gauge field on T^3 is the remnant of physics on the "other half" of the original K3

Metrics on half K3

- Write $X = X_+ \cup X_-$ with $X_+ \cap X_- = T^3 \times [0,1]$
-

$$\to H^1(T^3) \to H^2(X) \to H^2(X_+) \oplus H^2(X_-) \to H^2(T^3) \to$$

- $H^2(X_+)$ has rank 11, and a degenerate negative semi-definite intersection form with kernel of rank 3
- Moduli: $\Gamma \backslash Gr(16, \mathbb{R}^{0,8,3})$, with $\Gamma = \Gamma_{E_8}$; precisely the moduli of an E_8 gauge field on T^3

The 3-form field on half K3

- Morgan–Mrowka–Ruberman: L^2 gauge fields on a 4-manifold with boundary, whose boundary is T^3
- any gauge group G
- anti-self-dual connections correspond to flows in the space of G-connections; gradient flow for the Chern–Simons functional
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- thus, a 3-form with the properties we need can be obtained from an anti-self-dual connection A on the half K3, via

$$C = CS(A) - \frac{1}{2}CS(R)$$

where A and R are now connections on the 4-manifold.

A bold proposal

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- The three-form field in M-theory can be written in terms of a (non-propagating) E_8-connection A, with

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- The kinetic term for C becomes

$$\int \| \frac{1}{30} tr(F^2) - \frac{1}{2} tr(R^2) \|$$

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- The heterotic CS(A) matches across the two half K3's

- In the frozen singularity models, the 3-form flux should be interpreted as $CS(A)$ for an E_8 connection A

- This matches the observation in "Triples, fluxes, and strings" that 3-form flux at one half matches $CS(A)$ for corresponding data on T^3 (up to sign)

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  Preliminary evidence says yes

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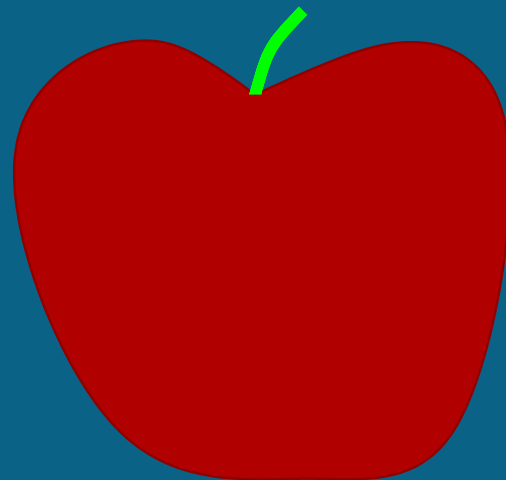
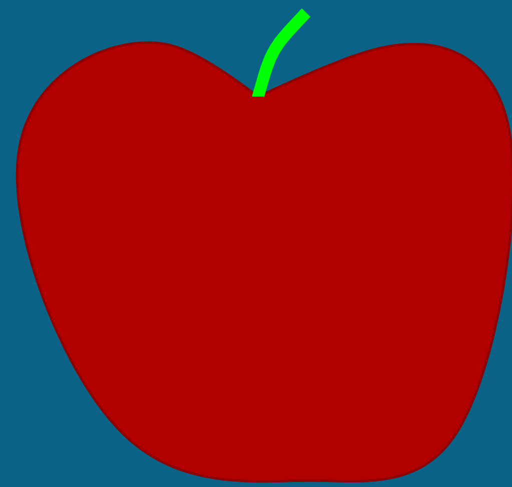
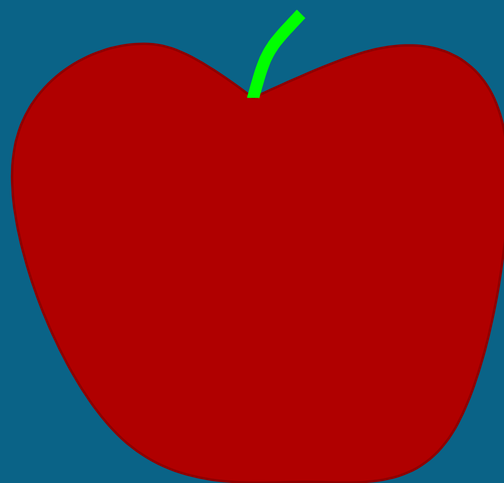
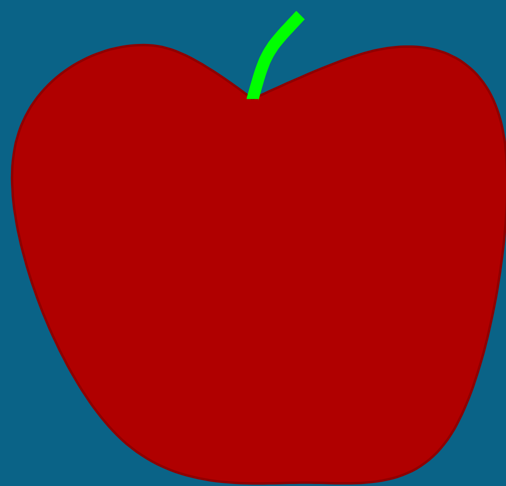
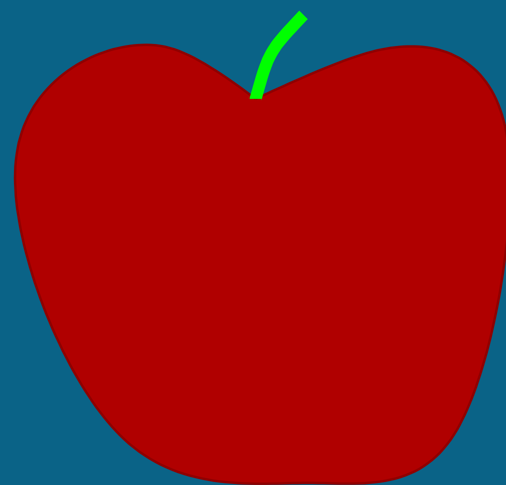
- Perhaps this only makes sense as a quantum theory?

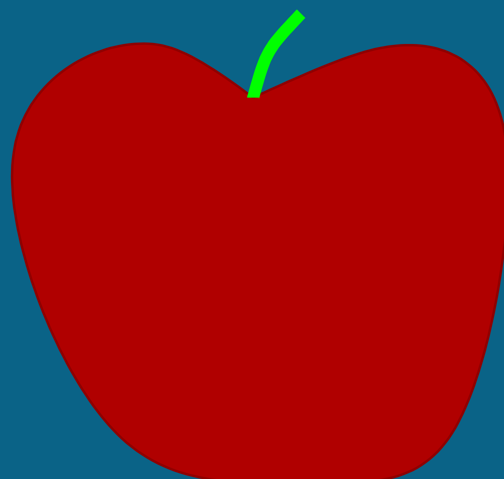


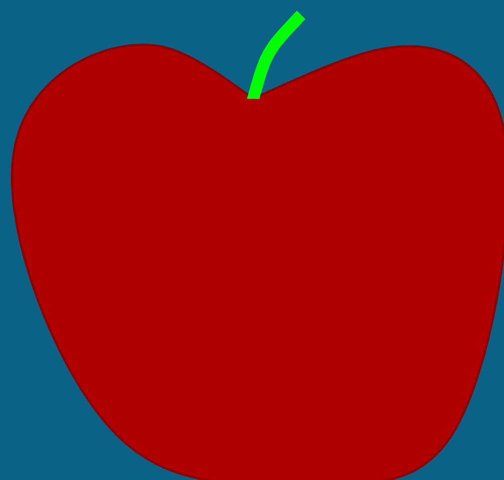


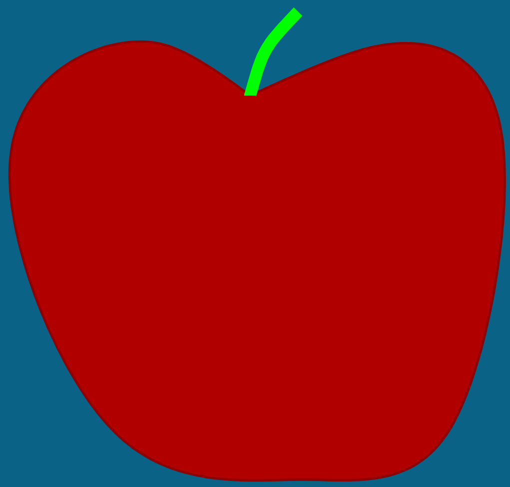


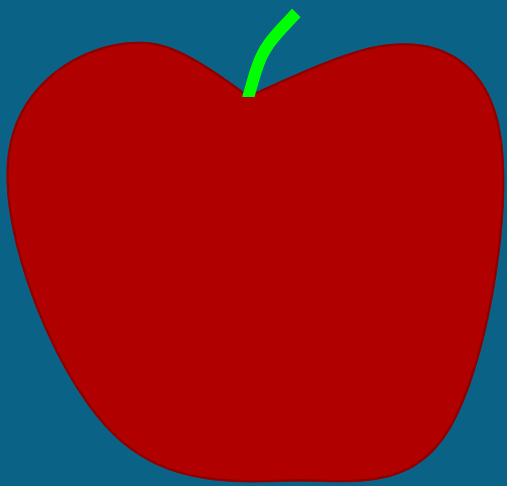


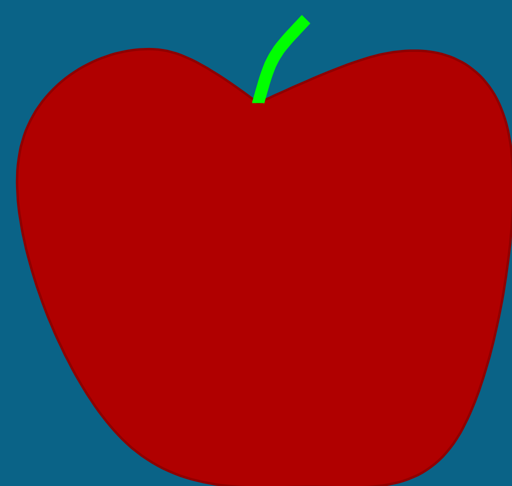


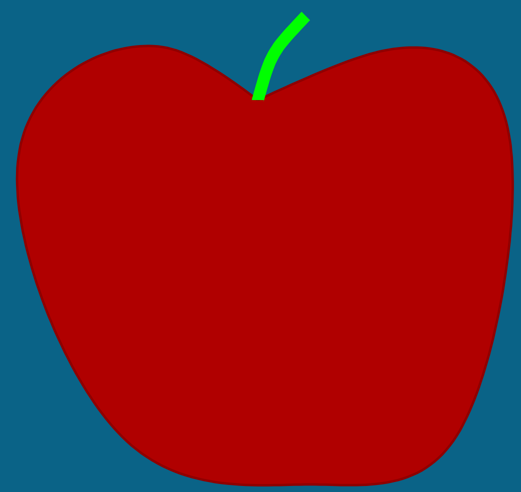












M