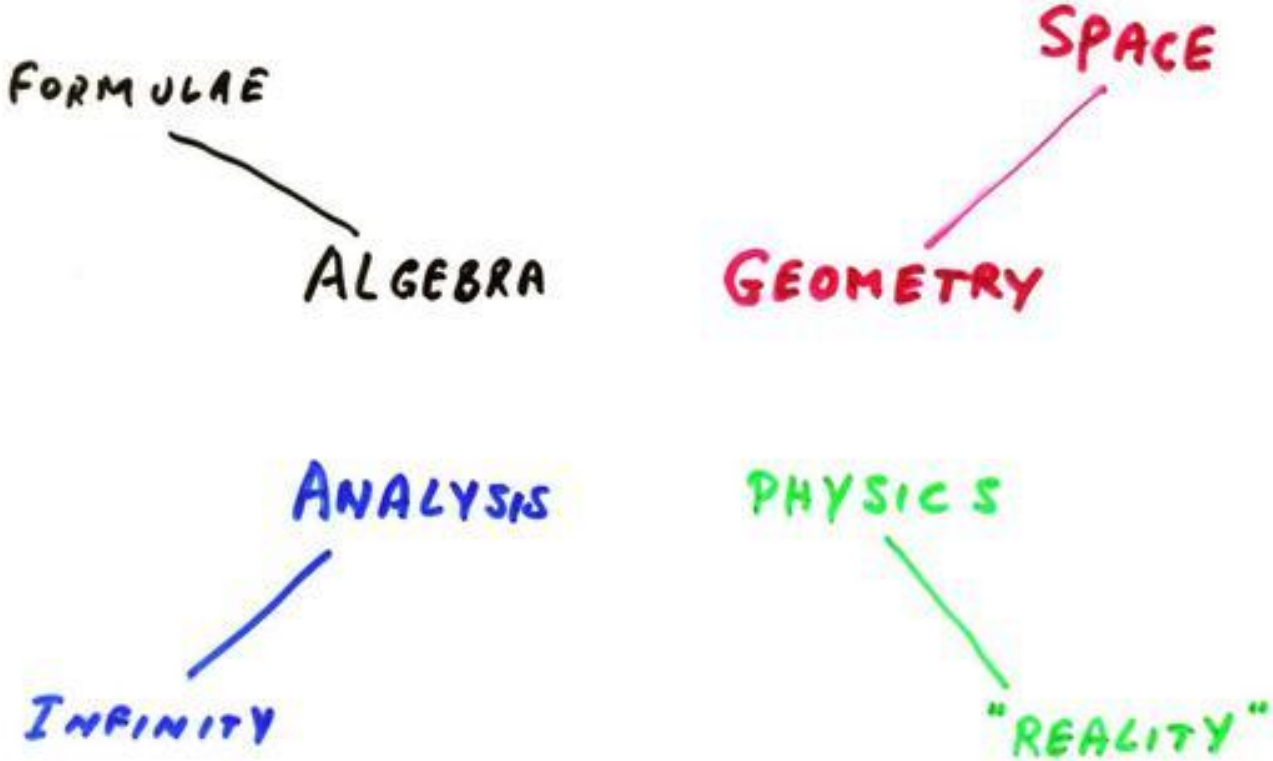


DIRAC (1939) LECTURE

THE RELATION BETWEEN

MATHEMATICS AND PHYSICS

COMPLEX NUMBERS

SQUARE-ROOT OF MINUS ONE

$$i^2 = -1$$

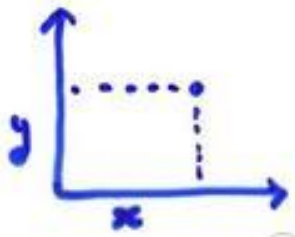
"IMAGINARY NUMBER"

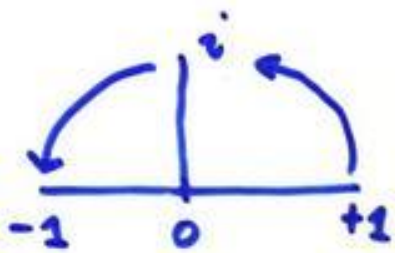
ALGEBRA

$z = x + iy$ COMPLEX NUMBER

$\bar{z} = x^2 + y^2 = (x + iy)(x - iy)$

GEOMETRY





ANALYSIS

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$$

THEORY OF FUNCTIONS OF A COMPLEX VARIABLE

$$\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y} = 0$$

(EX. z^n)

PHYSICS

COMPLEX NUMBERS USED IN ELECTRO-MAGNETIC THEORY

ALSO FUNDAMENTAL IN QUANTUM MECHANICS

GAUSS "THE TRUE METAPHYSICS OF $\sqrt{-1}$ IS NOT SIMPLE"

APPLIES TO MATHEMATICS

AND PHYSICS

$$e^{\pi i} = -1$$

TO BE OR NOT TO BE

QUATERNIONS

HAMILTON 3 SQUARE ROOTS OF -1

i, j, k $\quad i^2 = j^2 = k^2 = -1$

AND NON-COMMUTATIVE MULTIPLICATION

$$ij = -ji = k$$
$$jk = -kj = i$$
$$ki = -ik = j$$

$$(t^2 + x^2 + y^2 + z^2) = (t + ix + jy + kz) \times (t - ix - jy - kz)$$

GENERALIZED BY CLIFFORD TO MORE VARIABLES (CLIFFORD ALGEBRAS)

ALGEBRA

GEOMETRY

ROTATIONS IN 3-DIMENSIONAL SPACE

2 ROTATIONS ABOUT DIFFERENT AXES DO NOT COMMUTE

$$AB \neq BA$$

A ROTATE ABOUT NORTH POLE (N)
1/4 REVOLUTION

B ROTATE ABOUT LONDON (L)

AB : L → L → "MOSCOW"

BA : L → MOSCOW → N

PHYSICS

EINSTEIN SPECIAL RELATIVITY

4 DIMENSIONS OF SPACE-TIME

QUANTUM MECHANICS

WAVE FUNCTIONS

NON-COMMUTATIVE MULTIPLICATION

DIRAC FACTORIZED LAPLACE OPERATOR

$$\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$= \left(\frac{\partial}{\partial t} + i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times$$

$$\left(\frac{\partial}{\partial t} - i\frac{\partial}{\partial x} - j\frac{\partial}{\partial y} - k\frac{\partial}{\partial z}\right)$$

DIRAC OPERATOR

DIRAC OPERATOR

FREQUENCIES $\pm$

LAPLACE OPERATOR

FREQUENCIES ALL +

(VIBRATION MODES)

$\Rightarrow$ PREDICTION OF

POSITRONS [OPPOSITE CHARGE

TO ELECTRON]

REVOLUTION IN PHYSICS

↓

FERMIONS BOSONS

↓

SUPERSYMMETRY

GEOMETRY

DIRAC OPERATOR INVOLVES FUNCTIONS

WITH 4 REAL COMPONENTS (QUATERNIONS)

"

2 COMPLEX COMPONENTS = SPINORS

WHAT IS GEOMETRIC MEANING OF

SPINORS ?

" SPINORS LIVE IN THE SQUARE-ROOT OF THE GEOMETRICAL WORLD "

$(\text{SPINOR})^2$ = POINT + LINE + PLANE + ...

NUMBER LENGTH AREA

SPINORS AND DIRAC EQUATION CAN BE DEFINED ON CURVED SPACE-TIME

BACKGROUND I. INCORPORATE EINSTEIN'S THEORY OF GENERAL RELATIVITY]

MATHEMATICAL APPLICATION

DIRAC EQUATION IN DIMENSION 4

SUBSTITUTE FOR COMPLEX VARIABLE THEORY

ALGEBRAIC CURVES

$$x_1^n + x_2^n + x_3^n = 0 \qquad \text{(UP TO RATIOS)}$$

OVER COMPLEX NUMBERS $\Rightarrow$

RIEMANN SURFACE

$n = 1, 2$ SPHERE

$n = 3$ TORUS [ELLIPTIC CURVE]

$n \geq 4$ CONNECTED SUM OF TORI

ALGEBRAIC SURFACES

$$x_1^n + x_2^n + x_3^n + x_4^n = 0$$

$n \geq 5$ (ODD) CANNOT BE DECOMPOSED

[DONALDSON]

PROOFS USE IDEAS FROM PHYSICS

CLOSELY LINKED TO DIRAC

EQUATION.

SPECIAL PROPERTIES of 4 DIMENSIONS

WHAT WOULD GAUSS SAY

ABOUT SPINORS AND THE

DIRAC EQUATION ?