Dispersal of Mass and Circulation Following Shock-sphere and Shock-cylinder Interactions:

Effects arising from shock cavity collapse, vortex bilayers, density-gradient intensification and vortex projectiles

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Motivations

- Mass and vorticity dispersal in strong-shock-sphere and shock-cylinder interactions
- Secondary structures: vortex bilayers and vortex projectiles (VPs)
- Shock cavity implosion morphology and pressure enhancement
- Secondary baroclinic vorticity generation and instability: Density gradient intensification.
Brief Review:

Shock-sphere and shock-cylinder

Part I: Shock-sphere Interaction

2D Axisymmetric Compressible Euler Simulation with PPM

➢ Schematic of computation domain:

➢ Parameter domain:

<table>
<thead>
<tr>
<th>M (shock)</th>
<th>$\rho_1 / \rho_2$</th>
<th>resolution</th>
<th>$r_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5, 5.0, 10.0</td>
<td>10.0</td>
<td>1606(r)×246(z)</td>
<td>110 (zones)</td>
</tr>
</tbody>
</table>
**Abstraction of Structures : Bubble; Shocks & Shear Layers at Early and Intermediate Times**

$t_1$: $\omega_{-1}(1)$ is the primary vorticity deposition layer (Samtaney and Zabusky 1994), R(1) reflected shock wave, T(1) transmitted shock wave, I represents the incident shock

$t_2$: $\omega^1_{+1}(2)$ is a shear layer generated at the high curvature of transmitted shock T(2). R(2) is a reflected shock wave. Ms(2) is the Mach stem, and $\omega^2_{+1}(2)$ is the shear layer produced at the triple point of Mach shock reflection. Transmitted shock T(2) forms a shock cavity that will collapse and re-expand.

$t_3$: $\omega_{+2}(3)$ is secondary generated positive shear layer due to a Mach reflected shock $R_2(3)$; double Mach reflection is typified by Mach stems Ms(3), Ms_{1}(3) and Ms_{2}(3) together with complex shear layers and vortex ring.

(*The abstraction is dependent on Mach number*).
Mass & Vorticity Evolution and Dispersal, Vortex Projectile Formation I

Results for $M = 2.5$:

Vorticity (upper) and density (below) at normalized time
Mass & Vorticity Evolution and Dispersal, Vortex Projectile Formation II

- Results for $M = 5.0$:

Vorticity (upper) and density (below) at normalized time
Results for $M = 10.0$:

Vorticity (upper) and density (below) at normalized time

Vorticity (upper) and density (below) at normalized time
Y-integrated Vorticity Quantifies:
Vortex bilayer and double mach reflection

\[ M = 2.5, \quad tM = 0.32 \]

\[ M = 10, \quad tM = 0.32 \]
Circulation and normalization & scaling for different Mach numbers. $\Gamma \propto (M-1)(1+M^{-1}+2M^{-2})$ (Samtaney and Zabusky 1994) for circulation deposited on the interface. Secondary vorticity generation and double Mach reflection contributes to the circulation increase.
Divergence of velocity shows shock patterns, especially transmitted shock cavity implosion morphologies and double mach reflections. White represents expansion region.
Scaling Pressure Enhancement for Shock Cavity Collapse:

\[ p_{\text{max}} \left(1 + \frac{2\gamma}{\gamma + 1} (M^2 - 1)\right) \]

Scaling of maximum pressure \((M=2.5 \text{ to } 10, \ \eta=10, \ \gamma=1.209)\). One dimensional pressure jump condition serves as a satisfactory scaling of pressure enhancement, especially at high Mach number since shock cavity implosion approaches one dimension collision of transmitted shocks. Time is normalized by the duration for the transmitted 1D shock to cross a bubble diameter.
Part II: Shock Cylinder Interaction
2D Cartesian Compressible Euler PPM Simulations with finite interfacial transition layer

Schematic of Computational Domain

Note: \( R_0 \) is 1/4th the size of y-domain in all calculations. Figure not to the scale.

Parameter domain

<table>
<thead>
<tr>
<th>Mach No. (( M ))</th>
<th>Density Ratio(( \eta ))</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.095, 1.2</td>
<td>5.0</td>
<td>400<em>100, 800</em>200, 1200*300</td>
</tr>
</tbody>
</table>

Numerical Method

Compressible Euler Equations: Piecewise Parabolic Method
Mass and Circulation Evolution
Transition Layer with **Gaussian** Distribution: Comparison with Jacob’s 1993 Experiment

(Mach 1.095, η=5.0, 800*200)
Mass and Circulation Evolution (Cont.)

(Mach 1.095, η=5.0, 800°200)
Mass and Circulation Evolution

Transition Layer with a linear distribution

Density

Vorticity

Shadowgraph

Early time evolution of mass and circulation using a linear transition layer between air and SF6 bubble. Secondary roll-ups appear early in this configuration.

(Mach 1.2, η=5.0, 1200*300)
Mass and Circulation Evolution (Cont.)

Density

Vorticity

Shadowgraph

Late time evolution of mass and circulation using a linear transition layer between air and SF6 bubble. Vortex projectile formation and evolution at late time.

(Mach 1.2, η=5.0, 1200*300)
Vorticity Profile along a slice through vortex core at $t = 680 \mu s$

Vorticity along a y-slice at location of maximum vorticity in the diagnostic box at $t = 680 \mu s$. The peak correspond to the dominant vortex driving the flow after shock passage.

(Mach 1.2, $\eta=5.0$, 1200*300)
Global circulation calculated in the diagnostic box. Increase in net circulation after primary vorticity deposition by shock passage due to secondary baroclinic effects.

(Mach 1.2, $\eta=5.0$, 1200*300)
Density Gradient Intensification

Gradient of density \( \nabla \rho \) along a slice at \( j=25 \). Same color peaks in each graph show upstream and downstream interface along this slice for various times showing the intensification of gradient of density.

(Mach 1.2, \( \eta=5.0 \), 1200*300)
Density Gradient Intensification (Cont.)

Integrating $\nabla \rho$ and $\nabla \rho / \rho^2$ in the whole domain emphasizing the density gradient intensification.

(Mach 1.2, $\eta=5.0$, 1200*300)
Velocity distribution in the diagnostic box at intermediate and late times showing a peak followed by large tail at the higher velocities. Velocities plotted are in a frame moving with a velocity upstream of the cylinder at the axis after passage of primary shock and reflected shock.

(Mach 1.2, $\eta=5.0$, 1200*300)
Conclusions

Part I: Shock Sphere (2.5 ≤ M ≤ 10)

1. Observe intermediate time single vortex bilayer (VBL) for low Mach number and two VBLs for higher Mach number that evolves into a Vortex Projectiles (VPs).
2. Dominant contribution of VBLs to mass transport.
4. Shock cavity implosion pressure enhancement \( \propto 1 + \frac{2\gamma}{\gamma + 1}(M^2 - 1) \)

Part II: Shock Cylinder (M = 1.095 & 1.2)

1. Primary and secondary structures agree with Jacob’s Experiment (’93).
2. Initial finite Gaussian transition layer on gas cylinder motivated by experimental “setup” time
   - Initial Condition Parameterization: Interfacial gradient and core size.
3. **Opposite signed (+)** circulation evolves on downstream interface and is responsible for bubble elongation.
   - Secondary Baroclinic (+ / −) circulations (\( \Delta\Gamma_\approx 100\% \)) result from interfacial density-gradient enhancement.
   - Velocity distribution in diagnostic box.