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Presented at the 8th IWPCTM, Caltech, Pasadena, CA, 12/9-12/14/01.

This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48
Several aspects of mixing due to the Rayleigh-Taylor (RT) instability are investigated.

Analysis of 3D multimode simulations using the PPM code [D.H. Porter and P.R. Woodward, Astrophys. J. Suppl. 93, 309 (1994), and references therein.] show that there are regions of the parameter space of the initial conditions in which the growth rate is independent of variations in the initial conditions. The simulated growth rates are found to increase as the Navier-Stokes viscosity is increased. It is investigated whether this counterintuitive result is due to the suppression of material mixing at the molecular level for larger viscosities.

Analyses of two RT experiments, one in which water is accelerated by a compressed gas (E.E. Meshkov and N.V. Nevmerzhitsky, Proc. 3rd Int. Wkshp. on the Physics of Compressible Turbulent Mixing, 1991) and one in which an interface between gases of different density is decellerated in the post-shock region of a shock in an electromagnetic shock tube (A.M. Vasilenko et al., ibid.), are presented. Direct compressibility effects on the RT growth are shown to be negligible in the former. Various effects of the expansion of the gases in the region of the interface on the RT growth rates are investigated for the latter experiment, both analytically and with 1D simulations. These effects are found to be insufficient to reconcile the growth rates observed in the Vasilenko et al. experiments with some other experimental and simulation results.
• Nonlinear Rayleigh-Taylor evolution arises in
  – astrophysics - collapsed stars, supernovae
  – performance degradation of inertial fusion capsules

• Rayleigh-Taylor evolution is of general basic interest as a paradigm for phenomena in
  – atmospheric and space sciences
  – magnetic fusion
  – combustion
High-Resolution 3D PPM simulations of Rayleigh-Taylor instability give the following results.

- Lower growth rates ($\alpha$) than experiments and many other simulations.
- $\alpha$ increases with Navier-Stokes viscosity $\nu$.
- $\alpha$ is insensitive to changes in system size and spatial scale of initial perturbations.
- There appears to be a single large scale of the evolution; i.e.,
  - amplitudes from concentration thresholds and profile overlap, and perpendicular integral correlation length scales agree for most cases.
- Cases with very different $\alpha$ have very similar atomic mix fraction profiles.
THE PPM CODE IS AN ESTABLISHED HYDRO CODE USED EXTENSIVELY IN ASTROPHYSICS

- **PPM** = Piecewise Parabolic Method
  - Godunov-type code
  - Uses piecewise parabolic interpolation
  - Woodward-Porter version
    - Lagrange + remap to Euler mesh
    - 3D; Directionally split
    - Has Navier-Stokes dissipation

- **Robust for high-Mach-number flows**

- **Runs well on large parallel computers**
Baker, Meiron, and Orszag RT cases were used as validation tests of PPM.

- $A=0.33$ case.
- PPM and Baker et al. agree well for bubble and spike amplitudes.
- Qualitative appearance of interface is similar, apart from discrete polygon nature of Baker et al. results, and slightly more roll up and some evidence of residual secondary Kelvin-Helmholtz notches in the PPM case.
For $A=0.048$, PPM gives amplitudes slightly lower than Baker et al.

- Qualitative appearance of interface is similar, apart from discrete polygon nature of Baker et al. results and slightly more roll up in the PPM case.
THE SETUP MOST USED HERE HAS UNIFORM TEMPERATURES ABOVE AND BELOW THE INTERFACE

- ideal, single-$\gamma$ (=5/3) gas
- density discontinuity supported by a discontinuity in the temperature or the molecular mass
- exponential density and pressure profiles above and below interface
- near-incompressible low-level initial velocity perturbations
  - multi-mode – quasi-Gaussian with a specified peak $k$
A RANGE OF RESOLUTIONS, NAVIER-STOKES DISSIPATION VALUES, AND SYSTEM SIZES IN THE NEAR-INCOMPRESSIBLE REGIME WERE USED

- \( \rho_h / \rho_1 = 2-3 \) (A=1/3-1/2)
- \( \nu = \kappa = 0, 1/6.5, 1/2.3 \ g \Delta^{3/2} \)
- \( L = 0.02, 0.04, 0.08 \ c^2/g \)
- \( 256^3, 256^2 \times 512, 512^3 \)
- Low-level initial velocity perturbations
- Peak wavenumber varied

Initial ms \( \frac{v_z}{c_s} \)

Initial \( V_z \) energy per mode
PPM AND PPM+NS SIMULATIONS OF RAYLEIGH-TAYLOR EVOLUTION SHOW BUBBLES, SPIKES, AND ROBUST "BUBBLE MERGER"
• All spatial scales $l$ grow as $l(t) = F(f) \cdot g \cdot (t-t_0)^2$, where $f$ is the density ratio, and $t$ is a "movable" time origin.

• Common characterization:
  - bubble heights have $F(f) = \alpha \cdot A$
  - $A = (f-1)/(f+1)$ is the "Atwood number"
  - $\alpha_{\text{bubble}}$ is a key parameter for comparisons.

• $l_{\text{bubble}}$ is obtained as the height at which the volume fraction of fluid originating below the interface is 1%. 

AT LATE TIME (>0.4), PPM-NS CALCULATIONS ARE IN THE $t^2$ ASYMPTOTIC REGIME

• $\alpha$ is sensitive to the criterion for determining the envelope of the mixing layer.
3D PPM simulations give lower values of \( \omega_b \) than experiments and other codes.

<table>
<thead>
<tr>
<th>Code/Exp. Author</th>
<th>( A )</th>
<th>Resolution</th>
<th>( \gamma \frac{\Delta^2}{\nu} )</th>
<th>( \omega_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turmoil Youngs</td>
<td>0.50</td>
<td>( \leq 1025 \times 512^2 )</td>
<td>( \infty ) Van Leer</td>
<td>0.025–0.028</td>
</tr>
<tr>
<td>Turmoil Youngs</td>
<td>0.20</td>
<td>( 240 \times 160^2 )</td>
<td>( \infty )</td>
<td>0.027</td>
</tr>
<tr>
<td>Turmoil Youngs</td>
<td>0.90</td>
<td>( 240 \times 160^2 )</td>
<td>( \infty )</td>
<td>0.029</td>
</tr>
<tr>
<td>PPM-NS</td>
<td>0.33–0.5</td>
<td>( 256^3–512^3 )</td>
<td>2.3–18.4</td>
<td>0.012–0.026</td>
</tr>
<tr>
<td>PPM</td>
<td>0.33</td>
<td>( 256^3 )</td>
<td>( \infty ) - PPM</td>
<td>0.011–0.013</td>
</tr>
<tr>
<td>rocket</td>
<td>Read</td>
<td>0.23</td>
<td>na</td>
<td>0.06</td>
</tr>
<tr>
<td>LEM</td>
<td>Dimonte</td>
<td>0.22</td>
<td>na</td>
<td>0.06</td>
</tr>
</tbody>
</table>

- References:
  - Simulations by D.L. Youngs [Laser and Particle Beams 12, 725 (1994)].
$\alpha_b$ in PPM+NS simulations increases with $\nu$

$\alpha_b$ is insensitive to
- boundary position
- $A$
- compressibility
- $k_h$ of IC spectrum
CHANGING THE BOX SIZE AND $A$ DOES NOT CHANGE $\omega_b$ IN 3D PPM SIMULATIONS

<table>
<thead>
<tr>
<th>Model</th>
<th>Comment</th>
<th>$A$</th>
<th>Resolution</th>
<th>$g^{\frac{1}{2}} \Delta^{\frac{1}{2}} / \nu$</th>
<th>$\omega_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPM-NS</td>
<td></td>
<td>0.33</td>
<td>$256^3$, $k_p - 32$</td>
<td>6.5</td>
<td>0.018</td>
</tr>
<tr>
<td>PPM-NS incr. $L_x$</td>
<td>0.33</td>
<td></td>
<td>$256^2 \times 512$</td>
<td>6.5</td>
<td>0.018</td>
</tr>
<tr>
<td>PPM-NS change $A$</td>
<td>0.5</td>
<td></td>
<td>$256^3 \times 512$</td>
<td>6.5</td>
<td>0.018</td>
</tr>
<tr>
<td>PPM-NS lower $k_p$</td>
<td>0.33</td>
<td></td>
<td>$256^3$, $k_p - 16$</td>
<td>6.5</td>
<td>0.018</td>
</tr>
<tr>
<td>PPM-NS $L_x - 0.08$</td>
<td>0.33</td>
<td></td>
<td>$256^3$</td>
<td>6.5</td>
<td>0.018</td>
</tr>
<tr>
<td>PPM-NS incr. $\nu$, $\kappa$</td>
<td>0.33</td>
<td></td>
<td>$256^3$</td>
<td>2.3</td>
<td>0.028</td>
</tr>
<tr>
<td>PPM-NS decr. $\nu$, $\kappa$</td>
<td>0.33</td>
<td></td>
<td>$256^3$</td>
<td>18.4</td>
<td>0.012</td>
</tr>
<tr>
<td>PPM-NS incr. res.</td>
<td>0.33</td>
<td></td>
<td>$512^3$</td>
<td>2.3</td>
<td>0.022 *</td>
</tr>
<tr>
<td>PPM no NS</td>
<td></td>
<td>0.33</td>
<td>$256^3$, $k_p - 32$</td>
<td>$\infty$</td>
<td>PPM 0.0125</td>
</tr>
<tr>
<td>PPM</td>
<td></td>
<td>0.33</td>
<td>$256^3$, $k_p - 64$</td>
<td>$\infty$</td>
<td>PPM 0.011</td>
</tr>
</tbody>
</table>

- Increasing (decreasing) Navier-Stokes dissipation raises (lowers) $\omega_b$.
- Increasing compressibility has no effect on $\omega_b$.
- Run with lower initial $k_p (= 16)$ - $\alpha$ unaffected.
• In RT studies, the large scales commonly diagnosed are measures of the vertical scale, e.g.,
  – amplitudes from threshold concentration levels,
  – integral measures of overlap of concentration profiles.

• In much of fluid dynamics “integral” correlation length scales are used.
  – Integral scales measured at the position the interface would have in the absence of RT instability provide measures of the transverse scale.

• In high resolution 3D multimode PPM simulations of RT, these scales usually maintain a proportionality with ratios independent of time.
Bubble and spike amplitudes and integral scales track if NS dissipation is used.
Perpendicular integral scales track vertical scales less well for pure PPM.
A hypothesis that low α's are associated with high atomic mix fraction was tested.

- If the mixing fraction is large, bubbles and spikes are impure and experience small buoyancy relative to the surrounding fluid.
  - This might be expected to result in low growth rates.
Cases with very different $\alpha$ show very similar atomic mix fraction profiles.

- PPM and $R_g=6.5$ have similar mix fraction profiles, different $\alpha$ ($\alpha_b=0.013, 0.018$).

- $R_g=2.3$ has slightly lower mix fraction, highest $\alpha$ ($\alpha_b=0.028$).
Summary of Results From High Resolution 3D PPM Simulations of Rayleigh-Taylor Instability Evolution

- Lower growth rates ($\alpha$) than experiments and many other simulations.

- $\alpha$ increases with Navier-Stokes viscosity $\nu$.

- $\alpha$ is insensitive to changes in system size and spatial scale of initial perturbations.

- There appears to be a single large scale of the evolution; i.e.,
  - amplitudes from concentration thresholds and profile overlap, and perpendicular integral correlation length scales agree for most cases.

- Cases with very different $\alpha$ have very similar atomic mix fraction profiles.

- Demonstrated that simulations are in a regime where compressibility is negligible.
The experiments by E.E. Meshkov and N.V. Nevmerzhitsky were reexamined because they appeared to give large values of $\alpha$ and an apparent increase of $\alpha$ with acceleration.

- Rayleigh-Taylor experiment (3rd. IWPCTM, France, 7/91.)

- Water accelerated by compressed air or Helium

- Atwood number $A \approx 1$

- Membrane breaks; plywood plate detaches; water and plate accelerated downwards by compressed air

- Authors conclude that $\alpha_{\text{bubble}}$ increases with $p_c$
The results shown in the Meshkov-Nevmerzhitsky `91 IWPCTM paper do not demonstrate a systematic increase in $\alpha$ due to compressibility.

- Data shown for expts #422 and #446.

- Different curve fitting to #422 data can revise $\alpha_b$ down slightly from 0.128 to $\approx$0.1.

- $\alpha_b$ for #446 has large uncertainty.

- Compressibility effects are highest, but negligible, in #422.
Compressibility is negligible in Meshkov - Nevmerzhitovsky experiment #422.

- **Subsonic**
  - $M_{\text{bubbles}} \leq 0.04$, $M_{\text{spikes}} \leq 0.13$.

- **Bubble expansion due to change in relative pressure small**
  - $\delta l_{\text{exp}}/l \leq 0.03$ for isolated bubble
  - $\delta l_{\text{exp}}/l$ negligible for open-cell bubble structure

- **Change in drag due to bubble expansion results in < 2% change in bubble height.**
Conclusions on Meshkov - Nevmerzhitsky Experiments

• Expt. #422 has higher $\alpha_b$ than experiments by Read and by Dimonte and Schneider (but has higher density ratio).

• Difference not explained by compressibility
  – more likely connected with initial conditions.

• For the other experiments, $\alpha_b$ is highly uncertain (e.g., #446) or data is insufficient to demonstrate a trend.
The experiments of Vasilenko et al. were reexamined because α’s were found that were much larger than in other experiments and simulations.

• Miscible (gas) Rayleigh-Taylor experiment

• $S_c (≡ v/D) = O(1) (>> 1$ for liquids)

• $W = \alpha Ag t^2$ ; ($\alpha = \alpha_{\text{bubble}} + \alpha_{\text{spike}}$)

• $\alpha = 0.29 - 0.34$, (for $\rho_2/\rho_1 = 1.4 - 20$)

• Conflict with other data:
  – experiment: $\alpha_{\text{bubble}} \approx 0.06$, $\alpha_{\text{spike}} / \alpha_{\text{bubble}} \approx 1 + A$
  – simulation: $\alpha_{\text{bubble}} \approx 0.02 - 0.06$
- Rayleigh-Taylor experiment
  (3rd. IWPCTM, France, 7/91.)

- Electromagnetic shock tube

- Impulsive shock with rarefaction passing through 3 noble gases (e.g., He-Ar-He, He-Kr-He, He-Xe-Kr)
Expansion behind the shock front does not directly account for differences in $\alpha$.

- $A \ll 1$ - analytical estimate of expansion effect using 1D Sedov wave based model

- General $A$: 1D HAMR computations show that propagation of a Sedov wave through light-heavy-light gases can be modeled as a shift in origin of Sedov wave.
  - Same analysis applies for bounds on expansion effect.

- Analysis by Zhou and Dimits (Zhou et. al, this meeting) finds that experiments do not reach turbulence transition.
  - Expect significant influence of initial conditions.
• Approximate shock/rarefaction as 1D Sedov wave.

• \( u \approx u_0 \xi / \xi_0 \); \( \xi \equiv r \left[ \rho_0 / (E t^2) \right]^{1/(2+d)} \) (supported by 1D simulations); \( r_0(t) \equiv \) shock position; \( u_0 \equiv CV_s \); \( C \equiv (\gamma-1)/(\gamma+1) \); \( V_s \equiv \) shock speed.

• Then \( u \approx 2C/(2+d)(r/t) \) for \( 0 \leq r \leq r_0 \)

• Define \( r_1(t) \equiv \) Lagrangian trajectory of fluid element that coincides with shock at \( t= t_1 \) [i.e., \( r_1(t_1) = r_0(t_1) \)].

• Then \( r_1(t)/r_0(t_1) = (t/ t_1)^{2C/(2+d)} \)
Effect of expansion for $A<<1$

- Define deceleration path:
  $$R_a(t) = u_0 \cdot (t - t_1) - [r_1(t) - r_1(t_1)].$$

- Suppose layer width $h \approx \alpha R_a(t)$.

- Contribution of expansion to $h'$ is $h'_{exp} = h \frac{\partial u}{\partial r}$.

- Then $h'_{exp} / h'$ obtainable analytically.
RT growth enhancement by expansion is less than $\approx 1.3$

- $h'_{exp}/h' \leq 0.2 \Rightarrow$ less than factor 1.3 direct enhancement of layer width by expansion.

- Indirect effects? - need 2D or 3D simulation.
1D HAMR simulations support the application of the $A \ll 1$ analytical model to cases where $A \ll 1$ does not hold.
Summary and Conclusions Concerning Experiments by Vasilenko et al.

• Expansion behind the shock front does not account for differences in $\alpha$ between Vasilenko et al. experiments and other results.

• Estimated effect of expansion on $\alpha$ using a 1D Sedov-wave based model, valid for $A<<1$.

• 1D HAMR computations support the extension and application of this model to cases where $A<<1$ does not hold.

• Turbulence-transition analysis by Zhou and Dimits (Zhou et al., this meeting) suggests a significant influence of initial conditions on $\alpha$ as a possible explanation of the discrepancy.