Modeling Laser Material Strength Experiments

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Outline of poster

• Material strength model
  • Elastic-plastic flow
  • Steinberg-Guinan and Steinberg-Lund models
• VISAR velocity measurement
  • Experiment
  • Model
• Diffraction
  • Experiment
  • Model
• Sample recovery
  • Experiment
  • Decay of shock strength
• Summary and future developments
The constituitive properties of metals is of general scientific interest

Laser experiments give us access to new regimes
  - High pressures
  - High strain rates
How materials deform at strain rates $> 10^8$/s is unknown
Relevant for impact of micrometeorites on space hardware

Diagnostics
  - VISAR
  - X-ray diffraction
Recovery
Infer properties such as EOS, K, G, Y
Moderate shocks show both elastic and plastic waves.

- **Pressure** $P-\sigma_{zz}$
  - Elastic
  - Plastic flow and work-hardening
  - Elastic release

- **Strain** $\theta-\varepsilon_{zz}$

- **Hugoniot**
  - Plastic
  - Elastic limit

- **Volume**

- **Pressure wave**
  - Plastic
  - Elastic

- **Distance**
We use a material strength package in our code

Newton’s law

\[
\rho \mathbf{w}_r = -\frac{\partial}{\partial r} (P - \sigma_{rr}) + \frac{\partial}{\partial z} \sigma_{rz} + \frac{1}{r} \left(2\sigma_{rr} + \sigma_{zz}\right)
\]

\[
\rho \mathbf{w}_z = -\frac{\partial}{\partial z} (P - \sigma_{zz}) + \frac{\partial}{\partial r} \sigma_{rz} + \frac{1}{r} \sigma_{rz}
\]

Definition of strain

\[
\theta = \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r}
\]

\[
\varepsilon_{rr} = \frac{1}{3} \left(2 \frac{\partial v_r}{\partial r} - \frac{\partial v_z}{\partial z} - \frac{v_r}{r}\right)
\]

\[
\varepsilon_{zz} = \frac{1}{3} \left(2 \frac{\partial v_z}{\partial z} - \frac{\partial v_r}{\partial r} - \frac{v_r}{r}\right)
\]

\[
\varepsilon_{rz} = \frac{1}{2} \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)
\]

EOS with strain

\[
P = -K \theta - \mathbf{P}_\text{inelastic}
\]

\[
\sigma_{rr} = 2G \varepsilon_{rr} - \mathbf{\sigma}_{rr}^{\text{inelastic}} + 2\sigma_{rz} \omega + (\sigma_{zz} - \sigma_{rr})\omega^2
\]

\[
\sigma_{zz} = 2G \varepsilon_{zz} - \mathbf{\sigma}_{zz}^{\text{inelastic}} - 2\sigma_{rz} \omega - (\sigma_{zz} - \sigma_{rr})\omega^2
\]

\[
\sigma_{rz} = 2G \varepsilon_{rz} - \mathbf{\sigma}_{rz}^{\text{inelastic}} + 2(\sigma_{rr} - \sigma_{rz})\omega - 2\sigma_{rz} \omega^2
\]

\[
\omega = \frac{1}{2} \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right)
\]
We use a von Mises yield criterion for the onset of plastic flow

Deviatoric strain invariant \( J = \sqrt{\frac{4}{3} \left( \sigma_{rr}^2 + \sigma_{zz}^2 + \sigma_{rz}^2 + \sigma_{rr} \sigma_{zz} \right)} \)

Effective pressure \( P_e = P - \frac{3}{\sqrt[3]{\left( \sigma_{rr} + \sigma_{zz} \right) \left( \sigma_{rz}^2 - \sigma_{rr} \sigma_{zz} \right)/16}} \)

When \( J > Y(P_e) \), the elastic limit is exceeded and plastic flow begins

Uniaxial strain equations

\[
\rho \frac{\partial}{\partial z} \left( -P + \sigma_{zz} \right)
\]

\[
\theta^e = \frac{\partial v_z}{\partial z}
\]

\[
\sigma_{zz} = \frac{2}{3} \frac{\partial v_z}{\partial z}
\]

\[
P^e = -K \theta^e - P^{\text{inelastic}}
\]

\[
\sigma_{zz} = 2G \sigma_{zz} - \sigma_{zz}^{\text{inelastic}}
\]

\[
P_e = P - \frac{1}{4} \sigma_{zz}
\]

\[
J = |\sigma_{zz}|
\]

Sound speed \( c_{11} = \sqrt{(K + \frac{4}{3} G)/\rho} \)
Steinberg-Guinan Model

\[ G(P, T) = G_0 \left( 1 + \frac{1}{G_0} \frac{\partial G}{\partial P} \frac{P}{\eta^{1/3}} - \frac{1}{G_0} \frac{\partial G}{\partial T} (T - 300) \right) \]

\[ Y = Y_0 f(\varepsilon_p) \frac{G(P, T)}{G_0} \]

\[ Y_0 f(\varepsilon_p) = Y_0 \left( 1 + \beta (\varepsilon_p + \varepsilon_i) \right)^n \leq Y_{\text{max}} \]

\[ T_{\text{melt}} = T_0 \exp \left( 2a \left( 1 - \frac{1}{\eta} \right) \eta^{2(\gamma_0 - a - 1/3)} \right), \quad \eta = \frac{\rho}{\rho_0} \]

D.J. Steinberg, UCRL-MA-106439 (1991)
Steinberg-Lund Model

\[ Y = \{Y_T(\varepsilon_p, T) + Y_{A_f}(\varepsilon_p)\}G(P, T)/G_0 \]

\[ \varepsilon_p = \left\{ \frac{1}{C_1} \exp\left[ \frac{2U_K}{kT} \left( 1 - \frac{Y_T}{Y_p} \right)^2 \right] + \frac{C_2}{Y_T} \right\}^{-1} \]

\[ Y_{A_f}(\varepsilon_p) = Y_A \left( 1 + \beta(\varepsilon_p + \varepsilon_i) \right)^n \leq Y_{\text{max}} \]

\[ Y_T \leq Y_P \]

VISAR measures the surface velocity history

- An optical laser pulse is reflected from the free surface of the foil and injected into an interferometer
- The phase of the fringe is proportional to the velocity of the free surface
- Spatial resolution of the VISAR system provides data on the rear-surface motion with and without the LiF window
VISAR measurement of elastic-plastic wave breakout in Al-6061

- 195 µm Al-6061, LiF over half of the rear surface
- Omega shot #21382 - 19 J on target
The wave profile shows a pull-back at higher drive pressure

- 195 µm Al-6061, LiF over half of the rear surface
- Omega shot #21384 - 33 J on target
We use VISAR data to determine the shear modulus, bulk modulus and yield strength.

\[
\begin{align*}
\nu_e &= \frac{2P_e}{U_e \rho_0} = \frac{YU_e}{G} \\
t_e &= \frac{L_1}{u_e} + t_1 \\
t_p &= \frac{L_2}{u_p} + t_2 \\
U_e^2 &= \frac{K + \frac{4}{3}G}{\rho_0} \\
U_p^2 &= \frac{K}{\rho_e}
\end{align*}
\]
Shocks lose strength as they propagate

\[
\left(\frac{dP}{dx}\right)_{\text{shock}} = -\left(\frac{dP}{dx}\right)_{\text{rarefaction}} \left(\frac{u_{\text{material}} + c_s}{U_{\text{shock}}} - 1\right)
\]
The shear modulus, bulk modulus and yield strength affect the rise time and velocity of the VISAR data.

Change shear modulus

\[ G = 276 \text{ kb} \]
\[ 380 \text{ kb} \]

Change bulk modulus

\[ K = 722 \text{ kb} \]
\[ 849 \text{ kb} \]

Change yield strength

\[ Y = 2.9 \text{ kb} \]
\[ 5.4 \text{ kb} \]
The shock strength decreases more rapidly with increasing yield strength.

![Graph showing pressure and maximum pressure against distance (µm) with different yield strengths (0, 3.34 kb, 6.8 kb).]
We can match rise times and velocities by varying bulk modulus, shear modulus and yield strength.

\[ G = 320, \ K = 866, \ Y = 3.34 \text{ kb} \]

VISAR, shot 21382

\[ G = 320, \ K = 794, \ Y = 4.27 \text{ kb} \]

VISAR, shot 21384

Velocity (\(\mu\text{m/ns}\))

Strain rate = 4\times10^6

Time (ns)

Data

Simulation

Free surface

With LiF
The Steinberg-Guinan model by itself gives a spall time that is too late compared to the data.

Data
- Steinberg-Guinan
- Steinberg-Lund
- SG+Steinberg-Tipton failure model

\[ \varepsilon_{\text{max}} = 0.25, \quad (\rho/\rho_0)_{\text{min}} = 0.9665 \]
Steinberg-Tipton Failure Model

Damage ranges from 0 to 1

Broken material: $Y_b < P$, $G_b/G_0 = Y_b/Y_0$

$$\{P,G,Y\} = \text{damage}\cdot\{P_0,G_0,Y_0\} + (1-\text{damage})\cdot\{P_b,G_b,Y_b\}$$

$$\frac{d}{dt}\text{Damage} = \begin{cases} \frac{RC_s}{\Delta X_\text{zone}} & \sum_i \max(0, \frac{f_i}{f_{\text{max}_i}})^2 > 1 \\ 0 & \sum_i \max(0, \frac{f_i}{f_{\text{max}_i}})^2 < 1 \end{cases}$$

$$C_s = \sqrt{\frac{4G_0}{3\rho}}$$

$$f_i = \{\text{eps}, \rho/\rho_0-1, P, \sigma, \Delta\sigma\}$$
Parameters

• Steinberg-Guinan
  • $p_{\text{min}} = -30 \text{ kb}$
  • $\rho / \rho_0 = 0.9665$
  • $K = 940 \text{ kb}$
  • $G_0 = 325 \text{ kb}$
  • $Y = 3.335 \text{ kb}$
  • $\epsilon_{\text{max}} = 2.0$

• Steinberg-Tipton
  • $\rho / \rho_0 - 1 = -.0335$
  • $\epsilon = .25$
  • $R = 10^{20}$

• Steinberg-Lund
  • $Y = 1.5 \text{ kb}$
  • $c_1 = .71$
  • $c_2 = .12$
  • $u_k = .31$
  • $y_{\text{prl}} = 1.9 \text{ kb}$
Dynamic x-ray diffraction measures density and crystal structure

- In situ x-ray diffraction allows us to probe the material state by providing information on the lattice under compression
- Technique applied on laser experiments at Nova and elsewhere (Janus, Vulcan, Trident, OMEGA) and powder and gas gun facilities

Diffraction from shock compressed Si has been demonstrated on Nova

- Low intensity square laser pulse generates a single shock drive
- Displacement of the diffraction signal indicates a compression of the lattice spacing

![Graph showing lattice compression and time vs. shock breakout for Si (111) at 130 kbar and 320 kbar. The graph displays the change in lattice compression, $d/d_0$, over time (ns rel. shock breakout).]
Diffraction from orthogonal lattice planes provides information on the transition to plasticity

- Simultaneous measurements are made of compression of orthogonal lattice planes
- Shock compression above the HEL for Si and Cu show very different behavior on the ns time scale\(^1\)
  - Si responds uniaxially
  - Cu shows plastic deformation

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Simultaneous measurements of orthogonal planes indicate Si responds uniaxially on a ns time scale

- Si shock compressed along (400); probed along (400), (040)
- $P = 115-135$ kbar; HEL = 84 kbar, 40 $\mu$m thick Si
- Simultaneous measurements of Bragg and Laue diffraction

1-D compression in Si is due to high Peierls barrier
X-ray diffraction of 40 \( \mu \text{m} \) Si shows density features that vary with drive temperature.
Molecular dynamic simulations show that the Si longitudinal stiffness increases with pressure.

\[ C_{11} = K + \frac{4}{3} G = pr + 1650 \]

Simulation done by D. J. Roundy
The density structure depends in a complicated way on the drive temperature.

**Temperature (eV)**

- Time (ns)
  - 0
  - 2
  - 4

**Mass per density interval**

- Density (g/cc)
  - 2.4
  - 2.5
  - 2.6
By increasing the drive, we can match part of the data.
We have recovered samples to study the residual effects due to these high strain rate laser experiments.

- Single crystal Cu samples were shocked by direct laser irradiation and captured in a foam-filled cavity.
- Preliminary tests done at OMEGA; shock pressure is >1 Mbar, decays to ~50 kbar at the rear surface.
We see spall on a Cu sample driven by Janus
Shock strength falls roughly as distance$^{-3/4}$ in Cu

**Power (GW)**

- Time (ns)

**Maximum pressure (kb)**

- z (µm)

Strain rate decreases from $7 \times 10^7$ to $9 \times 10^5$
Summary and future work

• VISAR provides free surface velocity history
  • Gives shear modulus, bulk modulus and yield strength
  • Gives information on fracture model and spall
• X-ray diffraction provides information about lattice deformation

• Future work
  • Correlate VISAR with x-ray diffraction
  • Relate VISAR with post-shock recovery and residual deformation of structure