A General Buoyancy-Drag Model for the Evolution of the Rayleigh-Taylor and Richtmyer-Meshkov Instabilities

Y. Elbaz, Y. Srebro, O. Sadot and D. Shvarts

Nuclear Research Center - Negev, Israel.
Ben-Gurion University, Beer-Sheva, Israel.
Abstract

The growth of a single-mode perturbation is described by a buoyancy-drag equation, which describes all instability stages (linear, non-linear and asymptotic) at time-dependant Atwood number and acceleration profile. The evolution of a multi-mode spectrum of perturbations from a short wavelength random noise is described using a single characteristic wavelength. The temporal evolution of this wavelength allows the description of both the linear stage and the late time self-similar behavior. The model includes additional effects, such as shock compression and spherical convergence. Model results are compared to full 2D numerical simulations and shock-tube experiments of random perturbations, studying the various stages of the evolution.
Ideal Model Requirements

• Calculate mix region for:
  - general acceleration profile (RT and RM).
  - all instability stages (linear, early nonlinear, asymptotic)
  - general geometry (planar, cylindrical, spherical)
  - compressibility and coupling to 1D flow.
  - ablation.

• Describe internal structure of mixing zone:
  - density, temperature and pressure of every material.
  - degree of mixing.

• Feedback to 1D simulation:
  - material flow.
Definitions

Atwood number

$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$

$k = \frac{2\pi}{\lambda}$
Layzer model

(2D) \[ \frac{du_B}{dt} = \left( \frac{1 - E}{2 + E} \right) \cdot g(t) - \left( \frac{6\pi}{2 + E} \right) \cdot \frac{u_B^2}{\lambda} \quad , \quad E = e^{-3kh_B} \]

(3D) \[ \frac{du_B}{dt} = \left( \frac{1 - E}{1 + E} \right) \cdot g(t) - \left( \frac{2\pi}{1 + E} \right) \cdot \frac{u_B^2}{\lambda} \quad , \quad E = e^{-2kh_B} \]

- Single mode (periodic array of bubbles and spikes).
- Describes all instability stages.
- Valid for a general acceleration profile.
- Limited to A=1.
Buoyancy-drag equations

\[
\begin{align*}
(\rho_1 + C_a \rho_2) \frac{du_B}{dt} &= (\rho_2 - \rho_1) \cdot g(t) - \frac{C_d}{\lambda} \rho_2 \cdot u_B^2 \\
(\rho_2 + C_a \rho_1) \frac{du_S}{dt} &= (\rho_2 - \rho_1) \cdot g(t) - \frac{C_d}{\lambda} \rho_1 \cdot u_S^2
\end{align*}
\]

• Single mode (periodic array of bubbles and spikes).
• Describes only asymptotic stage.
• Valid for a general acceleration profile.
• Valid for every \( A \).
New model for single-mode perturbation

• We combine Layzer model with buoyancy-drag equations.

• $C_a$, $C_d$, $C_e$ are determined from Layzer model for $A=1$, and assumed to be Atwood independent.

\[
\left[ (C_a \cdot E(t) + 1)\rho_1 + (C_a + E(t))\rho_2 \right] \frac{du_B}{dt} = \\
(1 - E(t)) \cdot (\rho_2 - \rho_1) \cdot g(t) - \frac{C_d}{\lambda} \rho_2 \cdot u_B^2
\]

\[
\left( E(t) = e^{-C_e \cdot k \cdot h_B} \right)
\]
Multimode evolution

Mixing fronts (bubbles and spikes) are described by one characteristic wavelength: $<\lambda> = <\lambda_{BUB}>$.

- Linear stage: $\frac{d<\lambda>}{dt} = 0$

- Asymptotic self-similar behavior:

  $$\frac{h_B}{<\lambda>} = b(A) \quad \Rightarrow \quad \frac{d<\lambda>}{dt} = \frac{u_B}{b(A)}$$

- Transition from linear to asymptotic is at:

  $$h_B = <\lambda_0> \cdot b(A)$$
Model properties

- Linear stage:
  reproduces theoretical result (first order):
  \[ \ddot{h}(t) = A k g h(t) \]
- Early nonlinear stage:
  for \( A \to 1 \), correct to second order (Layzer model)
- Asymptotic stage:
  buoyancy-drag equation for all \( A \).

Limited to planar geometry and incompressible flow.
1D Hydrodynamic coupling

The dynamic front equation is solved coupled to the 1D lagrangian motion:

- Change in Atwood number:

\[ \rho_i = \frac{\int_{h_{1d}}^{h_i} \rho_i Vdx}{\int_{h_{1d}}^{h_i} Vdx} \quad i = 1, 2 \]

- 1D Lagrangian “drift” of the mixing zone boundaries:

\[ u_B \rightarrow u_B + U_{1d}(h_B) \]
\[ u_S \rightarrow u_S + U_{1d}(h_S) \]
Corrections required for non-planar geometry

Non-planar geometry introduces two effects:

- change in amplitude due to 1D motion (Bell-Plesset)
  - included in 1D coupling to lagrangian flow.
- Change in wavelength (conservation of wavenumber, $\ell = \lambda/R$).
  - geometric term added to wavelength equation:

$$\frac{d\langle \lambda \rangle}{dt} \to \frac{d\langle \lambda \rangle}{dt} + \left( \frac{d\langle \lambda \rangle}{dt} \right)_\text{geometry}$$

$$\left( \frac{d\langle \lambda \rangle}{dt} \right)_\text{geometry} = \langle \lambda(t) \rangle \frac{U_{1d}(t)}{R_{1d}(t)}$$
Shock tube experiments

Mach=1.2

50KHz Pulsed Nd:YAG Laser 532nm

Inlet

Compressed air

endwall

thick mylar membrane

piezoelectric transducers

shutter

delay system

oscilloscope

knife edge

thin membrane

high-speed camera

mirror

Test section

End Wall

SF₆

C.S

S.W

R.W

Air

[ms]

[mm]
Experimental results
(random initial conditions)

Incident shock

contact surface

reflected shock

refraction wave

air

SF₆

0.26ms

0.43ms

0.65ms

0.76ms

0.92ms

1.25ms

1.53ms

1.75ms

1.97ms

2.19ms
2D numerical simulations

shock wave

end wall

t=0.1ms

density [gr/cm³]

Air SF₆

shock wave

reflected shock

t=0.5ms

t=1.5ms

t=1.8ms

t=2.2ms

t=3.0ms
Good agreement between mix model and 2D simulation

- re-shock
- Rarefaction
- Bubble front
- 1D interface
- Spike front

- 2D Compressible Simulation
- Theoretical Model
Model agrees with experimental results

mix region [cm]
Summary

- Layzer model and buoyancy-drag equation have been combined to describe all instability stages for all Atwood numbers and a general acceleration profile.

- Multi-mode spectrum is described by one characteristic wavelength.

- 1D compressibility and scale change effects are introduced through Lagrangian “drift” of the mixing zone boundaries and by time dependant Atwood number.

- Model results have been compared to experiments and to full 2D numerical simulations.

- Non-planar geometry may be introduced by modifying characteristic wavelength.