NON LINEAR RT and RM
SINGLE MODES (ANALYTIC)

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OUTLINE

- Assumptions + Initial 2D/3D Perturbation
- Analytical Model
- Structure of the Interface
- Shape Factor
- Comp. Other Works.
ASSUMPTIONS:

- \( g = f(t) \) arbitrary
  \[
  \begin{align*}
  g &= \text{cst} \quad \text{RTI} \\
  g &= \delta(t) \quad \text{RMI}
  \end{align*}
  \]

- INCOMPRESSIBLE
  (compressible - linear)

- POTENTIAL FLOW
  \[
  \nabla^2 = \nabla \phi \quad \Rightarrow \quad \nabla^2 \phi = 0
  \]

- \( A_e = \frac{\rho H - \rho L}{\rho H + \rho L} \rightarrow 1 \)

- SINGLE MODE STUDY
  2D and 3D

- INFLUENCE OF THE LATTICE (PERIODIC)
  B3 (J6), B4 (J4), B6 (J3)
Perturbations

- 2D  
  Ripples

- 3D B4  
  3D Periodic Latt.

- 3D B3/B6  
  B4 - Egg Box
  B3/B6 Egg - Box
EQUATIONS

- INTERFACE
  \[ \begin{align*}
  &2D \quad z = \gamma(x, t) \\
  &3D \quad z = \gamma(x, y, t)
  \end{align*} \]

- PARTICLES OF FLUID ATTACHED TO THE INTERFACE
  \[ \frac{\partial \eta}{\partial t} = v_y - v_x \frac{\partial \eta}{\partial x} - v_y \frac{\partial \eta}{\partial y} \quad \text{3D} \]

- BERNOULLI EQUATION
  \[ \frac{\partial \phi}{\partial t} = \frac{1}{2} (\mathbf{v})^2 + g(t) \eta \]
MODEL

Extension of LAYZER Approach


Top of a bubble:

1) \( \eta(x,t) = \eta_0(t) + \frac{1}{2} \eta_1(t) x^2 + \ldots \)

2) monomode potential

\( \phi(x,3,t) \sim a(t)(\cos kx)e^{-kz} \)

Our work:

2D: \[ \phi(x,3,t) = \sum_{n=1}^{N} \phi_n(t) \cos(nx) e^{-nz} \]

\( \eta = \eta_0 + \sum_{n=1}^{N} K_n(t) \frac{x^{2n}}{(2n)!} \)

N-TRUNCATION NUMBER

(Upto to 6 for J, for the first time)

3D \[
\left\{
\begin{align*}
\phi & \sim \sum_{n,m} \phi_{nm}(t) \cos nx \cos my e^{-q_{nm}z} \\
\eta & \sim \sum_{n,m} K_{nm}(t) \frac{x^{2n} y^{2m}}{(2n)! (2m)!}
\end{align*}
\right.
\]
STAGNATION POINTS

- TOP OF BUBBLES
- TIP OF JETS
- SADDLE POINTS

for the first time
2D – Nonlinear Analytical Study (parallel ripples)

2D – Nonlinear Analytical Study (egg box type initial perturbation)

Detail of a Local System « Bubble+Jet+Saddle Point »
BLOW-UP
$B + S + J$

$h_B \approx 0.7$
$l_J \approx 1.4$

$l_J \approx 2h_B$  Highly NL

INTERFACE $\xi = 0$

$S$ in this plane
Numerical Simulation of the Rayleigh-Taylor Instability (B6, B4, B3)

Numerical Simulation of the Richtmyer-Meshkov Instability (B6, B4, B3)
\[ \frac{\Gamma}{s} = \frac{d(B-X)}{d(B-J)} \]

- **Num. Sim. (Oparin)**
  - **Theory** $N=1$
  - **Theory** $N=2$

**Good tendency** ($\Gamma$ is global)
RMI
\[ \Gamma = \frac{d(B-S)}{d(B-J)} \]

N=2 Good agreement.
Rather good for asymptotic values.
COMP. WNL APPROACH $q_{a t} \leq 1$

- RMI -

Bubble Curvat.: $\sigma = 3, 4$
$N = 4, 5$
$\sigma = 4$

Bub. decel.: $\sigma = 4$
$N = 4, 5$

Jet. accel.: $\sigma = 3, 4$
$N = 4, 5, 6$

- WNL - Pade - iTOB

RIGHT MODELING OF THE LINEAR PHASES.

$R_b = 2.6$

$V_b = \frac{\alpha}{t}$

$\alpha = 0.60$

WNL approach can only follow bubble curvature up to 40% of the asymptotic value.
CONCLUSION

- Analytical NL asymptotic stages (+ transients)
- Comparisons with 2D/3D codes with compute hydro in such NL regimes