The Dependence of the Shock Induced Richtmyer-Meshkov Instability on Dimensionality and Density Ratio

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Buoyancy - Drag Consideration for the Single Mode Case

Newton's second law (for the bubble):

\[
\left(\rho_1 \cdot \mathbf{V} + \mathbf{C_a} \cdot \mathbf{V} \cdot \rho_h \right) \frac{d\mathbf{U}}{dt} = (\rho_h - \rho_1) \mathbf{V} \cdot \mathbf{g} - C_d \cdot \rho_h \cdot S \cdot U^2
\]

For the bubble:

\[
(\rho_1 + C_a \rho_h) \dot{U} = (\rho_h - \rho_1) \cdot \mathbf{g} - \frac{C_d}{\lambda} \rho_h \cdot U^2
\]

For the spike: \(\rho_h \Leftrightarrow \rho_1\)

\[
(\rho_h + C_a \rho_1) \dot{U} = (\rho_1 - \rho_h) \cdot \mathbf{g} - \frac{C_d}{\lambda} \rho_1 \cdot U^2
\]
**Linear Stage Single Mode Velocity**

Atwood Number: \[ A = \frac{\rho_H - \rho_L}{\rho_H + \rho_L} \]

Using: \( g(t) = U_0 \delta(t) \) and \[ U^2 \ll \frac{(\rho_h - \rho_l) \lambda \cdot g}{\rho_h \cdot C_d} \]

\[ U_{\text{linear}} = U_0 \cdot k \cdot h_0 \cdot A \quad \text{Constant velocity} \]

\[ k = \frac{2\pi}{\lambda} \]
Asymptotic Single Mode Velocities:

\[(\rho_1 + C_a \rho_h) \dot{U} = (\rho_h - \rho_1) \cdot g - \frac{C_d}{\lambda} \rho_h \cdot U^2\]

\[g = 0\]

For the bubble:

\[U_B = \left( \frac{1 - A}{1 + A} + C_a \right) \cdot \frac{\lambda}{C_d} \cdot \frac{\lambda}{t}\]

For the spike: \(\rho_h \Leftrightarrow \rho_1\)

\[U_S = \begin{cases} 
(A + 1)/(A - 1) \cdot (3 - A)/(3 + A) \cdot U_B & \text{for 2D} \\
(A + 1)/(A - 1) \cdot U_B & \text{for 3D} 
\end{cases}\]

Using *:

<table>
<thead>
<tr>
<th></th>
<th>2D: (C_d = 6\pi)</th>
<th>2D: (C_a = 2)</th>
<th>3D: (C_d = 2\pi)</th>
<th>3D: (C_a = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D:</td>
<td>(\frac{1}{2\pi} \cdot \frac{\lambda}{t})</td>
<td>(\frac{1}{3\pi} \cdot \frac{\lambda}{t})</td>
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* Layzer (1955);
Hecht et al. (1994).
Experimental Apparatus

- End Wall
- Thin Membrane
- Thick Mylar Membrane
- Inlet
- Compressed Air
- Driven Section
- Test Section
- 80mm x 80mm
- 7.5m
Experimental Apparatus

- Oscilloscope
- Camera
- Piezoelectric transducers
- Thin membrane
- Thick mylar membrane
- Mirror
- Shutter
- Delay system
- 10KHz Pulsed Copper Vapor Laser (511nm)
- Driver section
- Test section
- End-wall
- Knife
- Compressed air
- Inlet
Experimental Setup - The Membrane

- **SF₆/Ar**
- **C.S**
- **S.W**
- **Air**
2D - Low and High Atwood number Experiments

A=0.2 (Air to Ar) $\lambda=26\text{mm}$

A=0.7 (Air to SF$_6$) $\lambda=26\text{mm}$
Experiment vs. Model for the Atwood Number Dependence

\[ U_{\text{asy}} = C \cdot \frac{\lambda}{t} \]

<table>
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<th>(A=0.2)</th>
<th>(A=0.7)</th>
</tr>
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<tr>
<td>(1/C)</td>
<td>2(\pi)</td>
<td>3(\pi)</td>
</tr>
<tr>
<td>(U_b/U_s)</td>
<td>0.77</td>
<td>0.28</td>
</tr>
</tbody>
</table>

\(c = \frac{1}{2\pi}\)
\(c = \frac{1}{3\pi}\)
**Dimensionality Investigation:**

$$U^{(asy.)}_{B_{A=1}} = \begin{array}{c|c}
2D: & \frac{1}{3\pi} \cdot \frac{\lambda}{t} \\
3D: & \frac{1}{2\pi} \cdot \frac{\lambda}{t} 
\end{array}$$

**The Effective Wavelength**

In 3D:
- $$k_i = \frac{2\pi}{\lambda_i}$$
- $$i = x, y$$
- $$|k| = \sqrt{k_x^2 + k_y^2}$$
- $$\overline{\lambda} = \frac{2\pi}{|k|}$$

In 2D:
- $$k = \frac{2\pi}{\lambda}$$
Experimental Setup - The Membrane - 2D/3D case

Periodic Initial Conditions:

2D

SF₆/Ar

C.S

S.W

air

3D

SF₆

C.S

S.W

air

(Bubble-3D Spike-2D)
Results of 2D vs. 3D Experiments

(M=1.20 Air to SF₆)

3D
$\bar{\lambda} = 28\text{mm}$
($\lambda_{x,y}=40\text{mm}$)

2D
$\lambda=26\text{mm}$

3D
$\bar{\lambda} = 57\text{mm}$
($\lambda_{x,y}=80\text{mm}$)

2D
$\lambda=80\text{mm}$
Dimensionality Dependence Results - Bubbles

Linear stage
\( k_{3D} \approx 0.1 \text{mm}^{-1} \)

Nonlinear stage
\( k_{2D} \approx k_{3D} \approx 0.2 \text{mm}^{-1} \)

* [Sadot et al. (1998)]
Completing the picture  3D Spikes

3D Spike, 2D Bubble

- 3D Spike Sim.
- 3D Spike Exp.
- 3D Bub Sim.
- 3D Bub Exp.

graph showing time [msec] vs. height [mm]

images of 3D Spike at different times:
- t=0.8mS
- t=1.1mS
- t=1.4mS
- t=1.7mS
Bubble Competition 2D

M=1.2  A=0.67  \lambda_1=10\text{mm}  \lambda_2=25\text{mm}  

[Sadot et al. (1998)]
Bubble Competition in 3D - Experiment

(M=1.20 Air to SF$_6$)
Bubble Competition in 3D

- Experiment
- Same bubbles without competition
- Simulation

The graph shows the height of bubbles over time, with data points and curves indicating experimental results and simulations for bubbles in competition and without competition.
Experiments were performed to investigate the dependence of the RM instability (Bubbles and Spikes) on the dimensionality and the Atwood number. Good agreement was found between the results of the experiments and the prediction of the model as well as with the results of full simulations. Bubble Competition was shown to exist also in the 3D case.

<table>
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<th>Dimensional effect</th>
<th>Inertia of the bubble</th>
<th>2D:</th>
<th>3D:</th>
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\[ U_B =\]
Summary: Implications for Multimode Theory for the RM Instability


Two key elements govern the multimode bubble front evolution:

1) The asymptotic velocity of a single bubble. \( \leq \) Shown as depended on Dimensionality and Atwood.
2) The merger rate between two neighboring bubbles. \( \leq \) Merging was observed in 3D as well.

\[
\begin{align*}
    h_B &= a_0 \cdot t^{\theta_B(2D/3D,A)} \\
    h_S &= b_0 \cdot t^{\theta_s(2D/3D,A)}
\end{align*}
\]

Present work verifies the:
2D/3D and Atwood Number Dependence of Single-Mode Bubble/Spike

\[
\theta_{2D} \cong 0.4 \neq \theta_{3D} \cong 0.2
\]