Suppression of the Richtmyer-Meshkov instability in the presence of a magnetic field

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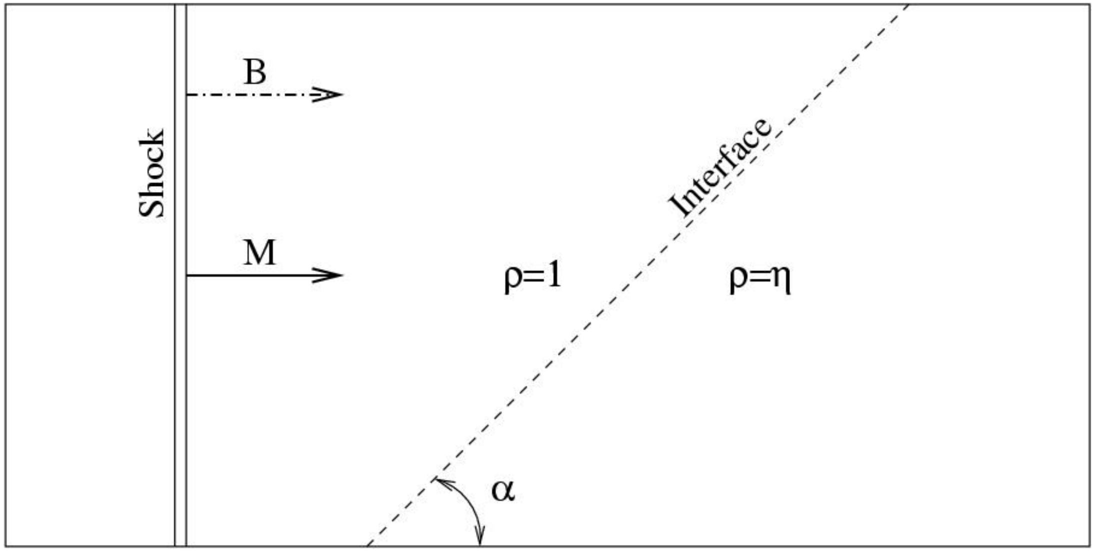
Outline

- Motivation; MHD shock refraction simulations of Samtaney (2003)
- Analysis; shock refraction at a density interface; formulation & solution technique
- Comparison with simulation results
- Transitions in solution type with decreasing magnetic field strength
- Approach to the hydrodynamic triple-point: a singular limit
- 2D single-mode interface RM simulations
- Conclusions

Introduction

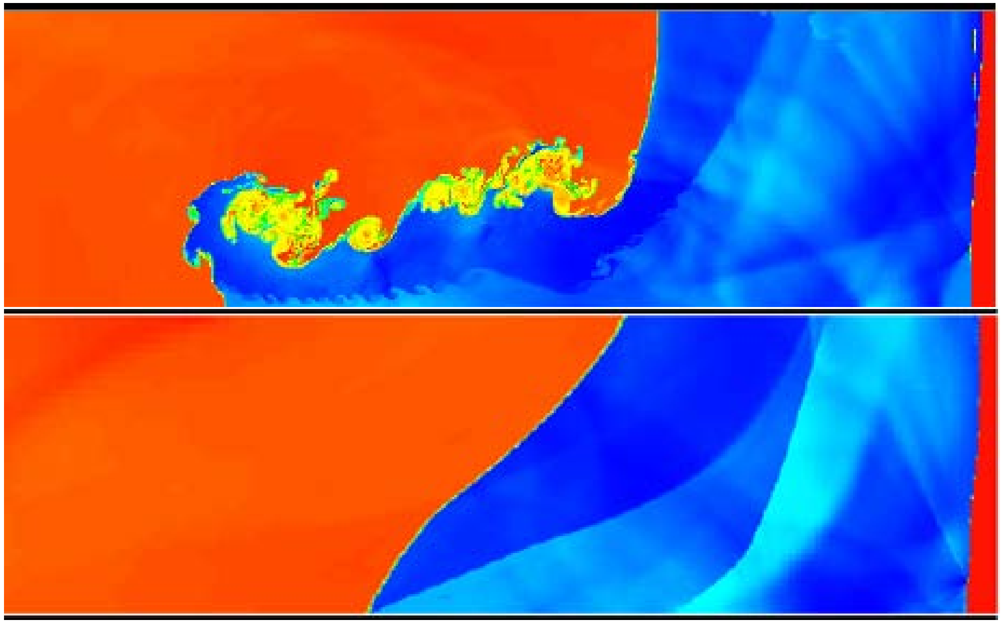
- In ideal MHD, Samtaney (Phys. Fluids, 2003) numerically demonstrated that magnetic fields suppress RM instability
- Flow studied: shock interacting with oblique planar contact discontinuity separating conducting fluids of different densities
- Flow characterized by:
  - *M:* incident shock sonic Mach number
  - η: Density ratio
  - α: Angle between incident shock normal and contact
  - $\beta^{-1}=B^2/2\mu_0 p_0$: Non-dim strength of the applied magnetic field



Introduction

$\beta^{-1} = 0.0$

$\beta^{-1} = 0.5$

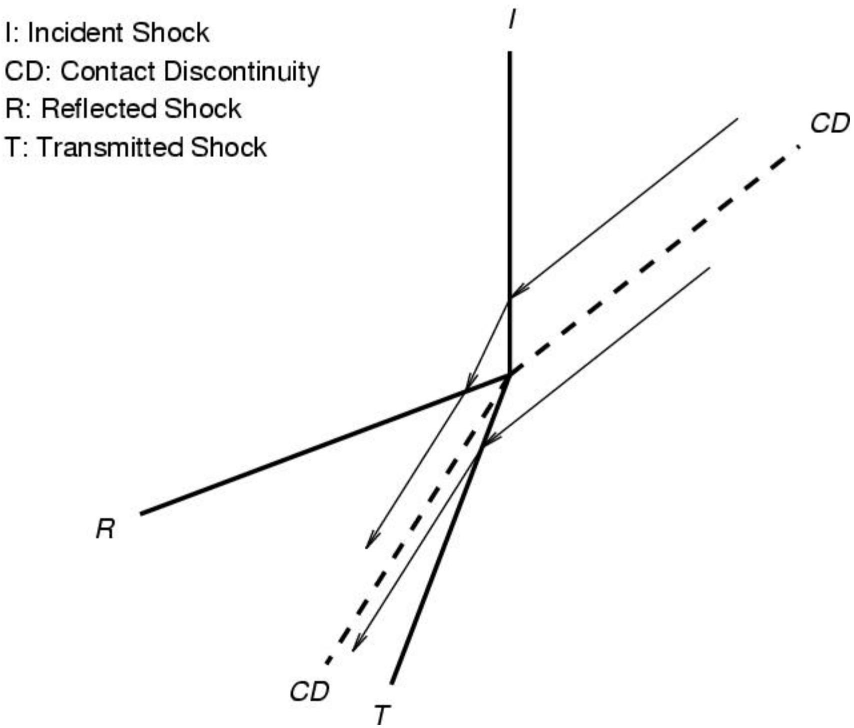


Density fields from Samtaney's Richtmyer-Meshkov simulations with M=2, η=3, $\alpha=\pi/4$, γ=1.4 and β^{-1}=0 (top) or β^{-1}=0.5 (bottom)

- $\beta^{-1} = 0.0$: *CD* is a vortex layer that rolls up
- $\beta^{-1} = 0.5$: *CD* remains smooth & no roll-up observed

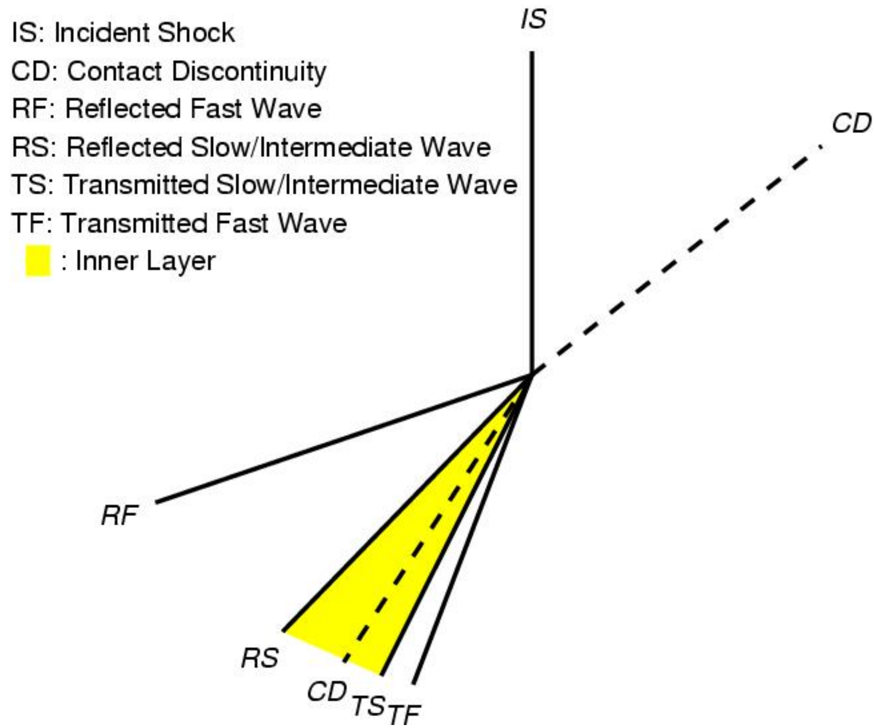
This can be understood by examining how the shock refraction process changes with the application of a magnetic field

β^{-1}=0 shock refraction process:



- *CD* is Kelvin-Helmholtz unstable

β^{-1}=0.5 shock refraction process:



- Shear across *inner layer*, but *CD* vorticity free, stable

- Wave configuration is referred to as a *quintuple-point*
- MHD shocks support tangential velocity jumps; vorticity deposited on *RS* & *TS* instead of contact surface

Present work:

- Demonstrate that the quintuple-point is an entropy-satisfying weak solution of the equations of ideal MHD
- Investigate how solution converges to the hydrodynamic triple-point as *B* tends to zero
- Ideal MHD simulations of canonical RM flow: initial interface perturbation a single-mode sinusoid

Formulation: Governing Equations

- Steady ideal MHD equations for quasi-neutral conducting fluid
- Viscosity, thermal conductivity, Hall effect, and electrical resistivity neglected

$$\nabla \cdot (\rho \mathbf{u}) = 0 \, ,$$

$$\rho \left(\mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla p + \frac{1}{\mu_0} \left(\nabla \times \mathbf{B} \right) \times \mathbf{B} \, ,$$

$$\rho \left(\mathbf{u} \cdot \nabla \right) e_T = \frac{1}{\mu_0} \left(\nabla \times \mathbf{B} \right) \times \mathbf{B} \cdot \mathbf{u} \, ,$$

$$\nabla \cdot \mathbf{B} = 0 \, ,$$

$$\nabla \times \left(\mathbf{u} \times \mathbf{B} \right) = 0 \, .$$

- ρ is density, p is pressure, $\boldsymbol{u}$ is velocity, $\boldsymbol{B}$ is the magnetic field, μ_0 is magnetic permeability, h is enthalpy, & $e_T = h + \boldsymbol{u} \bullet \boldsymbol{u}/2$
- Assume plasma behaves as an ideal gas with constant specific heats

Formulation: Rankine-Hugoniot Relations

- MHD Rankine-Hugoniot (RH) relations govern discontinuous weak solutions to ideal MHD equations
- Coplanar, shock stationary RH relations:

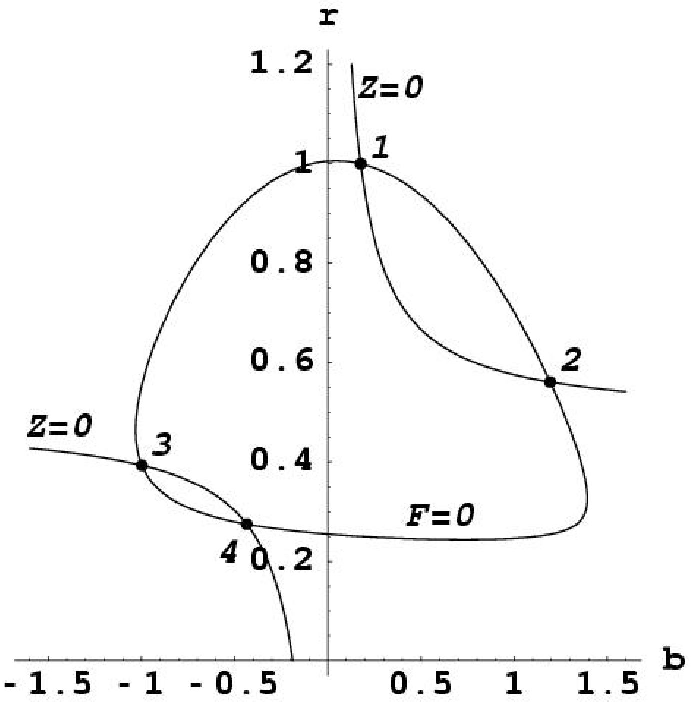
$$[\rho u_n] = 0 \ ,$$
$$\left[\rho u_n^2 + p + \frac{B_t^2}{2\mu_0}\right] = 0 \ ,$$
$$\left[\rho u_n u_t - \frac{1}{\mu_0} B_n B_t\right] = 0 \ ,$$
$$\left[\frac{\rho u_n}{2}\left(u_n^2 + u_t^2\right) + \frac{\gamma u_n p}{\gamma - 1} + \frac{1}{\mu_0} u_n B_t^2 - \frac{1}{\mu_0} u_t B_n B_t\right] = 0 \ ,$$
$$[u_n B_t - u_t B_n] = 0 \ .$$

- *n* & *t* denote vector components normal and tangential to shock
- $[A] \equiv A_2 - A_1$ denotes difference in A between states upstream (1) and downstream (2) of shock.

Formulation: Rankine-Hugoniot Relations

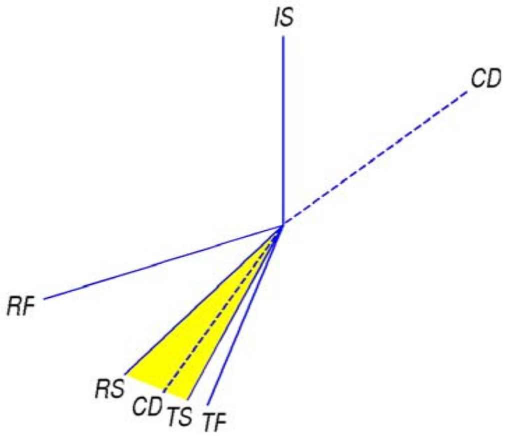
In terms of $r \equiv u_{n2} / u_{n1}$ and $b \equiv B_{t2} / B_1$, mass, momentum, and energy jump conditions reduce to the form $F(r,b) = 0$, while final jump condition can be expressed as $Z(r,b) = 0$.

- F=0 and Z=0 have up to 4 intersections labeled 1-4 in order of increasing entropy
- u_n for these states related to the fast (C_F), intermediate (C_I), and slow (C_{SL}) characteristic speeds
- Allows 6 transitions corresponding to entropy increasing shocks



Formulation: Participating Waves

- *Fast shocks*: Correspond to 1→2 transitions
- *Slow shocks*: Correspond to 3→4 transitions
- *Intermediate shocks*: Correspond to 1→3, 1→4, 2→3, and 2→4 transitions and are denoted I1-3, I1-4, I2-3, & I2-4 respectively



- *180⁰ rotational discontinuities* (RDs): These are a special case of 2→3 intermediate shocks with $r = 1$ and $u_{n1} = C_{I1}$
- *Slow-mode expansion fans*
- *Slow (C_1) compound waves*: 2→(3=4) intermediate shock followed immediately downstream by slow-mode expansion fan

Formulation: Admissible Waves

To assess the admissibility of MHD discontinuities we follow FK (Falle & Komissarov, 2001) and differentiate between:

Planar Flow: No gradients in the *z*-direction (2D-3C)

- Alfven waves exist and RDs admissible
- Intermediate shocks and compound waves inadmissible

Strongly Planar Flow: u_z and B_z also zero (2D-2C)

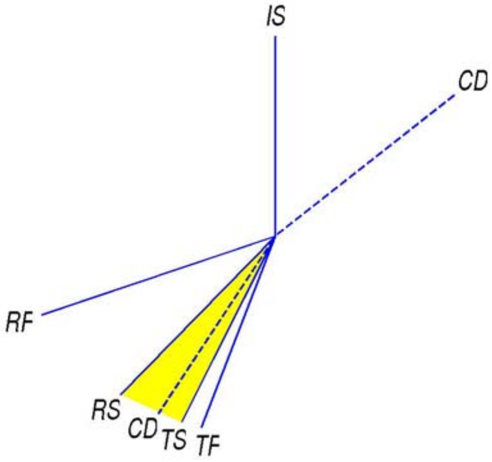
- Alfven waves do not exist (require non-zero u_z and B_z)
- $1 \rightarrow 3$, $1 \rightarrow 4$, $2 \rightarrow 4$ intermediate shocks, C_1 compound waves admissible
- RDs inadmissible as require non-zero u_z and B_z internally

Fast and slow shocks, expansions always admissible

Solution Technique

Solutions to MHD shock refraction problem found by:

- Specifying combination of waves radiating from intersection point
- Iterating on 4 unknown wave angles using secant method until *p*, ***u***, & *B* continuous across the contact discontinuity

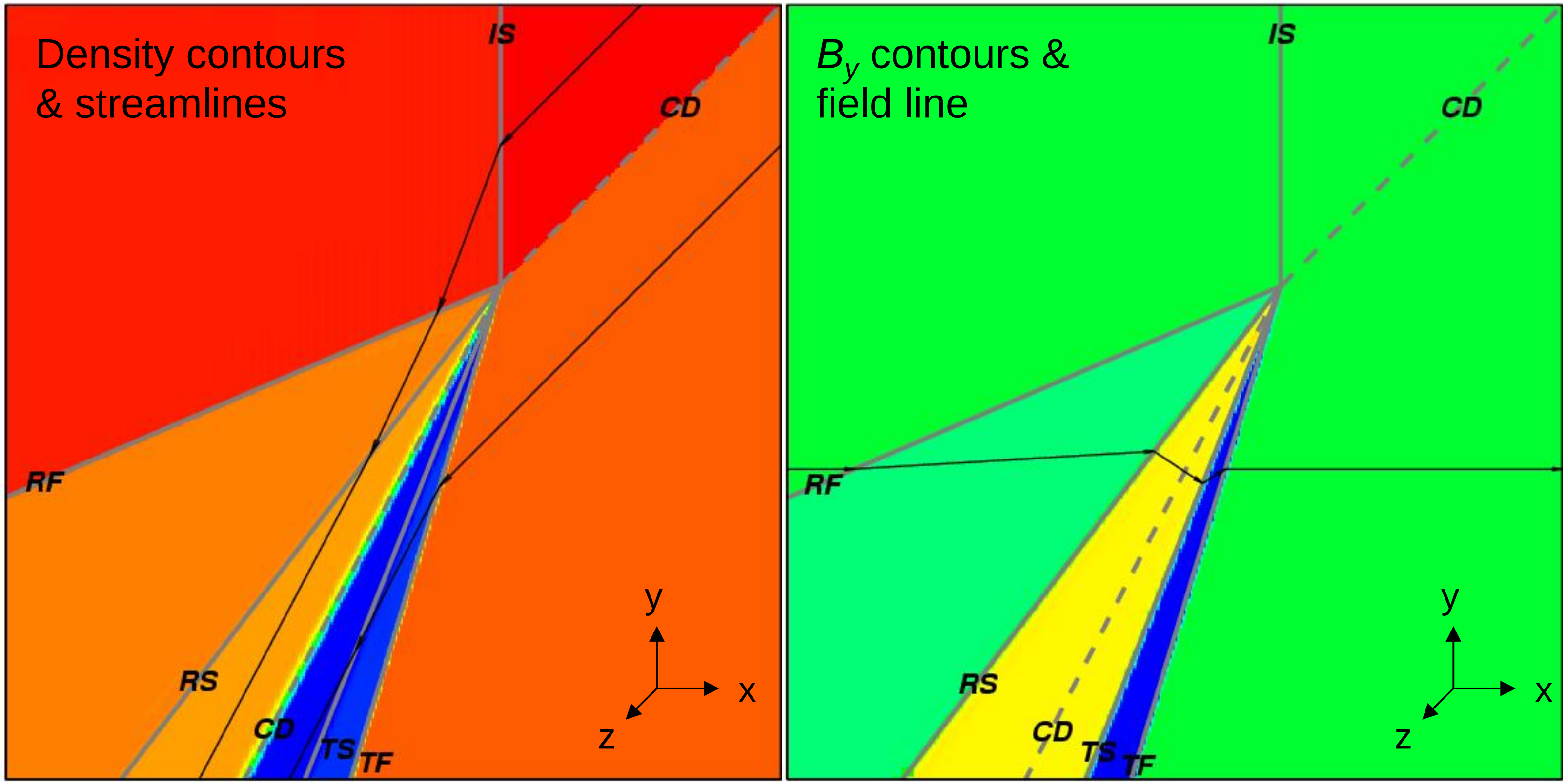


- Check consistency by ensuring angle of ***B*** matched to same tolerance

Following Torrilhon (2003) we classify our ideal solutions as:

- Regular (*r*-) solutions if all waves FK-admissible in planar system
- Irregular (*c*-) solutions if all waves FK-admissible in strongly planar system

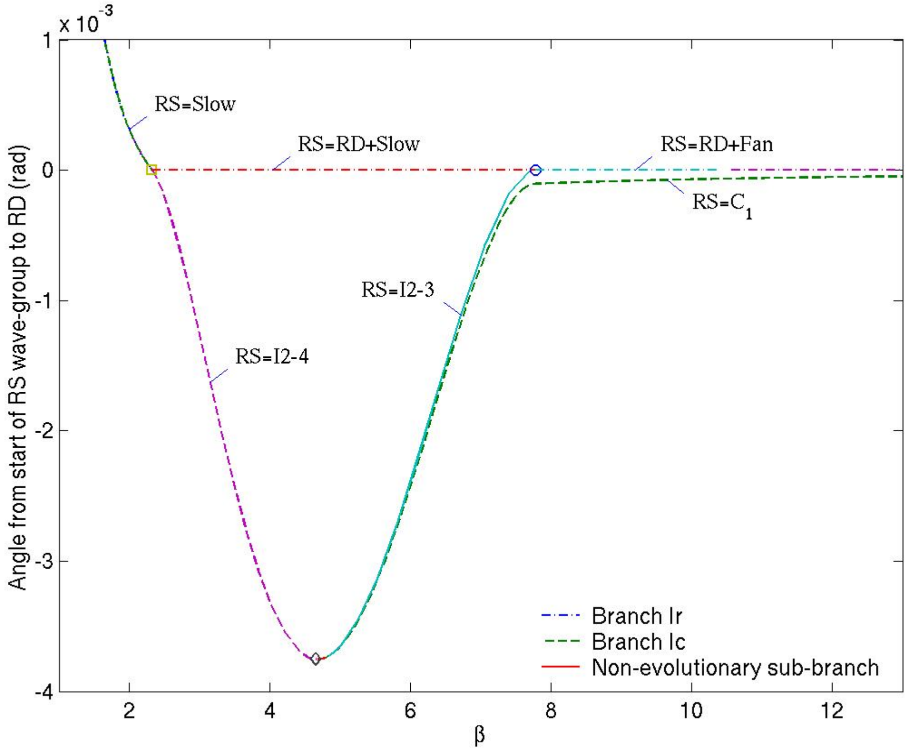
Comparison to Simulation Results (Samtaney, 2003)



- Close agreement indicates quintuple-point is an entropy-satisfying weak solution of ideal MHD equations
- It is a *c*-solution as *TS* is a I2-4 shock, an *r*-solution also exists in which *TS* is a RD followed by a slow shock

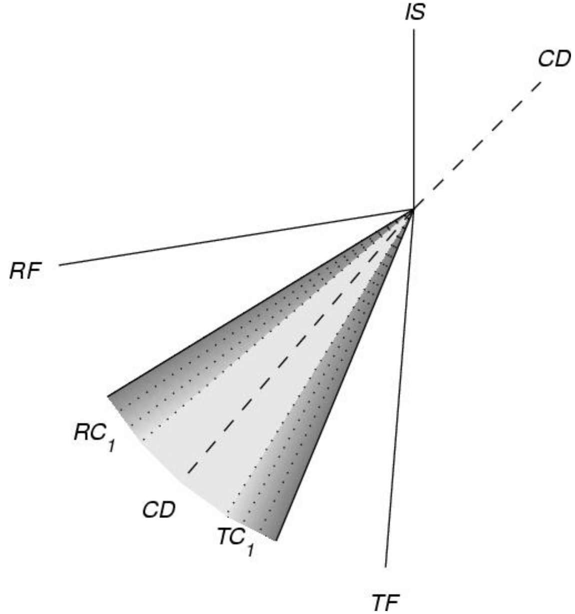
Transitions in Solution Type with Increasing β

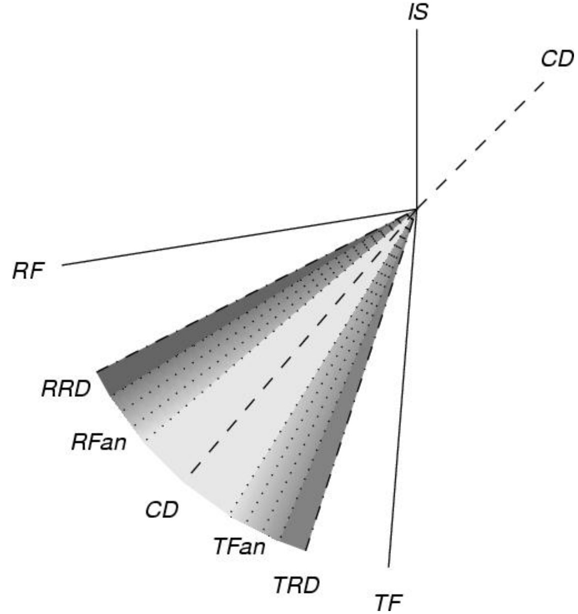
- As $\beta = 2\mu_0 p_0/B^2$ increases, both *RS* & *TS* undergo one of the following sets of transitions in wave type:
  - Slow shock → I2-4 shock → C_1 compound wave (*c*)
  - Slow shock → RD + slow shock → RD → RD + slow-mode fan (*r*)
- Solutions generally not unique
- Transitions to inadmissible waves also satisfy equations
- Leads complex solution branch structure



Singular Approach to Hydrodynamic Limit

- For β greater than all transition points, we have identified two possible flow structures that may arise from the shock refraction process:





- Quintuple-point *c*-solution consisting of 3 fast shocks and 2 C_1 compound waves
- Seven wave *r*-solution, called a *septuple-point*, with 3 fast shocks, 2 RDs, and 2 slow-mode expansion fans

Singular Approach to Hydrodynamic Limit

- $\beta \to \infty$: solutions $\to$ hydrodynamic triple-point, except shocked hydrodynamic contact replaced by an inner layer, with angular width $\propto \beta^{-1/2}$
- *B remains* finite within layer and scales like $\sqrt{(\mu_0 p)}$ as $\beta \to \infty$, thus MHD contact cannot support jump in u_t
- Necessitates presence of expansion fans, which support finite jumps in u_t, p, and B_t even though angular extents $\to 0$
- To verify findings, equations governing leading order asymptotic solution of shock refraction problem derived then solved iteratively
- Agreement between full and asymptotic solutions was excellent

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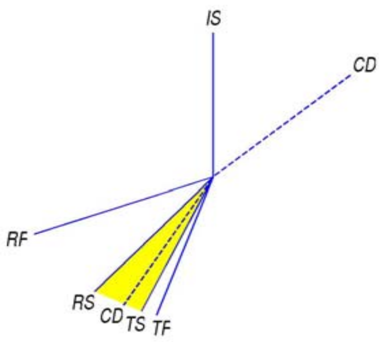

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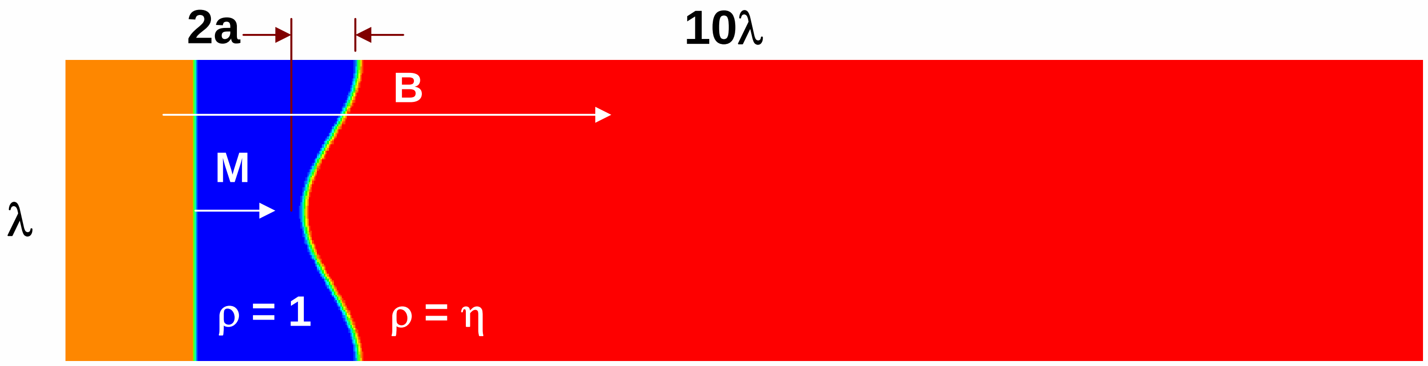
2D Single-mode RM Simulations

- Demonstrate suppression of instability for canonical flow
- Quantify growth-rate reduction due to magnetic field for simple config.
- Use theory to select simulation parameters such that only fast and slow shocks produced by shock refraction process

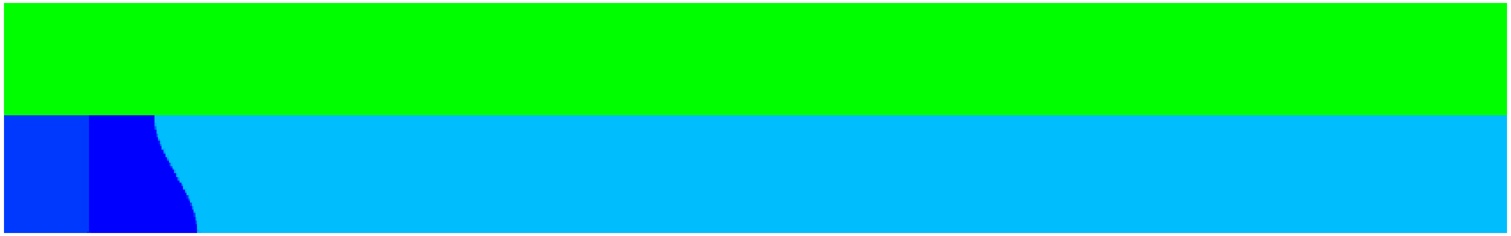
Setup:



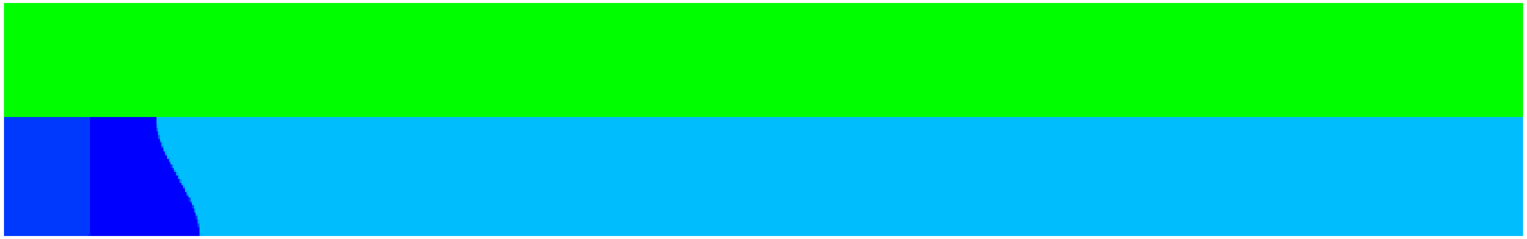
Parameters: $M = 2$ $\eta = 3$ $\gamma = 5/3$ $\lambda/a = 10$ $\beta = 2\mu_0 p/B^2 = 1$ (for $B \neq 0$)

Simulation Results

- Code (Torrilhon & Deiterding): 2nd order in space, HLLE approx Riemann solver, van Leer limiter, div**B** = 0 maintained by flux redistribution



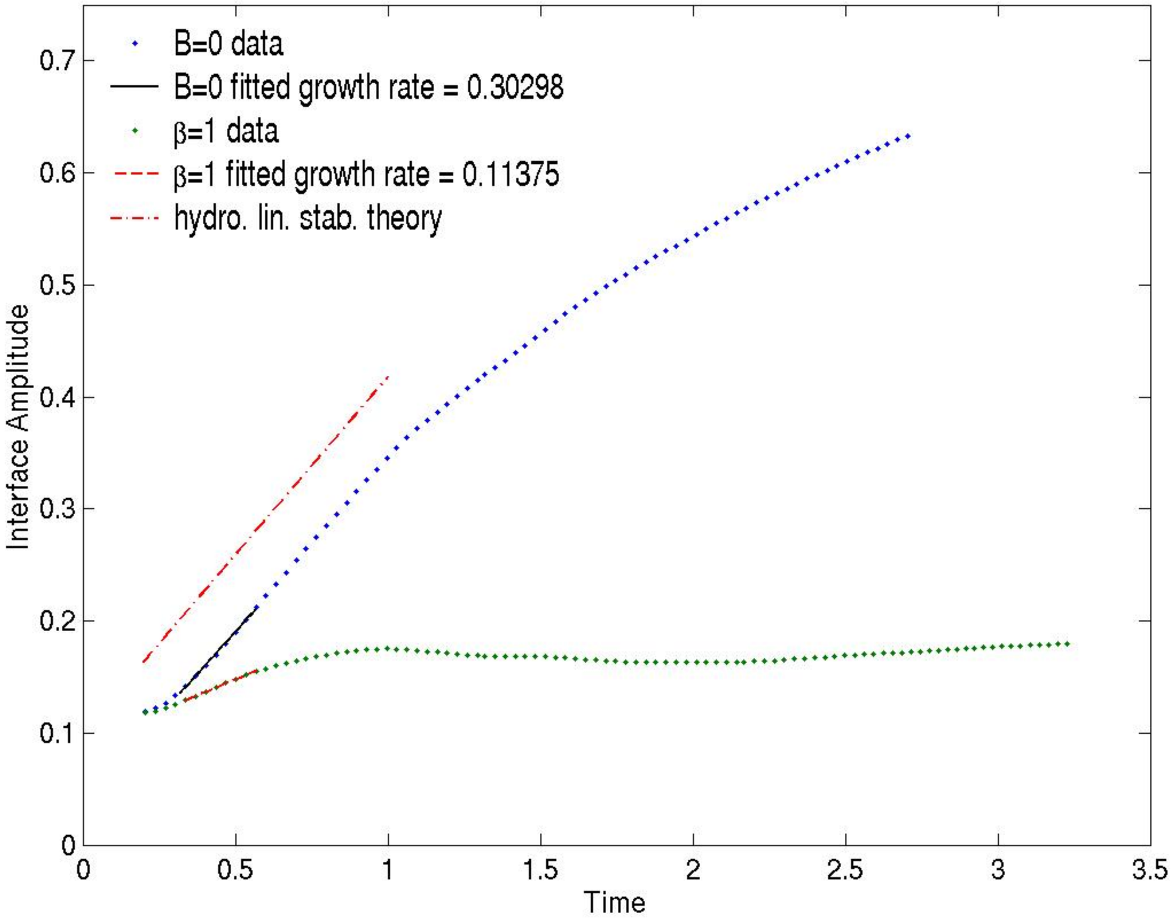
Vorticity (top) and density (bottom), no magnetic field



Vorticity (top) and density (bottom), magnetic field present

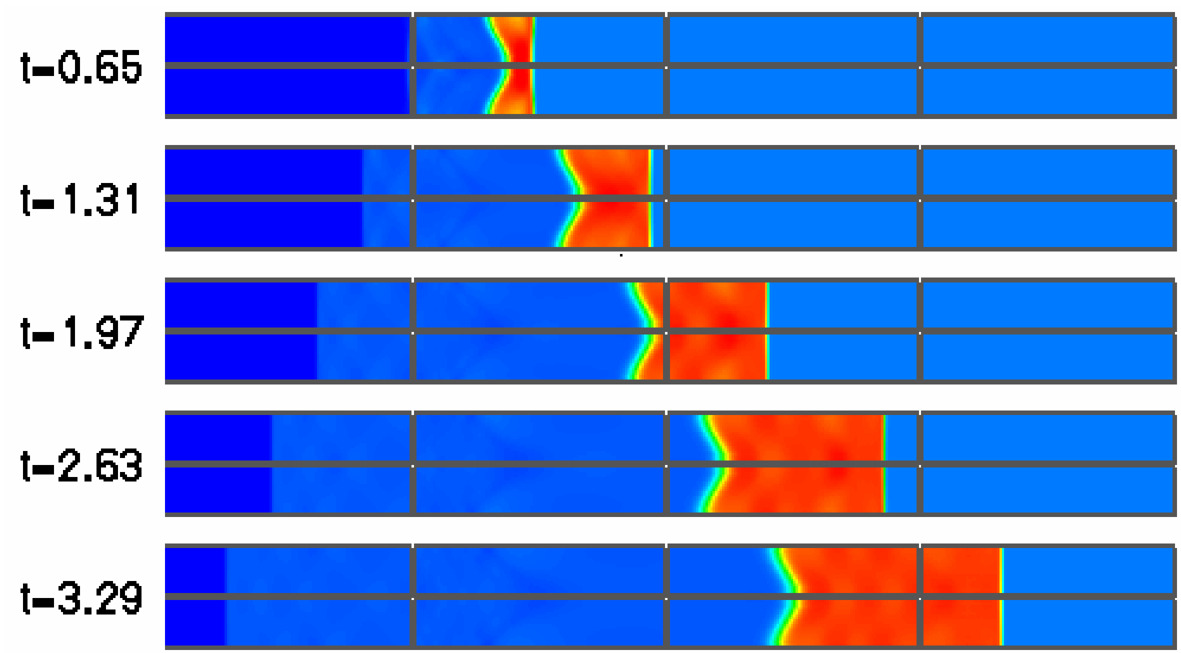
Growth Rates

- Early-time growth rate reduced by factor of ≈2.7 by presence of magnetic field
- Remaining growth due to non-uniform pressure and ***B*** fields.



3D Single-mode RM Simulations

- Same setup as 2D simulations with additional perturbation (single sinusoidal mode) in the *z*-direction
- Code (Samtaney): Roe type approx Riemann solver (8-wave formulation), unsplit upwinding method of Colella, div**B** = 0 maintained by projection method
- Demonstrates suppression of instability for canonical flow in 3D



Center-plane density fields

Conclusions

- Developed iterative procedure for determining flow structure produced by regular refraction of MHD shock at oblique planar density interface
- Reproduced quintuple-point structure seen in numerical simulations, confirming mechanism for suppressing instability is valid
- In the limit of $\beta \rightarrow \infty$:
  - solutions identified tend to the hydrodynamic triple-point
  - exception: shocked hydrodynamic contact replaced by singular structure called the inner layer
- Behavior of 2D, 3D RM flow with sinusoidally perturbed interface in agreement with prediction
- Early-time growth rate greatly reduced by presence of magnetic field in 2D RM simulations