Visualization of harmonics of 2D internal waves generated by bodies of different shapes in a stratified fluid

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Abstract: With the aid of the synthetic schlieren technique and a convenient image processing, new insight is attained into the visualization of internal waves produced by bodies of different cross-section shapes oscillating in stratified fluids. In particular, quantitative whole-field density measurements allow inferring information about the amplitude and phase of the internal waves at the fundamental frequency and at its harmonics.

Key-Words: - Internal waves, harmonics, stratified environment, visualization, image processing

1 Introduction

We deal with the internal gravity waves that propagate in a stably and uniformly stratified medium. These waves are of special interest in meteorology and oceanography since the atmosphere and ocean are stratified due to changes in temperature, composition and pressure. Gravity waves take place in stable zones of the atmosphere, for example near the tropopause, as response of the development of intense vertical motions by convection, storms or the forced rising of air parcels in mountainous regions. The undular phenomena associated with these oscillations, commonly radiated from the perturbed region, transport energy and momentum in relatively fast time scales, representing a process by which the fluid motions in different levels can be matched.

The internal waves theory is well known [1]. Basically, if a fluid element is perturbed vertically in a density stratified fluid, it experiences a buoyancy force directed to restore it to its original position. This buoyancy force, in combination with the inertia of the fluid element, acts as a simple harmonic oscillator with a frequency (the buoyancy frequency)

\[ N = \left( \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \right)^{1/2}, \]

where \( \rho \) is the fluid density, \( \rho_0 \) is a reference density, \( g \) is gravity and \( z \) is the coordinate oriented vertically upward. The oscillation in a direction at an angle \( \theta \) to the vertical reduces the restoring force, allowing the formation of waves of angular frequency \( \omega = N \cos \theta \).

The fluid motion and group velocity occurs at an angle \( \theta \) to the vertical, while the phase velocity is perpendicular to them (the sign of the vertical component of the phase velocity is opposite to that of the group velocity). Summarizing, a linearly stratified fluid \((N = \text{const})\) is therefore able to support waves of frequencies \(0 \leq \omega \leq N\).

A number of studies were devoted to the understanding of the mechanisms that are responsible for the generation, interaction and decay of the internal waves. Particularly, the analysis of the wave field produced by the vertical oscillation of a body in a stratified fluid and its comparison with the linear theory of internal gravity waves was firstly considered by Mowbray & Rarity [2]. They demonstrated theoretically and experimentally that for waves forced by an oscillating cylinder, the fluid motion was confined within a region of fluid resembling a St. Andrew's Cross. This is in sharp contrast to the flow found when the fluid is unstratified.

The process of internal gravity wave generation by a simple harmonic flow of a stably stratified fluid over an obstacle was studied by Bell [3]. The general solution obtained was successfully compared with experimental data. Attention was primarily directed to the behaviour of the solution in various limiting cases, and to estimating the flux of energy into the internal wave field. Bell [3] also found that waves are generated not only at the fundamental frequency \( \omega_0 \) but also at its harmonics.

Different experimental techniques have been used to visualize qualitatively the flow in the experimental


While schlieren has been used for many years to visualize flows containing variations in the refractive index, its application has often been limited by the cost of the optical components, the high mechanical stability needed and the difficulty in extracting quantitative information. A number of recent advances have been reported in the use of schlieren techniques to visualize the density gradient field in stratified flows. In particular, synthetic schlieren ([7], [9]) is relatively easy to use and may be scaled up to visualize large regions of the flow. It is also particularly advantageous because it can be used to make non-intrusive quantitative measurements of the full wave field.

We assume that the body motion produces only a small perturbation to the flow field that would exist in its absence. Then disturbance energy is propagated away from the source region in the form of an internal gravity wave field. In this study, we are interested in the visualization of the higher harmonics of these waves. The essential elements that define the problem are the stable density stratification and the body motion which varies harmonically with time. From the density gradient field and its time derivative it is possible to construct the perturbation fields of density and horizontal and vertical velocities. Thus, in theory, momentum and energy fluxes can be determined. Here, we examine the amplitude and phase of internal gravity waves generated by parallelepipeds of circle-, square- and rhomb-shaped cross-sections oscillating vertically, paying particular attention to the visualization of the higher harmonics.

Adjustable speed motor and eccentric cam

**Fig. 1. Experimental setup.**

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### 2 Experimental description

Figure 1 shows the arrangement of the experimental setup. The light generated by a diffuse source passes through a mask and the tank containing stratified saltwater, and is captured by a camera (not shown) located at a fixed position far away (3.00 m) from the tank. A fabric with small circular holes of 1 mm diameter is used as a mask. The camera (JAI CVM4+MCL, 24 images/s, 1372×1024 pixels) registers the images which are stored directly in a PC.

By using the “double-bucket” method, the tank (0.30 m deep, 0.91 m long and 0.08 m wide) is filled with salt water in order to establish a 0.275 m deep uniform stratification. The maximum density of the fluid, measured with a Reichert-TS refractometer and a 5500 Antor-Paar densimeter, varies between 1.0645 and 1.0225. Then the buoyancy frequency is \( N^2 = 1.43 \).
± 0.07 s⁻². A body with its centre at 0.15 m over the tank bottom is immersed with its axis aligned horizontally and spanning the full width of the tank. It is suspended from a thin metallic supporting arm which is driven by an adjustable speed eccentric cam, forcing the body to undergo sinusoidal vertical oscillations. The amplitude of the body oscillation (that is, the maximum distance between the lower and higher positions of the body axis) is 0.0081 ± 0.0001 m. The flow around the body is laminar for the oscillation frequencies and amplitudes generated.

After the completion of an experiment, a segment of a vertical column of pixels passing through the lowest point of the oscillating object was extracted from the image sequences to form a time series image as shown in Fig. 2. The vertical axis is proportional to the vertical position, while the abscissa is proportional to time. The upper and lower zones clearly distinguished in the figure correspond to the body and the fluid, respectively. The vertical position of the lowest part of the body, \( h = h(t) \), is obtained from the contour between these zones. The fundamental frequency \( \omega_0 \) is determined by means of two methods: (a) the best fit sinusoidal curve in the table \( (t,h) \), and (b) the spectral analysis of frequencies of the time series. The uncertainty in each case is less than 0.1\% while the difference between both values is smaller than 1\%.

![Figure 2. Time series of a fraction of a column of pixels. The evolution of the interface between the two colours indicates the height of a point belonging to lower contour of the oscillating body (vertical axis) as a function of time (horizontal axis).](image)

Once the object is located into the stable stratified fluid, a reference image of the initial situation is saved. Then any small change in the refraction index gradient inside the environment produces the deflection of the light beams. During each experiment, the deviations of the intensities with respect to the reference image corresponding to each mask hole are detected and quantified by applying the synthetic schlieren technique [9]. DigiFlow software [10] allows the images to be processed easily and the fast attainment of qualitative information on-line. Quantitative results are also obtained with an additional processing where the light intensity deviations are converted to changes of the vertical and horizontal density gradients, provided the density and the refraction index of a salt solution follow an almost linear relationship [11].

3 Results

3.1 Visualization of internal waves

Figure 3 shows the perturbations at a given time generated by three bodies of similar sizes but different cross-section shapes, which oscillate vertically with the same frequency \( \omega_0 = 0.298 ± 0.003 \) s⁻¹. Applying the synthetic schlieren technique, quantitative information of the variation of the instantaneous density gradient field \( \rho^{-1} \partial \rho / \partial y \) is obtained starting from the intensity distribution of the images.

Internal waves are evidenced as beams forming an angle \( \theta = \cos^{-1} \omega / N \) to the vertical in agreement with theory. In the case of the cylinder (figure 3a), four beams at angle \( \theta \) are seen far from the body. Each of these beams is composed by two thinner beams of opposite phases forming the known St. Andrew Cross. For the other two bodies such a configuration is less clear. But, since the symmetry axis of the body cross-section and the motion axis coincide with the direction of gravity, in all cases the intensity distributions look symmetrical with respect to a vertical axis, and to a horizontal axis passing through the centre of the oscillation. This suggests that the used experimental setup avoids spurious horizontal motions that might give place to asymmetries of the flow.

The beams evolve as expected [1, 2]. They appear above (below) the cylinder with increasing intensities, then move down (up) following a direction that is perpendicular to the respective beam while their intensities decrease, and finally vanish. The angle \( \theta \) and the direction of the motion of the beams are similar for the three bodies; in fact they depend on the oscillation frequency which is the same in the three cases. However, the location of the points where the beams appear and the intensity evolution seem to depend on the cross-section shape of the body.

3.2 Visualization of higher harmonics

The comparison among the evolution of the internal waves generated by bodies of different cross-section shapes is not a simple task. In addition, the internal waves corresponding to the harmonics are also difficult to visualize because of their very small intensities and their superposition with the waves of frequency \( \omega_0 \). We use the fact that the density gradient...
in each point of the region of interest varies in time with a given frequency. This information is found in the time variations of the intensity distributions that appear in the sequences of images registered during $n$ periods of oscillation.

Fig. 3: Vertical density gradients in arbitrary units obtained with oscillating bodies of: (a) circle-, (b) rhomb- and (c) square-shaped bases. The colour scale (d) ranges between black (negative values) and white (positive values).

Fig. 4: Modulus of $\Re$ at frequency $\omega_0$ for the three cases presented in fig. 2. The false colour scale is also the same.

Therefore, for each pixel located at $(x,y)$ in the image, we calculate the complex parameter

$$\Re(x,y) = \int_0^{nT} \frac{1}{\rho} \frac{\partial \rho}{\partial y} e^{i\omega_t} dt,$$

where $\omega = \omega_0$, $2\omega_0$, $3\omega_0$, ... are the fundamental frequency and its harmonics. Thus the value of $|\Re|$ is maximum if the frequency of the variations of the pixel intensity coincides with $\omega$, and is minimum if it
is different from $\omega$.

Figures 4 and 5 show the images corresponding to the modulus and phase of $\Re(x,y)$, respectively, calculated with the corresponding sequence of images of the instantaneous density gradient as those shown in figure 3. The white regions in figure 4 are associated with the maximum amplitude of the time variations of the density gradient. The light blue regions, for which $|\Re| \approx 0$, do not have significant time variations at the fundamental frequency.

Figure 5 shows a $180^\circ$ change of phase at the horizontal axis of symmetry, and the inferior half-plane is nearly a mirror reflection of the superior one but with an opposite phase. Then the density gradients at points located below the oscillating objects are in the opposite direction with respect to the mirror points located above. This may also be seen in fig. 3.

As said before, the angle formed between the beams of internal waves and the vertical is the same for the three bodies because the oscillation frequency is the same. However, a different spatial distribution is clearly observed for each body. Figure 4a shows that internal waves are originated above the cylinder (for the sake of clarity, the waves originated below the bodies are not mentioned from now on). We find beams directed upwards and others directed to the horizontal symmetry axis. This last beam has an intensity greater (about 50%) than the first one, and it is superposed to the other beam coming from below so that it seems to be reflected at such an axis. For the squared cross-section body (Fig. 4c), the internal waves also are generated above the object; however, the waves directed to the horizontal symmetry axis have an intensity that is lower (about 50% or less) than those directed upwards. For the rhomb-shaped cross-section body (Fig. 3b) the waves are mainly generated along the lateral edge. These differences among the waves produced using bodies of different cross-section shape are maintained when other oscillation frequencies are used.

Figures 6 shows the modulus of $\Re$ for the first harmonics ($\omega = 2\omega_0$) obtained using the same image sequence of the cases represented in fig. 4. The beams now form a smaller angle to the vertical when compared with those of fig. 4, as expected from the double frequency $2\omega_0$. It is found that the harmonic waves are generated in the same regions around the body where the waves of frequency $\omega_0$ are produced. The amplitude of the beams is about one fifth of that corresponding to the beams formed at the main frequency. The distribution of the phase of the function $\Re$ for the first harmonics shows the same main features of fig. 5.

4. Conclusion

We present measurements of the density gradient fields produced when oscillating bodies excite internal gravity waves in stratified salt water solutions. These measurements are obtained using a non-intrusive optical technique suitable for determining the density gradient fluctuations in two-dimensional unsteady flows. Unlike the traditional schlieren technique, the synthetic schlieren is simpler to apply and to extract.
information. The experimental methodology and the specialized image processing employed allows visualizing and measuring higher harmonics of internal waves produced by different oscillating bodies immersed in the bulk of a stratified fluid.

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References:

Fig. 6. Modulus of $\Re$ at frequency $2\omega_0$ for the three cases presented in fig. 2. The intensity scale is amplified by a factor 5 with respect to the values of fig. 4.