Mathematics IA: Differential Equations

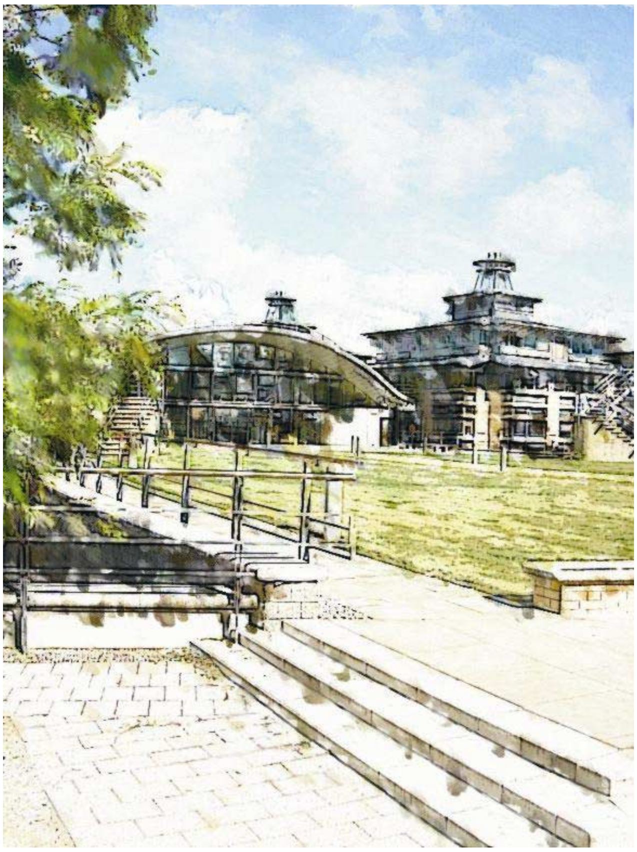
© Stuart B Dalziel

Department of Applied Mathematics and Theoretical Physics
University of Cambridge
Centre for Mathematical Sciences
Wilberforce Road
Cambridge CB3 0WA

s.dalziel@damtp.cam.ac.uk

http://www.damtp.cam.ac.uk/lab/people/sd103/lectures/part1a/

Examples sheets will be found at http://www.damtp.cam.ac.uk/user/examples/



10:00 Mon/Wed/Fri
Cockroft Lecture Theatre
Michaelmas Term 2004

CONTENTS

0. Introduction

0.1 Health & Safety

0.2 Why differential equations

Central to most of mathematics. The screwdriver of your toolkit.

Key tool to solving real-world problems as well as an essential ingredient of *pure* mathematics.

0.3 Variable backgrounds

Students have different backgrounds.

Modula A level

0.4 Books

W.E. Boyce and R.C. DiPrima Elementary Differential Equations and Boundary-Value Problems.

Wiley 7th edition 2001 (£34.95 hardback). 8th ed. due for publication in May 2004

D.N. Burghes and M.S. Borrie *Modelling with Differential Equations.* Ellis Horwood 1981 (out of print).

W. Cox *Ordinary Differential Equations.* Butterworth-Heinemann 1996 (£14.99 paperback).

F. Diacu An introduction to Differential Equations: Order and Chaos. Freeman 2000 (£38.99 hardback).

N. Finizio and G. Ladas Ordinary Differential Equations with Modern Applications. Wadsworth 1989 (out of print).

D. Lomen and D. Lovelock *Differential Equations: Graphics-Models-Data.* Wiley 1999 (£80.95 hardback).

R.E. O'Malley *Thinking about Ordinary Differential Equations.* Cambridge University Press 1997 (£19.95 paperback).

D.G. Zill and M.R. Cullen *Differential Equations with Boundary Value Problems.* Brooks/Cole 2001 (£37.00 hardback).

All these books should be in your college library

0.5 Greek

Greek symbols are used a lot in mathematics, both by convention and in order to maintain a compact notation. The following table lists the Roman and Greek letters we are likely to encounter in this course. For some of the Greek letters I have indicated the order in which I do the strokes. Note: I make no promises to be consistent!

Lower case	Upper case	Lower case	Upper case	Name
A	a	α	A	alpha
B	b	β	B	beta
C	c	χ	X	chi
D	d	δ	Δ	delta
E	e	ε	E	epsilon
F	f	ϕ	Φ	phi
G	g	γ	Γ	gamma
H	h	η	H	eta
I	i	ι	I	iota
J	j	φ		curly phi
K	k	κ	K	kappa
L	l	λ	Λ	lambda
M	m	μ	M	mu
N	n	ν	N	nu
O	o	o	O	omicron
P	p	π	Π	pi
Q	q	θ	Θ	theta
R	r	ρ	P	rho
S	s	σ	Σ	sigma
T	t	τ	T	tau
U	u	υ	Y	upsilon
V	v			
W	w	ω	Ω	omega
X	x	ξ	Ξ	xi
Y	y	ψ	Ψ	psi
Z	z	ζ	Z	zeta

0.6 Useful formulae

0.6.1 Series

Arithmetic $$a+(a+d)+(a+2d)+\cdots+\left(a+(n-1)d\right)=\frac{n}{2}\left(2a+(n-1)d\right)$$

Geometric $$a+ar+ar^2+\cdots+ar^{n-1}=\frac{a\left(1-r^n\right)}{1-r};\quad r\neq 1$$

$$\text{if } |r|<1 \text{ then } S_\infty=\frac{a}{1-r}$$

Algebraic $$\sum_{r=1}^{n} r=\frac{n(n+1)}{2}$$

$$\sum_{r=1}^{n} r^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{r=1}^{n} r^3=\frac{n^2(n+1)^2}{4}$$

Binomial expansion $$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots$$

- ◊ If n is positive integer, the series terminates and is true $\forall\ x$.
- ◊ If n is not a positive integer, the series is infinite and is valid for all $|x| < 1$ and sometimes for $|x| = 1$.

Exponential $$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Logarithmic $$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots \qquad \text{for } -1<x\leq 1$$

0.6.2 Trigonometry

$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$

$\cos(A\pm B)=\cos A\cos B\mp\cos A\sin B$

$2\sin A\cos B=\sin(A+B)+\sin(A-B)$

$2\cos A\cos B=\cos(A+B)+\cos(A-B)$

$2\sin A\sin B=\cos(A-B)-\cos(A+B)$

$\sin A+\sin B=2\sin\tfrac12(A+B)\cos\tfrac12(A-B)$

$\sin A-\sin B=2\cos\tfrac12(A+B)\sin\tfrac12(A-B)$

$\cos A+\cos B=2\cos\tfrac12(A+B)\cos\tfrac12(A-B)$

$\cos A-\cos B=2\sin\tfrac12(A+B)\sin\tfrac12(A-B)$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$\sin 2A=2\sin A\cos A \qquad \cos 2A=2\cos^2A-1=\cos^2A-\sin^2A=1-2\sin^2A$

If $t=\tan\tfrac12 x$, then $\sin x=\dfrac{2t}{1+t^2}$, $\cos x=\dfrac{1-t^2}{1+t^2}$, $\tan x=\dfrac{2t}{1-t^2}$

0.6.3 LECTURE NOTES

It is expected that students will attend lectures and take their own notes. This electronic version of the notes provides an additional resource to students, and does not necessarily include everything that is lectured. Conversely, these electronic notes may include some material that is not lectured. Ultimately, the lectures in conjunction with the Schedules determine what is examinable, not these electronic notes.

A yellow background indicates that the corresponding material was not presented in lectures. Absence of the yellow does not mean that it was presented.

0.6.4 COPYRIGHT