The Hydraulics of Doorway Exchange Flows

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The exchange flow through an opening (such as a doorway) in a partition, separating spaces containing fluid of different densities, is described in terms of two-layer hydraulics. The approach is based on the development, by Dalziel [Ph.D. thesis, Cambridge University 1988; J. Fluid Mech. (in press)], for a two-layer system of the functional formalism of Gill [J. Fluid Mech. 80, 641-671 (1977)]. It is shown that the opening can act as a hydraulic control in a fashion analogous to a weir in single-layer channel flows. It is shown that hydraulic theory predicts the observed phenomenon that the upper (warmer) layer in doorway exchange flow is thinner than the lower (cooler) layer. The predictions for exchange rates and interface heights are compared with laboratory experiments. The effect of a forced net flow through the opening is also considered.

1. INTRODUCTION

If a door is opened between rooms whose temperatures differ then there will be an exchange flow through the doorway: a temperature difference of a few degrees will produce a heat flux of around 1 kW. In modern "tight" buildings heat fluxes of this order are significant. As well as heat, the exchange flow will advect other tracers, e.g. pollutants, and therefore such flows are important in a variety of ventilation contexts. In this paper we shall describe the basic features of the exchange flow through a single opening in a plane, vertical wall separating spaces containing fluids of different density. We shall restrict our attention to rectangular openings such as doorways and windows. The denser fluid will flow towards the space containing the lighter fluid and occupy the lower part of the opening to a depth \( h_1 \), say, while the lighter fluid will flow in the reverse direction and occupy the upper part of the opening in a layer of depth \( h_2 \), say. Using \( H \) to denote the height of the opening, this gives

\[
x_0, x_e \quad \text{value of } x \text{ at: control, point of arrested flow (2If)}
\]
\[
z \quad \text{vertical distance from midheight of doorway}
\]
\[
\alpha \quad \text{ratio of } \lambda \text{ to the aspect ratio of the doorway}
\]
\[
\varepsilon \quad \text{parameter describing difference between doorway and corridor height}
\]
\[
\eta \quad \text{height of free surface above reference level (1If)}
\]
\[
\eta_0 \quad \text{value of } \eta \text{ far upstream}
\]
\[
\lambda \quad \text{ratio between height and width spreads for model geometry}
\]
\[
\rho, \rho_1, \rho_2 \quad \text{densities: of fluid (1If); of lower, upper layer (2If)}
\]
\[
\beta, \beta_0 \quad \text{mean of, difference between } \rho_1 \text{ and } \rho_2 \text{ (2If)}
\]
\[
\xi \quad \text{non-dimensional measure of } \eta
\]
\[
\chi \quad \text{dependent variable for general hydraulic functional, } J
\]

1. INTRODUCTION

If a door is opened between rooms whose temperatures differ then there will be an exchange flow through the doorway: a temperature difference of a few degrees will produce a heat flux of around 1 kW. In modern "tight" buildings heat fluxes of this order are significant. As well as heat, the exchange flow will advect other tracers, e.g. pollutants, and therefore such flows are important in a variety of ventilation contexts. In this paper we shall describe the basic features of the exchange flow through a single opening in a plane, vertical wall separating spaces containing fluids of different density. We shall restrict our attention to rectangular openings such as doorways and windows. The denser fluid will flow towards the space containing the lighter fluid and occupy the lower part of the opening to a depth \( h_1 \), say, while the lighter fluid will flow in the reverse direction and occupy the upper part of the opening in a layer of depth \( h_2 \), say. Using \( H \) to denote the height of the opening, this gives

\[
h_1 + h_2 = H.
\]
Pl

Fig. 1. Schematic diagram of two-layer exchange flow through a doorway.

Fig. 2. Representation of rapidly varying doorway geometry as a slowly varying channel.

Plan

Elevation

bounding supercritical regions [1, 2, 4]. Lane-Serff [5] gives a detailed description of the density and velocity profiles for doorway flows, and instabilities associated with them. Here we approximate the properties of the fluid by assuming two discrete layers: the lower layer of density \( \rho_1 \) has a velocity \( u_1 \) directed towards the space containing lighter fluid; the upper layer of density \( \rho_2 \) has a velocity \( u_2 \) directed towards the space containing denser fluid. This situation is outlined in Fig. 1. Writing \( q_1 \) and \( q_2 \) for the fluxes in the lower and upper layers, respectively,

\[
q_1 = u_1 h_1 W, \\
q_2 = u_2 h_2 W,
\]

where \( W \) is the width of the doorway. It will be assumed that the fluid is inviscid and incompressible. The Boussinesq approximation will be used, i.e. \( \Delta \rho = \rho_1 - \rho_2 \ll \frac{1}{2}(\rho_1 + \rho_2) \equiv \bar{\rho} \), and \( g' = g \Delta \rho / \bar{\rho} \) will denote the reduced gravitational acceleration (the acceleration experienced by a parcel of fluid of density \( \rho + \Delta \rho \) surrounded by fluid of density \( \rho \)). Note that for air \( \Delta \rho / \rho \) is equal to \(-A \Delta T/T\), where \( T \) is the temperature measured in Kelvin. The transient features, such as the effects of initially making an opening, will be discussed in the concluding section. Here we will assume that the flow is steady. The net flow between the spaces will be denoted by \( Q \),

\[
Q = q_1 - q_2,
\]

so \( Q \) is the net flow from the space containing denser fluid. If either one (or both) of the spaces is sealed then there can be no net flow. However, the case of non-zero net flow is important in ventilation problems, whether the flow is driven by mechanical ventilation or wind pressure, and both zero and non-zero net flow will be considered. A measure of the exchange flow rate is given by

\[
\dot{q} = \frac{1}{2}(q_1 + q_2).
\]

Note that for no net flow \( Q = 0 \), and thus

\[
\dot{q} = q_1 = q_2.
\]

First consider the case of no net flow: if the opening is a window, relatively far from the floor or ceiling of both spaces, then the flow will be vertically symmetric and so \( h_1 = h_2 = H/2 \). However, if the opening is a doorway then there is a clear geometrical asymmetry. Nevertheless, if it is assumed that the flow is dissipationless, so that the rate of release of potential energy is equal to the rate at which kinetic energy of the mean motion increases, then it can be shown that \( h_1 = h_2 = H/2 \). Indeed, until recently, most workers considering doorway exchange flow have made this assumption, e.g. [6, 7], despite their own experimental results which show that \( h_1 \) is greater than \( H/2 \) at the doorway. Some of these results are quoted in Sections 6 and 7. With the dissipationless assumption Bernoulli’s equation can be used to give a velocity profile \( (2g'z)^{1/2} \), where \( z \) is the distance above or below the midpoint. This gives a flux of \( (g'H)^{1/2}HW \) through the doorway from each room. In practice the flux is found to be less than this, the discrepancy accounted for by an ‘orifice coefficient’ normally denoted by \( C_d \). Note that this coefficient is larger, by a factor of three, than the constant we define below in equation (33).

The exchange flow through a doorway or window can be regarded as a two-layer flow in a channel with a sharp contraction in height and width representing the opening (see Fig. 2). Farmer and Armi [8] demonstrated that, for a channel varying slowly in height in the flow direction, the interface between the two layers was generally above the midpoint, with \( h_1 \) tending to approximately 0.625\( H \) as the difference in height between the channel at the contraction and that away from the contraction increased. They asserted that the interface would however be at the midpoint if there were any contraction in width. Dalziel [1], using a functional approach suggested by Gill [3] and outlined below, showed that this latter
assertion was incorrect: where there is a contraction in both height and width the interface will be above the midpoint. This analysis assumes that the channel varies gradually in height and width along the flow direction, which is clearly not the case for a doorway. However, it has been found that such hydraulic flows are relatively insensitive to the gradually-varying assumption, at least in their qualitative features [9].

The aim of this paper is to outline the hydraulic description of exchange flows and demonstrate that the basic features of exchange flows through doorways can be described in this way (following [1, 2, 5]). In particular it will be shown that the doorway acts as a hydraulic control, whether or not the net flow rate is zero, and that the interface between the two layers is above the midpoint when there is no net flow through the doorway. Also the effect of the geometry of the two spaces on the interface height and exchange flow rate, with and without a net flow, will be discussed. We present experimental studies supporting these results from [5].

2. SINGLE-LAYER HYDRAULICS

To assist understanding of the two-layer case we will begin with a review of the flow of a single layer of fluid in an open channel and use it to illustrate a general theory of hydraulic problems, following [3]. Consider a rectangular section channel of varying width, \( w(x) \), and floor level \( d(x) \), with the level of the free surface of the fluid given by \( q(x) \) where \( x \) is the along-flow coordinate. We assume that variations in \( d \) and \( q \) occur over length-scales that are large compared with the depth of the fluid, the 'shallow water' approximation. The types of channel considered will be those having a simple contraction in depth or width or both, with the contraction at \( x = 0 \) where \( d(x = 0) = 0 \), as shown in Fig. 3. The depth of the fluid in the channel is given by
\[
\eta(x) = q(x) + d(x).
\]
(6)

Assuming the flow to be steady, irrotational and inviscid (and gradually-varying) gives a velocity \( u(x) \) uniform across a section, and the flow rate
\[
q = uhw,
\]
(7)
is constant for all \( x \). Bernoulli's equation gives
\[
\frac{1}{2}u^2 + g\eta = g\eta_\infty.
\]
(8)

where \( \eta_\infty \) is a constant, equal to the surface elevation where the velocity is zero. For a flow from a large reservoir
\[
hw \to \infty, u \to 0, \text{ and } \eta \to \eta_\infty \text{ as } x \to -\infty,
\]
(9)

In terms of the dimensionless quantities
\[
\hat{h} = (gw^2/q^2)^{1/3}h \text{ and } \hat{d} = (gw^2/q^2)^{1/3}(d + \eta_\infty).
\]
(10)

Equations (6), (7) and (8) can be reduced to
\[
\frac{d}{dx} = \frac{\hat{h}}{\hat{d}} h^{-2}.
\]
(11)

Consider a geometry for which \( \hat{d} \) decreases along the channel to a value \( \hat{d}_m \) at \( x = 0 \), given by
\[
\hat{d}_m = \min_x \hat{d}.
\]
(12)

For \( x > 0 \) \( \hat{d} \) increases again. The description in terms of \( \hat{d} \) applies to channels with variations in width or depth or both: the conditions on \( \hat{d} \) are the same for all three. We shall consider the particular case
\[
\hat{d} = \hat{d}_m + x^2,
\]
(13)

though this involves no loss of generality since (11) only involves \( x \) implicitly; solutions for other \( \hat{d}(x) \) can be obtained by suitable stretching of the \( x \)-axis.

A measure of the surface elevation \( \eta \) is given by
\[
\xi = \hat{d}_m + \hat{h} - \hat{d} = \hat{d}_m + (gw^2/q^2)^{1/3}(\eta - \eta_\infty).
\]
(14)

For the geometry defined in (13)
\[
\xi = \hat{h} - x^2,
\]
(15)

and so
\[
\xi + \frac{1}{2}(\xi + x^2)^{-1} = \hat{d}_m.
\]
(16)

Figure 4 shows the family of solutions of equation (16) for various values of \( \hat{d}_m \). The surface slope is given by
\[
\frac{d\xi}{dx} = (\hat{h}^2 - 1)^{-1}(d\hat{d}/dx),
\]
(17)

and so is infinite where \( \hat{h} = 1 \) except at \( x = 0 \) where \( d\hat{d}/dx \) vanishes. Further differentiation gives
\[
3(d\xi/dx)^2 = (d^2\hat{d}/dx^2),
\]
(18)
Fig. 5. Controlled flow over broad-crested weir, with a hydraulic jump downstream of the weir (following Gill [3]).

which demonstrates the regularity condition \( \frac{d\bar{\alpha}}{dx} = 0 \) must represent a local minimum in \( \bar{\alpha} \).

There is only one solution (that with \( \bar{\alpha} = 3/2 \)) which can match differing values of the surface height far upstream and downstream of the contraction. Whenever the surface level far downstream is below the upper \( \bar{\alpha} = 3/2 \) curve the upstream level is given by \( \bar{\alpha} = 3/2 \), and is said to be 'controlled' by the contraction at \( x = 0 \). The final matching to the downstream level is achieved by a hydraulic jump (see Fig. 5), conserving the momentum flux downstream of the contraction. If the downstream level is above the upper \( \bar{\alpha} = 3/2 \) curve then the flow is everywhere sub-critical, the far upstream and far downstream levels are equal and the control is said to be 'flooded'.

Another feature of such flows is related to the phase speed of long-wave disturbances of the surface. This speed is given by

\[
c = u \pm \sqrt{gh},
\]

and thus the propagation of disturbances can be characterized by the Froude number, defined as

\[
F = \frac{u}{\sqrt{gh}}.
\]

For a controlled flow, \( F \) is less than unity upstream of the contraction, equal to unity at the contraction and greater than unity downstream, until the hydraulic jump. Hence the channel geometry is significant upstream of the contraction, where the flow is subcritical and information can be transmitted by surface disturbances in both directions, whereas disturbances can only be transmitted downstream in the supercritical region beyond the contraction and so the geometry there cannot affect the flow at the control. This is an important concept which has an analogue in the two-layer flow.

### 3. HYDRAULIC THEORY

Gill gives three criteria for defining a hydraulic type problem. First, the flow must be specified by one dependent variable, \( \chi \), whose dependence on \( x \) is implicit in terms of the geometry, so that it is possible to construct a functional, \( J \) such that

\[
J(\text{geometry}(x), \chi) = \text{constant}.
\]

For the single-layer case this can be achieved by putting \( \chi = h \) and \( J \equiv \bar{d} - \bar{h} - \frac{1}{2} \bar{h}^{-2} = 0 \). Second, equation (21) must have multiple solutions for some part of the channel. In the single-layer case there were two possible values of \( \bar{h} \). Finally there must be some sort of constriction in the sense that

\[
G = (\partial J/\partial \chi)(d\bar{w}/dx) + (\partial J/\partial \bar{d})(d\bar{D}/dx) + \ldots = 0,
\]

where \( w, D, \ldots \) are geometric parameters. In the single-layer case the equivalent condition is \( \bar{d} \) having a minimum, \( \bar{d}_m \).

The control occurs where the solution can switch smoothly between the possible branches of values of \( \chi \) satisfying equation (21) as \( x \) varies. This process requires

\[
\partial J/\partial \chi = 0,
\]

the flow is not controlled unless it changes solution branches. Differentiating (21) gives

\[
(\partial^2 J/\partial \chi^2)(d\bar{w}/dx)^2 = - (\partial G/\partial \bar{d}).
\]

A fundamental feature of such flows is that long wave disturbances are stationary at these critical points since, if \( \partial J/\partial \chi = 0 \) and \( J(\chi_0, \chi) = 0 \), then \( J(\chi_0 \chi + \delta \chi) = 0 \).

### 4. TWO-LAYER HYDRAULICS

Now consider a rectangular section channel having a horizontal floor and a varying ceiling height, given by the distance \( h(x) \) above the floor. The channel plan is symmetric about the centreline and is of width \( w(x) \). Dense fluid will flow from some reservoir at \( x = -\infty \), occupying a layer of depth \( h_1(x) \) and having velocity \( u_1(x) \); light fluid will flow in the opposite direction in a layer of depth \( h_2(x) \) with velocity \( u_2(x) \) (see Fig. 6). The sum of the layer depths is equal to the ceiling height, i.e.

\[
h = h_1 + h_2.
\]

Fig. 6. Sketch of elevation of a two-layer exchange flow through a closed channel of varying height.
The height at the contraction (doorway) is given by \( H \), so \( h(x = 0) = h_1(0) + h_2(0) \).

In the Boussinesq limit the constant difference in Bernoulli potential across the interface is given by
\[
\Delta B = \frac{1}{2}(\mu^2 - \mu_1^2) + h_1. \tag{26}
\]
Since \( \tilde{q} \) and \( Q \) are also constant, \( u_1 \) and \( u_2 \) can be expressed in terms of \( \tilde{q} \), \( Q \), the geometry of the channel (which determines \( w \) and \( h_1 + h_2 = h \) and \( h_1 \)). Thus the Bernoulli difference can be considered as a function of the form:
\[
\Delta B = f(\text{geometry}(x), Q, \tilde{q}; h_1), \tag{27}
\]
with possible flows given by \( \Delta B = \) constant. Now if \( \Delta B_1 \) is the value of \( \Delta B \) for some particular realisation of the flow, then Dalziel showed that a functional, \( J \), can be constructed,
\[
J(\text{geometry}(x), \Delta B_1, \tilde{q}, Q; h_1) = \Delta B_1 - f(\bullet; h_1), \tag{28}
\]
with solutions \( h_1(x) \) satisfying the equations of motion given by \( J = 0 \). This functional has the necessary properties for hydraulic-like flows outlined in the previous section (\( \chi \) has been identified with \( h_1 \)). We shall specify the net flow \( Q \) and treat \( \Delta B_1 \) and \( \tilde{q} \) as parameters whose values need to be determined by considering the total problem. It can be shown that only one choice of the pair \( \Delta B_1 \) and \( \tilde{q} \) will give a functional, \( J \), such that the solutions to \( J = 0 \) give branches \( h_1(x) \) that satisfy the boundary conditions. These boundary conditions will be discussed further below. For the single-layer flows recall that the general solution was given by equation \(16 \), but only one choice of the parameter \( d \) satisfied the boundary conditions.

As in the single-layer case a Froude number can be defined, giving information about the propagation of long wave disturbances on the interface between the two layers. This composite Froude number, in the Boussinesq approximation, is
\[
F^2 = F_{\text{1,2}}^2, \tag{29}
\]
where \( F_{\text{1,2}} = u_{\text{1,2}}/(g(h_1)_{1/2} \). Dalziel gives a more revealing form for \( F^2 \):
\[
F^2 = 1 + (h/g(h_1)_{1/2})c_1c_2, \tag{30}
\]
where \( c_1 \) and \( c_2 \) are the two possible phase velocities of the disturbances relative to the geometry. From \(30 \) it can immediately be seen that where \( F > 1 \) ("supercritical") the phase velocities have the same sign, so disturbances can propagate in one direction only, whereas for \( F < 1 \) ("subcritical") they have opposite signs and disturbances can propagate in both directions. When \( F = 1 \) one or both of the phase velocities must vanish. It can be shown, from \(30 \), that
\[
F^2 = 1 + \partial J/\partial h_1, \tag{31}
\]
a useful form when considering the functional \( J \).

We will not solve the full problem, which essentially involves finding the values of \( \Delta B_1 \) and \( \tilde{q} \) by guessing the position of control sections (where \( F = 1 \)) and adjusting them iteratively to maximize the exchange flow rate. This solution process is beyond the scope of this paper: complete details may be found in \([1,2]\). Once \( \Delta B_1 \) and \( \tilde{q} \) have been found \( J(\text{geometry}(x); h_1) \) can be considered as a set of functions of \( h_1 \), with parameters entirely dependent on \( x \). Examples of \( J(\text{geometry}(x); h_1) \) are sketched in Fig. 7 for \( x \) increasing from upstream of the controls, with respect to the lower layer, to downstream of the controls.

The functional \( J \) has been plotted as a function of \( \Delta = (h_1/h)^{-1} \), the difference of the interface from the midheight of the channel non-dimensionalized by the local channel depth, \( h \). This figure demonstrates how, following solutions to \( J = 0 \), an interface level close to the roof on the upstream side can be matched to a level near the floor on the downstream side. These boundary conditions are appropriate if the channel connects large reservoirs containing fluid of different densities. Also shown in this figure are sketches of \( J(\text{geometry}(x); h_1) \) with different values of \( \Delta B_2 \) and \( \tilde{q} \) (but the same geometry dependent on \( x \)). In these cases it is impossible to match smoothly between an interface level near the roof upstream of the constriction with a level near the floor downstream. Smooth matching from one branch of the solution to another can only occur at a section \( x = x_0 \) along the channel if there is a value of \( h_1 \) such that \( J(\text{geometry}(x_0); h_1) = \partial J/\partial h_1 = 0 \). Critical conditions occur at such sections which are known as hydraulic controls. Here there are two distinct controls: \( J \) has the form shown in Figs 7d and 7f at these controls, though for some geometries and net flow rates the controls will coincide and \( J \) will have a point of inflection.

Note the sign of \( \partial J/\partial h_1 \) (which is the same as the sign of \( \partial J/\partial A \)) at the points \( J = 0 \): using equation \(31 \) this can be used to ascertain the nature of the flow. Between the controls the flow is subcritical, outside this region the flow is supercritical with disturbances propagating away from the controls. It is possible for there to be hydraulic jumps on the supercritical flows between the controls and the reservoirs, similar to the hydraulic jumps allowed on the downstream side of the control in the single-layer case (such a situation is shown in Fig. 8). Such jumps are necessary to match on to a wide range of reservoir conditions, though note that the parameter \( \tilde{q} \) will be the same either side of the jump. It is possible for the reservoir conditions to be such that one or other of the controls is flooded, in a similar fashion to the single-layer case described above. See \([2]\) for details. Compare Figs 4 and 5, which pertain to the single-layer case, with Figs 7 and 8, the equivalent figures for the two-layer case. For geometries with a contraction in height and width at the same point, as is the case for a doorway, it can be shown that one of the controls is at this point \([1]\). For zero or moderate net flow rates the other control is on the upstream side with respect to the upper layer. In some cases this control may be in the reservoir, far from the constriction. For doorways between rooms this implies that it is the geometry of the warm room and the doorway that is significant in controlling the flow, whereas the geometry of the cool room plays no part. The layer of warm air is next to the sill formed by the top of the doorway whereas the cool air responds to this sill only indirectly via the layer of warm air.

5. THEORETICAL RESULTS

Once the interface height at the doorway is known, the exchange flow rate can be calculated. The flow is critical at the doorway, so at this point (i.e. at \( x = 0 \),
1 = u_1/(g' h_1) + u_2/(g' h_2), \quad (32)

from equation (29). For zero net flow this gives [since \( h_1 + h_2 = H \) at the doorway, and using equation (2)]

\[
\dot{q} = u_1 h_1 W = u_2 h_2 W = (g' H)^{1/2} HWk(A_0),
\]

where \( A \equiv (h_1/h) - 1/4 \); as before, \( A_0 \) denotes the value of \( A \) at the doorway, and \( k(A) \) is given by

\[
k^2 = (1/2+A)^3(1/2-A)^3/(1/2+A)^3+(1/2-A)^3.
\]

In Fig. 9, \( k \) is plotted as a function of \( A_0 \) from which it can be seen that \( k \) varies from one quarter, when the interface is at the midpoint, to 0.208 when the interfaces is 0.125\( H \) from the midpoint. This will be shown to be the extreme deviation from the midpoint when there is no net flow. For a window relatively far from floor or ceiling the flow is symmetric, \( h_1 = h_2 = H/2 \), and so \( k \) is equal to one quarter.

First consider the case of a contraction in height with no contraction in width. This case models a doorway in a corridor, where the doorway occupies the full width of the corridor. In the real case the upper layer flow will separate at the upper horizontal edge of the doorway as sketched in Fig. 10. The free streamlines coming from this edge may be regarded as the boundary of the channel. The flow is insensitive to the shape of this boundary, providing the shape is slowly varying in the subcritical region of the flow, and depends only on the difference between the height at the doorway and that far from the doorway. Compare this case with the single-layer case where the \( x \)-coordinate could be stretched arbitrarily; this can be done here as well since \( w(x) \) is constant. Figure 11 shows \( A_0 \) plotted against \( H_{(w)}/H \), where \( H_{(w)} \) is the height of the corridor. An asymptotic analysis of this case in [1, 2] shows that

\[
A_0 = \varepsilon^4 + 7/8 + O(\varepsilon^5),
\]

for \( \varepsilon \to 0 \), where \( H_{(w)}/H = 1 + \varepsilon^3 \).

Now consider the case of a doorway between rooms wider than the doorway, as shown in Fig. 12. The width of the doorway will be denoted by \( W \) as before, and the width of the room on the upstream side of the doorway with respect to the upper layer by \( W_0 \). The flow in each direction separates at the vertical edges of the doorway, and the upper layer separates at the upper horizontal edge (as for the corridor case). Again the hydraulic approach may be used if we put the channel boundaries on the free streamlines. The \( x \)-coordinate may be stretched arbitrarily, as before, but it is essential to keep the relationship \( h(w) \) defined parametrically by \( h(x) \) and \( w(x) \). Though the \( x \)-coordinate may be stretched the same transformation must be used for both \( h(x) \) and \( w(x) \). It may be possible to determine \( h(w) \) empirically. However, the observed departures of doorway flow from the hydraulic assumptions, in particular the steep interface at the doorway, make accurate, quantitative analysis on this basis pointless. Instead we will pick a model geometry (first considered in [5]) to investigate the qualitative features of the flow as the geometry of the door and rooms is altered. The main relevant feature of the geometry chosen is the relationship \( h(w) \) that it defines, and since the free streamlines are smooth a simple geometry is
Fig. 7. continued
appropriate. The geometry chosen is given by

\[
    w(x) = \begin{cases} 
    1 + |x|, & \text{if } |x| < (w_u - 1), \\
    w_u, & \text{otherwise}; 
    \end{cases} \\
\]

and \( h(x) = \begin{cases} 
    1 + |x|, & \text{if } |x| < (h_u - 1), \\
    h_u, & \text{otherwise}. 
    \end{cases} \) (35)

Fig. 8. Sketch showing the formation of hydraulic jumps on the supercritical flows on either side of the contraction. These jumps are necessary to match onto the conditions away from the contraction.

\[
\begin{array}{c}
0.25 \\
0.20 \\
0.15 \\
0.10 \\
0.05 \\
0.00 \\
\end{array}
\]

Fig. 9. The flow coefficient \( k \) in the exchange flow rate equation (33) plotted as a function of \( A_0 \), the fractional difference of the interface from the mid-height at the contraction. The curve plotted is for no net flow. The shaded region indicates the portion of the curve representing valid solutions over the range of different doorway geometries.

This geometry is shown in Fig. 13, but recall that the \( x \)-coordinate can be stretched arbitrarily, so the results apply to any geometry which widens and deepens away from the contraction, providing the shape is the same in the horizontal and vertical directions. Note that here \( h_u = H_u/H \) and \( w_u = W_u/W \). In Section 7 we shall explore the sensitivity of the flow to the particular choice of geometry. We shall do this by considering the effects of varying the relative rates of increase of the channel

Fig. 10. Exchange flow through a doorway in a corridor of constant width.

Fig. 11. Variations in \( A_0 \) with \( H_u/H \) when there are no variations in width near the doorway.

Fig. 12. Sketch of exchange flow through a doorway between rooms wider and taller than the doorway.
width and height, e.g. by allowing the height to vary more rapidly than the width. We would expect the relationship between the effective channel width and height to be a function of the doorway aspect ratio.

If there is no contraction in height and no net flow, the flow is symmetric and the interface at the doorway will be at the midheight whatever the contraction in width. The interface height when the contraction is in height alone was discussed above. When there is a contraction in width also and no net flow, the interface height at the doorway is closer to the midpoint that when there is no contraction in width.

We will first describe the results for no net flow. Figure 14a shows $A_0$ plotted against $H_w/H$ for various fixed values of $W_0/W$, and Fig. 14b shows the contours of $A_0$ plotted against $H_w/H$ and $W_0/W$. It can be seen that for this model geometry the value of $A_0$ lies between 0.054 and 0.055 for much of the possible range of values for $H_w/H$ and $W_0/W$. Note that the value of $A_0$ on the lines $H_w/H = 1$ and $W_0/W = 1$ will be the same regardless of the exact nature of the geometry, provided only that $w$ and $h$ increase monotonically away from $x = 0$. Thus we expect the form of the contours shown in Fig. 14b to be similar for other geometries with a continuous relationship $h(w)$, and thus for doorway exchange flows, though the value of $A_0$ in the ‘plateau’ region may vary.

Now we will consider the case where a net flow is imposed so that $Q \neq 0$. Figure 15 shows $A_0$ plotted against $Q/(g'H)^{1/2}HW$ for a fixed geometry of the type described above with $w_0 = h_0 = 1.2$. (Note that here $H = W = 1$.) The discontinuity in the value of $A_0$ is due to the position of the second control jumping from one side of the doorway to the other in response to changes in the net flow $Q$. Full details of this complex behaviour are beyond the scope of this present paper (see [1]).

Of particular importance in a variety of ventilation problems is the value of $Q$ that needs to be imposed to ensure that the flow through the doorway is in one direction only. This is necessary for example at the entrances to ‘clean rooms’ and operating theatres, where it is desired to keep unwanted contaminants out of a space, and quarantine wards, where the intention is to keep infectants in. If the imposed flow is such that the Froude number of one of the layers, assuming it to occupy the full depth of the opening, is at least unity at the opening, then there will be no flow in the opposite direction. That is, if

$$Q = \pm (g'H)^{1/2}HW,$$  \hfill (36)

flow will be unidirectional with the stationary layer having a depth which vanishes at the constriction as shown in Fig. 16. If the imposed flow is stronger than this then
the stationary layer will vanish further downstream (with respect to the imposed flow), at the point in the channel where the local Froude number is unity, i.e. at the point $x = x_c$ where $Q = \pm \left( g h(x_c) \right)^{1/2} h(x_c) w(x_c)$. In theory it is not necessary for the flow to be as strong as was stated in equation (36) to arrest the flow. In [1, 2] it is shown that the dense layer is arrested for $Q$ equal to $(2/3)^{3/2}$ times the value given in (36), while, if $H_c/H$ is less than $3/2$, it is theoretically possible to arrest the light layer with $Q$ equal to some value lying between one times and $(2/3)(H_c/H)^{3/2}$ times the value given in (36). In both these cases the point where the stationary layer is arrested is not at the constriction.

The application of this theory (of arrested flows) to a doorway flow is uncertain: the boundaries of the channel were assumed to be the free streamlines of the flow, which do not exist for the stationary fluid. We do not expect to be able to make useful predictions of the position of the arrested fluid in the case of doorway flows. In particular we think that equation (36) provides a reliable estimate for the imposed flow needed to arrest the flow in one direction, and do not believe it possible to arrest a wedge of fluid that already intrudes into the space.

6. EXPERIMENTS

Two sets of experiments were carried out to investigate the variation of interface height with geometry when there is no net flow. For one set a long channel was used and for the other set a box. In both cases the fluid used was water and density differences were produced by adding salt. The dimensions of the channel were 199 cm long by 9.4 cm wide and it could be filled to a depth, $H_c$, of up to 9.5 cm. A sharp sill, made of 0.2 cm perspex set in a plasticine base, was placed on the floor of the channel at about the midpoint, as shown in Fig. 17a. Compared with a doorway in a corridor this geometry is inverted so the label Subscript 2 will refer to the lower layer in these experiments.

A removable barrier was placed in the channel to one side of the sill. Salt, and sometimes dye, was dissolved in the water on one side of the barrier. Samples of the fluid were then taken from each side of the barrier to determine the density difference. Typical values of the reduced gravity, $g^*$, were 5 to 10 cm s$^{-2}$. Thus density differences were much less than the mean density so the free surface did not move significantly due to the exchange flow over the sill. However, slopping motions started by removing the barrier were significant and therefore expanded polystyrene blocks were placed on the surface of the water and loosely attached to the channel sides to damp these oscillations. The barrier was removed and the flow observed by eye and sometimes photographed. The transients associated with the initial release and flow over the sill died away quickly, typically over a few seconds. A photograph of a typical flow is shown in Fig. 17b. Note the hydraulic jump downstream of the sill with respect to the lower layer. The height of the interface at the sill was measured by eye or from photographs and the results are plotted in Fig. 20a, together with the results from the next set of experiments where there is a contraction in width as well as height.

The other set of experiments was carried out in a perspex box of length and width both 60.8 cm and capable of being filled to a depth of up to 40 cm. The box was divided in two by a vertical wooden partition having a doorway cut into it with a height of 15.0 cm and a width of 7.0 cm. On one side of the partition a vertically sliding perspex door was mounted which entirely covered the doorway when in its lowered position. The box and partition are shown in Fig. 18a. Two free-standing perspex rectangles, about 10 cm wide by 25 cm high, were used to adjust the effective geometry of the room on the opposite side of the partition to the door (see Fig. 18b). The important features of the geometry are summarized in Fig. 19.

Salt and dye were added to the space not containing the movable barriers. After samples were taken, the door was slid vertically clear of the doorway and the flow observed. As noted earlier it is the geometry of the warmer room, here represented by the space with movable barriers, that is significant in controlling the flow. The interface is steep at the doorway and also oscillates. These features of the flow make measurement of the interface height uncertain, and account for some of the errors associated with the measurements. The results for
Fig. 17. (a) Apparatus used for investigating two-layer flow over a sill (b) Shadowgraph of two-layer flow over a sill.

Fig. 18. 60 x 60 x 40 cm box with partition showing (a) vertically sliding door (shaded) on runners and (b) movable perspex barriers (shaded).
experiments with a large fixed value of $W_\omega/W$ and various values of $H_\omega/H$ are plotted in Fig. 20a. Notice the similarity in form between this figure and Fig. 14a. In [7] it is found that the average interface height is at 0.55 $H$ for doorways of a height 1.83 m and widths of between 0.24 and 0.99 m. The results for smaller values of $W_\omega/W$ with a fixed large value of $H_\omega/H$ are shown in Fig. 20b.

**Flux measurements**

The flow rate through a contraction connecting two spaces containing fluid of different densities was also measured. The apparatus used was the 60 × 60 × 40 cm perspex box described above, with the vertical partition having a vertically sliding door. The box was filled with water to a depth of 33 cm and, with the door closed, salt was dissolved in the water on one side of the partition. Samples were taken from both sides of the partition. The door was opened for a length of time (measured by stopwatch), the door was then closed, the fluid either side of the partition mixed up and new samples taken. The experiment was repeated several times. The volumes of the two spaces, $V_1$ and $V_2$, were obtained by measuring the dimensions of each space, and this was checked against estimates of the ratio of the two volumes given by considering the densities in the two spaces. [Note that the total mass of fluid is constant, equal to $E$, say, and so $\rho_1 = (E/V_1) - (V_2/V_1)\rho_2$.]

The density of the samples was measured using an Anton PAAR density meter. Two estimates of the volume exchanged for each individual experiment can be calculated from the densities before and after the experiment. In Fig. 21 the volume exchanged is plotted against the length of time the door was opened. The times have been non-dimensionalized by the timescale $(H/g)^{1/2}$, the volumes by the scale $WH^2$. The length of time for which the door was opened was varied and so by estimating the flux by at least squares fit (not presuming that the best fit line should pass through the origin), any unsteady volume fluxes associated with the initial opening of the door and any systematic timing errors are eliminated. The best fit line does not, however, pass significantly far from the origin, suggesting that the initial unsteady part of the flow gives a mean flux not significantly different from the final, relatively steady flux. The value of the constant $k$ in equation (33) estimated from these experiments is $0.207 ± 0.005$. In [6] a value of 0.2 was found.
We shall consider models with $h_\infty \gg 1$ and $w_\infty \gg 1$, such as may occur for flow between rooms where the height and width of the doorway are small compared with the height and width of the rooms.

We have shown that the basic features of doorway exchange flow can be described in terms of two-layer hydraulic theory. In particular, the incorrectness of assuming the exchange flow to be dissipationless, leading to the assertion that the interface between the warm and cool air should be at half the doorway height when there is no net flow, has been demonstrated. The observed phenomenon that the upper layer is thinner than the lower one is explained.

We assume a fixed parametric relationship between variations in the effective channel width and height as described by equation (35). In this section we generalize the parametric relationship for channels where $h_\infty \gg 1$ and $w_\infty \gg 1$, such as may occur for flow between rooms where the height and width of the doorway are small compared with the height and width of the rooms.

We shall consider models with

$$h(w) = 1 + \lambda(w - 1),$$

(37)

to explore the sensitivity of the flow to the particular model geometry chosen in Section 5 [defined in equation (35)] and to justify its choice. Openings the full height of the adjoining spaces will be modelled by $\lambda = 0$, while the geometry used in Section 5 has $\lambda = 1$. The parameter $\lambda$ is a measure of the rate at which the total depth, relative to the opening, increases compared with the rate of increase in channel width, again relative to that at the opening. If we assume that the actual free streamlines from an edge of the doorway take a form independent of the width or height of the doorway, then their relative spread will be a function of aspect ratio of the doorway.

In the simplest model we might assume the free streamlines spread at a constant angle away from the door, giving the parameter $\lambda$ as $W/2H$.

When there is no net flow, hydraulic theory predicts one control at the doorway and the second slightly inside the warm room. The inflowing cold air will spread laterally and almost two-dimensionally over the floor of the warm room, setting the lateral bounds of any effective channel. In contrast the removal of warm air from the room will extend over a depth much greater than the height of the doorway. In an ideal fluid the streamlines will extend all the way to the ceiling adjacent to the door (they will however remain essentially confined laterally to the region where two layers are present). For real fluids, however, the withdrawal process will lead to the formation of a free streamline from the top of the door,

with the possibility of a recirculating region above it next to the wall. If a simple parametric model of the form described by equation (37) were valid, we would therefore expect a value of $\lambda$ larger than $W/2H$, but one which still depends on the aspect ratio of the doorway. We postulate

$$\lambda = zW/H,$$

(38)

with $z > \frac{1}{2}$.

Note that details of the motion in the cold room are unimportant as the flow is supercritical just away from the doorway leading into the cold room. Details of the angled plume which forms are beyond the scope of this paper but may be found in [5].

Figure 22 plots curves showing the predicted value of $A_0$ as a function of $W/H$ for a number of different values of $z$. Note that as the width of the doorway is increased, relative to its height, the relative increase in the width of the channel away from the doorway is less and so the increase in depth is more pronounced leading to the interface being positioned further above the midpoint of the doorway. Similarly, the departure from $A_0 = 0$ is more pronounced for larger values of $z$. In the limit of $z \to \infty$, $A_0 \to 0.125$ and the exchange flow rate is decreased from $k$ equal to one quarter to $k = 0.208$.

Also plotted in Fig. 22 are the experimental observations from [7] as a function of the aspect ratio of the doorway. While precise agreement between these experiments and the present theory is not to be expected, the theory does illustrate the overall trend with variations in the doorway aspect ratio. Note that the channel model used in Section 5 (with $\lambda = 1$) is a reasonable match to the data from [7] for a typical doorway ($W/H \approx 0.4$), supporting the choice of model geometry in Section 5.

An analysis of the effects of net flow through the model geometry of this section was undertaken by Dalziel [1] but is beyond the scope of this paper. The results are similar in form to those given in Section 5 for $w_\infty \gg 1$ and $h_\infty \gg 1$, and the discussion on their validity identical.

### 8. CONCLUSIONS

We have shown that the basic features of doorway exchange flow can be described in terms of two-layer hydraulic theory. In particular, the incorrectness of assuming the exchange flow to be dissipationless, leading to the assertion that the interface between the warm and cool air should be at half the doorway height when there is no net flow, has been demonstrated. The observed phenomenon that the upper layer is thinner than the lower one is explained.

The hydraulic theory has also shown that, in general, it is the geometry of the warm room and the doorway that controls the flow; the geometry of the cooler room plays no part, except for strong flows from the cool room. It should be noted that for $H_\infty/H$ and $W_\infty/W$ both greater than about 1.2 there is little further effect of the geometry on the flow: the walls or ceiling of the warmer room must be close to the doorway to significantly affect the flow. The aspect ratio of the doorway can influence the positioning of the interface, and hence the exchange flow.
rate, by changing the importance of variations in the width between the free streamlines compared with the expansion in depth of the flow in the warm room. The analysis in Section 7 suggests, however, that the flow depends only weakly on the aspect ratio of typical doorways. In many situations the approximation $h/H = 0.55$ will be sufficiently accurate to describe exchange flows. This insensitivity to geometry explains the success of widely used methods for estimating doorway flows based on a single-valued constant (orifice coefficient).

We have measured the exchange flux through a doorway and found it to be given by

$$\dot{q} = kHW(g'H)^{1/2},$$

(39)

with $k = 0.207 \pm 0.005$. This is of somewhat smaller than the theoretical value of $k = 0.243$ predicted from the observed interface height. We expect the value of $k$ for a window where the interface is at the midpoint, to be somewhat smaller than the predicted value of one quarter.

An important result is that the use of a hydraulic theory, based on the assumption that the flow is gradually-varying in the along-flow direction, gives reasonable predictions even though the gradually-varying assumption is violated. This is also the case for single layer flow, with theory for broad-crested weirs being applied to sharp-crested weirs. In that case the theory underestimates the flow rate: predicting a flow rate of $(gh)^{1/2}hv$ whereas the real flow is 5% greater than this (see for example [10]).

It is possible to halt the flow in one direction by an imposed flow in the opposite direction. We do not think that the predictions based on hydraulic theory for arresting the flow other than at the doorway are reliable: it is not possible to use the free streamlines of the arrested flow to define a channel boundary. Indeed it is arguable that the appropriate boundary is the walls of the room into which the imposed flow is directed. This would imply that the flow is arrested at the doorway since $h$ and $w$ vary between their maximum and minimum there, and so will satisfy the Froude number condition. (If the imposed flow is so strong that the Froude number is greater than unity even if the moving fluid were to occupy the whole depth and width of the room into which it is directed, then the ‘stationary’ fluid will be driven from the room by a ‘plug’ flow.)

The experiments demonstrated that the flow through a doorway is not steady, even after the initial acceleration period, and that the use of time-averaged velocity and density profiles will underestimate the mass flow rate due to the correlation between the variations of density and velocity from the mean (see [5]). To accurately determine the exchange flow through an opening it is necessary to measure the time-dependent profiles or to measure the volume that has been exchanged during a known length of time by measuring a conserved marker. Here the latter technique was used, measuring the densities in the spaces either side of a door before and after a period when the door was open. It would be possible to use some marker other than mass, whose concentration is easily measured, and this may be easier, especially for experiments in air.

The initial acceleration of the fluid on first opening the door takes place over a timescale $(H/g')^{-1}$. For a typical doorway this would be a few seconds. If the door is open for periods less than this the transient effects of opening the door may be important though the effect on the mean flux appears to be insignificant.

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