Self-similarity and Internal Structure of Turbulence Induced by Rayleigh-Taylor Instability

S.B. Dalziel, P.F. Linden and D.L. Youngs
1 DAMTP, University of Cambridge, Silver Street, Cambridge CB3 9EW, England
2 Atomic Weapons Establishment, Aldermaston, Reading RG7 4PR, England

Abstract: A detailed comparison between simple experiments and three-dimensional numerical simulations of Rayleigh-Taylor instability has been undertaken. Two layers, one salt water and the other fresh, are initially separated by a rigid barrier which is removed at the start of the experiment. The aim of this work is to understand the significance of the perturbation imposed by the removal of the barrier and utilise this understanding to provide improved agreement between the experiments and numerical simulations of them. We investigate both the growth of the width of the mixing zone and the structure of the concentration field within it. Our results demonstrate that, at scales small compared with the confining geometry, the flow rapidly adopts self-similar turbulent behaviour with the influence of the barrier-induced perturbation confined to the larger length scales. Concentration power spectra and the fractal dimension of iso-concentration contours are found to be representative of fully developed turbulence and there is close agreement between the experiments and simulations.

1. Introduction

Recent work [1, 2, 3] has shown that it is necessary to model experimental initial conditions in an appropriate manner if close agreement is to be found between experimental and numerical studies. For simple laboratory experiments on Rayleigh-Taylor instability, where two layers of miscible liquid are separated initially by a barrier, the loss of memory is incomplete and the growth of the boundaries of the mixing zone is dominated by the energetic, low wavenumber components of the initial perturbation to the velocity field. This perturbation is produced by the removal of the barrier. The three-dimensionality of the resulting flow arises from higher wavenumber components with a more random configuration. For a simulation to provide close agreement for external measures of the flow, such as the width of the mixing zone, only the two-dimensional component of this initial perturbation is important. However, if the molecular mixing and other characteristics of the internal structure are to be simulated accurately, a three-dimensional perturbation must be present also.

In the case of the almost shear-free composite barrier, described initially by [4], the low wavenumber components of the initial velocity field are essentially two-dimensional and may be described in terms of a vortex sheet left behind the barrier producing an irrotational flow through the remainder of the tank. This flow field may represented as the streamfunction of the form:

\[ \Psi(x, y, z) = \Psi_0 U_{\text{Barrier}} L h_{\text{Barrier}} \frac{H}{H} \sum_{n=1}^{N} a_n \sin \frac{n\pi x}{L} \sinh \frac{n\pi y}{2L} \left( 1 - \frac{2\|z\|}{H} \right) \]

where \( \Psi_0 \) is an order one function of the Reynolds number and \( a_n \) \((n = 1, 10)\) are coefficients determined from experiments. Here \( x, y, z \) are the along, across and vertical tank coordinates with the origin in the centre of the tank of length \( L = 400 \text{ mm} \), width \( L/2 \) and height

More information - Email: sdl03@amtp.cam.ac.uk or s.dalziel@damtp.cam.ac.uk
H = 500 mm. The barrier of thickness $h_{\text{Barrier}} = 2.4$ mm is removed at a speed $U_{\text{Barrier}}$ in the negative x direction. In the simulations presented here, we replace the higher three-dimensional wavenumber components of the initial perturbation with a random perturbation to the initial position of the interface. If the amplitude of this random perturbation is small then it may be described as a random perturbation to the density field, $\rho_{\text{Noise}}$, added to the density field at $z = 0$. The experiments reported here were performed using salt water and alcohol/water solutions (the alcohol was added to match refractive indices) to give an Atwood number of $A = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) = 0.002$, where $\rho_1$ and $\rho_2$ are the initial densities of the upper and lower layers, respectively.

2. Growth of the mixing zone

Figure 1 summarises the initial growth of the mixing zone by presenting the concentration fields in the $x-z$ plane at $y = 0$. For the experiments (column I), only the lower half of the tank is visualised and the concentration measurements are obtained by image processing techniques from a fluorescent dye illuminated by a simple light sheet from an arc lamp. Column II shows the concentration field for a two-dimensional simulation (Settle - see [3], this volume) with the initial conditions described by (1). The full barrier simulations, performed using TURMOIL3D [5, 6, 7] with the initial conditions given by (1) and including $\rho_{\text{Noise}}$ is in column III while idealised simulations (also computed by TURMOIL) starting with only $\rho_{\text{Noise}}$ to initiate the flow is in column IV.

Visual comparison between the four columns in figure 1 shows a remarkable level of similarity for the width of the mixing zone between the experiments, two-dimensional simulations and barrier simulations. In contrast, the idealised simulations exhibit a substantially lower growth rate and a less coherent structure to the flow. This demonstrates clearly the dominant role of the two-dimensional low wavenumber component of the initial perturbation. The much more rapid growth down the right-hand wall is the result of the left-right asymmetry in the barrier removal process producing a vortex sheet, while the dominant wave length is believed to result from instabilities on this vortex sheet. The agreement between these flows has been quantified by [3] and measurements of the quadratic component of the temporal evolution are given in [2].

Visual comparison of the internal structure of the mixing zone again shows favourable agreement between the experiments and barrier simulations, and indeed some agreement with the idealised simulations. Before showing this agreement to be quantitative, we note that we would not expect the two-dimensional simulations to show any of the internal gradients and structure of the found in the experiments due to the absence of vortex stretching. The existence of concentrations intermediate between $C = 0$ or $C = 1$ are the result of numerical diffusion and not nonlinear, three-dimensional processes.

3. Concentration power spectra

The internal structure of the experimental and numerical flows may be analysed in a number of ways. Here we shall restrict ourselves to the concentration power spectra. We shall not present spectra for the two-dimensional simulations as these clearly lack the internal structure in which we are interested. To ensure a consistent treatment of the flows, all processing was based on images of the concentration field. In all cases the spectrum was determined for individual vertical planes in the window $x/L \in [-\frac{1}{2}, \frac{1}{2}]$ and $z/H \in [-0.1, 0]$ by calculating one-dimensional horizontal FFT after suitable windowing and padding of the data. The individual horizontal spectra within the window were then averaged. An ensemble of sixteen experiments were used with the spectra located at $y = 0$. The resulting data set was averaged over the
ensemble. For the simulations the spectra were calculated for each vertical plane across the width of the tank and these spectra then averaged. In the case of the barrier simulations

\[ \tau = 1 \]

\[ \tau = 2 \]

\[ \tau = 3 \]

Figure 1. Comparison of concentration fields for experiments (column I), two-dimensional simulations (column II), three-dimensional “barrier” simulations (column III) and three-dimensional “idealised” simulations (column IV). The concentration fields are shown for (a) \( \tau = 1 \), (b) \( \tau = 2 \) and (c) \( \tau = 3 \).

an ensemble of three sets of coefficients \( a_n \), each obtained from a different experiment where the initial conditions were measured, and the result of the cross-tank averages were averaged over the ensemble as well. Figure 2 shows typical power spectra at time \( \tau = 2 \) for the three flows. All show a region in wavenumber space where a power law is in reasonable agreement. The solid lines represent a least squares fit (over an appropriate sub-range of wavenumbers \( k \)) giving spectral slopes of \(-1.49\), \(-1.63\) and \(-1.79\) for the experiments, barrier simulations and idealised simulations, respectively. The dot-dash lines show fits of a \( k^{-5/3} \) spectrum to each of the three cases to show that the spectra are indeed close to the \( k^{-5/3} \) expected during
the self-similar growth phase, and the overall power levels are similar. Note that the range of self-similarity is less for the simulations which have a much lower effective resolution than the experiments. [5] showed that the roll-off in the spectra for homogeneous, isotropic turbulence calculated with TURMOIL3D occurred at around $6\Delta x$, where $\Delta x$ is the mesh size, making the smallest scales simulated approximately a factor of fifteen larger than the experiments where the pixel size was comparable with the Kolmogorov length scale. Figure 3 plots the temporal

![Figure 2: Concentration power spectra at $\tau = 2$ for (a) experiments, (b) barrier simulations and (c) idealised simulations.](image)

The time evolution of the slope of these spectra. Of the three, the experiments reach a consistent slope fastest. This is due primarily to the existence of disturbances at all wavenumbers at the beginning of the experiments. Note that the spectral slope prior to $\tau = 0.4$ are misleading as they are contaminated by the barrier being present in the field of view. Closer examination of the individual spectra show that at early times, where a slope of around $-2$ is obtained using the least squares fitting procedure, the range of wavenumbers following a power law behaviour is somewhat smaller than seen in figure 2a. Indeed, a slope of $-5/3$ could reasonably be fitted over much of this range if data were restricted to a smaller range of wavenumbers. With the two simulations the initial perturbation is more confined in wavenumber space. For

![Figure 3: The time evolution of the spectral slope for (a) experiments, (b) barrier simulations and (c) idealised simulations.](image)

the barrier simulations nonlinear interactions fill in the gap between the low wavenumber two-
dimensional barrier perturbation and the high wavenumber noise significantly more rapidly than can happen in the idealised simulations where only the high wavenumber noise component is initially present. As a result the barrier simulations achieve a spectral slope of $-5/3$ by $\tau \approx 1$ compared with closer to $\tau \approx 2$ for the idealised simulations. As with the experiments, a more direct comparison of the individual spectra shows them to be consistent with a $k^{-5/3}$ slope over a wider range of wavenumbers than is suggested by this figure. One of the conclusions we may draw from the examination and comparison on the spectral slope of the concentration power spectra is that the internal structure of the Rayleigh-Taylor instability, which is dominated by the processes occurring at higher wavenumbers, rapidly loses its memory of the initial conditions. For the experiments, the existence of a less structured initial perturbation over a broader range of wavenumber, in combination with the higher effective Reynolds number, means that the $k^{-5/3}$ spectrum characteristic of turbulent flow is attained much faster than in the simulations. In drawing a comparison between the slope of these concentration power spectra and the $k^{-5/3}$ slope of the Kolmogorov velocity power spectra we are making use of the fact that we are studying an accelerating flow where the distortion of material surfaces will be dominated by the most recent characteristics of the velocity field.

4. Fractal dimension

Another approach to characterising the internal structure is the fractal dimension of iso-concentration contours. Here we use the Kolmogorov capacity, obtained through the normal box-counting algorithm, to determine the dimension. Figure 4 shows the relationship between the number of boxes and the size of boxes required to cover a $C = 0.5$ concentration contour is well modelled by a power law for all three cases. The exponent for this power law is then the fractal dimension. The time evolution of these dimensions is plotted in figure 5 showing close agreement in all three cases for $0.7 \leq \tau \leq 2.5$. Indeed the dimension is approximately constant at $D_2 \approx 1.47$ over this period. The disparity for $\tau \leq 0.7$ is due again to the need for the simulations to fill the gaps in wavenumber space not present in the initial conditions, and for $\tau < 0.4$ the presence of the barrier in the field of view prevents the experimental dimension starting from closer to unity. At later times the increases in the experimental dimension is the result of an unmodelled three-dimensional component of the experimental initial conditions [3] and the establishment of a globally stable stratification. A relationship $s = D_2 - 3$ has been proposed [8] between the slope $s$ of the velocity power spectrum and the fractal dimension $D_2$. 

![Figure 4. Power-law relationship between contour length and coverage using box-counting algorithm for (a) experiments, (b) barrier simulations and (c) idealised simulations.](image)
Figure 5. Time evolution of the $C = 0.5$ fractal dimension. The experimental data is shown as the solid line, the barrier simulations with the dot-dash line and the idealised simulations with the dashed line.

of the iso-concentration contour. Our results for the slope of the concentration power spectra and fractal dimension of iso-concentration contours is consistent with this.

5. Conclusions

In this paper we have shown that both the external structure of the mixing zone and the structure of the concentration field within it for a set of simple laboratory experiments may be modelled accurately if an appropriate description of experimental initial conditions is used. Two-dimensional simulations are able to model the growth of the mixing zone due to the dominance of the low wavenumber two-dimensional component of the barrier-induced perturbation. [4] has shown that the timescale for loss of memory of an initial perturbation to the velocity field (by the introduction of vorticity) may be substantially larger than the timescale for loss of memory for a perturbation to the density field where the two perturbations contain a similar energy. More recently [3] have shown that the structure is more important than the amplitude of such a velocity-field perturbation with the development of the flows presented here only weakly dependent on the amplitude of this barrier-induced perturbation. While the flow retains a memory of the low wavenumber components of these initial conditions over a period much longer than the characteristic time for the flow, the high wavenumber components soon become independent of the precise form of the initial conditions and evolve into a classical turbulent flow during the so-called self-similarity phase of the growth of the instability. Both concentration spectra and iso-concentration contour fractal dimensions are consistent with the presence of a Kolmogorov velocity spectrum. SBD wishes to acknowledge the financial support of AWE Aldermaston and the Issac Newton Trust.
References