Rayleigh-Taylor instability in complex stratifications

Stuart B. Dalziel¹ & Jeff Jacobs²

¹Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK
s.dalziel@damtp.cam.ac.uk

²Department of Aerospace and Mechanical Engineering, University of Arizona, Tucson, AZ 85721, USA
jacobs@ame.arizona.edu

1. Abstract

Rayleigh-Taylor instability has received considerable attention over recent years due to its fundamental rôle in the mixing processes of many stratified flows, and due to the close relationship between it and Richtmyer-Meshkov instability. Recent advances in numerical models and improved experimental diagnostics have shown a convergence in our ability to model the development of the instability, despite there being a number of significant areas where our understanding is still limited. However, to date both numerics and experiments have been largely confined to the simplest possible case: two layers of different but uniform density, and quiescent (or as near as may be achieved experimentally) initial conditions.

This paper presents results from an experimental study of flows one level greater in complexity. In particular, we investigate the effect of introducing a third layer to the problem. The densities of the three layers, \( \rho_1, \rho_2 \) and \( \rho_3 \) (from top to bottom) are chosen so that one interface is unstable with \( \rho_1 > \rho_2 \), and the second interface stable with \( \rho_2 < \rho_3 \). To understand the development of the flow also requires knowledge of the relationship between \( \rho_1 \) and \( \rho_3 \) and the relative depths of the layers on either side of the unstable interface. We present results for the globally stable \( \rho_1 < \rho_3 \), neutral \( \rho_1 = \rho_3 \) and unstable \( \rho_1 > \rho_3 \) cases.

2. Introduction

Rayleigh-Taylor instability of central importance to mixing processes in a broad variety of density stratified flows, ranging from atmosphere and ocean dynamics, to industrial processes and inertially confined fusion. Despite its ubiquitous and pivotal rôle in the mixing of density-stratified fluids, there is still much not understood about the instability.

To date, almost all studies of Rayleigh-Taylor instability have been limited to the simplest possible scenario: an interface, nominally normal to the destabilising acceleration, between a homogeneous layer of density \( \rho_1 \) overlying a homogeneous layer of lesser density \( \rho_2 (\rho_1) \) in a gravitational field. For most purposes, the high Reynolds number limit, where molecular effects become unimportant, is of primary interest, allowing the instability to be parameterised by a single dimensionless parameter, the Atwood number

\[
A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}.
\]

Experimental studies have considered Atwood numbers spanning almost the entire range 0 to 1 for both miscible and immiscible fluids. Many studies have used an initially stable stratification accelerated downwards by rocket motors (\( 10^2 \)g, e.g. Read 1984), linear electric motors (\( 10^3 \)g, e.g. Dimonte & Schneider 1996) and compressed gas (\( 10^5 \)g, e.g. Kucherenko et al. 1991). More simply, gravitational acceleration of layers initially separated by a barrier has provided more detailed diagnostics at a significantly lower cost (e.g. Linden et al. 1994, Dalziel 1993, Dalziel et al. 1999).
The current work is motivated by the realisation that Rayleigh-Taylor instability seldom exists in ‘ideal’ initial conditions with a single horizontal interface between to motionless fluid layers. This paper addresses the question of how the instability develops in a more complex stratification, while the companion paper by Holford & Dalziel (elsewhere in this volume) takes an initial look at the behaviour when the interface is not horizontal.

In this paper we consider a three-layer stratification such as that illustrated in figure 1. The densities of the three layers are arranged such that the upper interface is statically unstable (i.e. \( \rho_1 > \rho_2 \)) and the lower interface is statically stable, at least in the local sense that \( \rho_2 < \rho_3 \).

From these three densities we may define two dimensionless quantities analogous with the two-layer Atwood number:

\[
A_{12} = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}, \tag{2}
\]

\[
A_{13} = \frac{\rho_1 - \rho_3}{\rho_1 + \rho_3}. \tag{3}
\]

For this paper, our choice of densities the 1:2 interface is always unstable, giving \( A_{12} > 0 \), whereas the 2:3 interface is stable. However, the relationship between \( \rho_1 \) and \( \rho_3 \) has not yet been specified, and we shall consider both cases where the flow is globally stable with \( \rho_1 < \rho_3 \) (thus \( A_{13} < 0 \)), globally neutral with \( \rho_1 = \rho_3 \) (\( A_{13} = 0 \)), and globally unstable with \( \rho_1 > \rho_3 \) (\( A_{13} > 0 \)).

In this paper, we present some preliminary experiments on these three-layer stratifications in a Boussinesq system. The tank, of length \( L = 400\text{mm} \), width 200mm and working depth \( H = 500\text{mm} \), is filled with a combination of fresh and salt water, with propan2ol used to match refractive indices. The Atwood number \( A_{12} \) is held fixed at \( 2 \times 10^{-3} \), with the upper and middle layers initially separated by a barrier.

In section 3 we consider the statics of the flow before presenting the results for the globally neutral flow in section 4. The development in globally stable and unstable cases is shown in section 5, while the results of simple two-dimensional numerical modelling are described in section 6. Finally, our conclusions are presented in section 7.

3. Statics and Richardson number

Some insight may be gained by considering the instability of the upper interface and its subsequent evolution separately from the processes controlling the lower interface.

If the upper interface instability proceeds without mixing, then fluid of density \( \rho_1 \) is brought in contact with the lower layer. Ignoring the kinetic energy this fluid may contain, the lower interface will be unstable if \( \rho_1 > \rho_2 \), corresponding to \( A_{13} > 0 \).

Complete mixing between the upper and middle layer releases less potential energy and also reduces the density the lower layer is exposed to. With the configuration shown in figure 1, the post-mixing density just above the lower interface is \( \rho_{\text{mix}} = (H_1\rho_1 + H_2\rho_2)/(H_1+H_2) \), giving an effective Atwood number for the lower interface of
with $A_{\text{mix}} > 0$ indicating static instability.

This is, of course, only part of the story. The flow will have significant kinetic energy by the time it reaches the lower interface. This suggests that even if the flow is stable by the above criteria, it may penetrate into or erode away the lower layer. In such a case the controlling parameter for this penetration or erosion process will be a Richardson number. Using the standard

$$h = \frac{\alpha \Delta g \tau^2}{(5)}$$

growth for the instability of the upper interface, we may show that the Richardson number at the moment when the mixed region first contacts the lower layer depends on the ratio $A_{13}/A_{12}$ and does not depend on the depth $H_2$. This parameterisation, however, is insufficient by itself to fully describe the flow.

4. Globally neutral flow

Figure 2 illustrates the growth of the instability in this three-layer system with the globally neutral condition $A_{13} = 0$. Images from two different but nominally identical experiments are presented. The left-hand column shows the development of the lower layer, while the right-hand column shows the development of the middle layer. The times are expressed nondimensionalised by the timescale $(H/A_{12} g)^{1/2}$.

In these experiments, shear between the barrier and the fluid is almost completely eliminated through the use of fabric wrapped around the barrier that is withdrawn through the centre of the barrier during the removal process. However, the finite volume of the barrier still leads to vorticity generation during the withdrawal process through an inviscid mechanism (see Dalziel et al. 1999). In figure 2 the barrier is withdrawn towards the left, depositing negative vorticity through an inviscid mechanism, predominantly towards the right-hand end of the tank. This vorticity results in a jet propagating down the right-hand wall. This jet can be seen to impinge on the 2-3 interface in the $\tau = 1$ image in figure 2.

The growth of the instability can be divided into three distinct stages. During the initial stage the growth, the instability follows the pattern seen by Dalziel et al. (1999) for two-layers, with the only up-down asymmetry being the result of the barrier-induced initial
conditions. This initial stage stops when the mixing zone reaches the lower interface, after which the middle-layer fluid continues to rise while erosion of the lower-layer commences. During the final stage, middle-layer fluid reaches the top of the domain, effectively shutting off the generation of further kinetic energy, and the system gradually runs down.

Figure 2 concentrates on the second stage, where the lower layer is being eroded by the kinetic energy provided by the release of potential energy in the instability above. The density difference across the interface is small but stable as a result of the mixing during the Rayleigh-Taylor instability phase. Ignoring the jet near the right-hand end that can be seen in the \( \tau = 1 \) panel, the instability penetrates into the lower \( \rho_3 \) layer more or less uniformly along the length of the tank. Lower-layer fluid is entrained into the turbulent zone above, lifting some of it far up the tank. As can be seen by comparing the two images at \( \tau = 1 \) and at \( \tau = 2 \), the penetrating fluid is a mixture of \( \rho_i \) and \( \rho_2 \) that has a density less than that of the lower layer (recall that \( \rho_1 = \rho_3 > \rho_2 \)), indicating the crucial role played by the kinetic energy of the instability. As the instability continues to develop above the lower interface, the fluid reaching the interface becomes denser, gradually rising from \( \rho_2 \) towards \( \rho_1 \). However, the at the same time the kinetic energy released by the instability decreases due to mixing and dissipation, increasing the Richardson number of the lower interface and eventually preventing any further mixing.

The evolution of the density field is illustrated in figure 3 which shows the vertical profile of the concentration of middle-layer fluid averaged over the length of the tank and an ensemble of 12 experiments. Each of the curves is separated by 0.2 nondimensional time units.

Figure 4 re-plots the data presented in figure 3, but after scaling on the width of the mixed zone and maximum concentration. Each curve has also been shifted so that their centroids coincide. The clear collapse of these curves demonstrates the fundamental self-similarity of the development of the instability and the resulting mixing. Unfortunately, this self-similarity by itself is not sufficient. While a similarity theory can be constructed to cover both the growing instability above and the penetration into the lower layer below, the theory needs to be closed using a parameterisation of the flux across the lower interface.

Figure 3: Evolution of the horizontal mean concentration of middle-layer fluid.

Figure 4: Scaled density profiles, demonstrating the self-similar behaviour of the mixing process.
5. Globally stable and unstable

Figure 5 shows the development of the instability under both globally stable (left-hand column) and globally unstable (right-hand column) conditions with $A_{13}$ set to $-A_{12}$ and $\frac{1}{2}A_{12}$, respectively. In the globally stable case, there is relatively little erosion of the lower layer. While there is sufficient kinetic energy at the time of first contact for entrainment to occur, this situation does not last long. In contrast, with the globally unstable case, the higher density found in the upper layer is insufficiently diluted during the development of the instability, thus allowing it to penetrate unhindered into the lower layer.

6. Numerical simulations

Dalziel et al. (1999) have demonstrated that when appropriate initial conditions are employed, three-dimensional numerical simulations are capable of modelling the qualitative and quantitative details of the development of Rayleigh-Taylor instability between two layers. They also saw that the overall growth of the instability in the experimental apparatus used here was dominated by the two-dimensional perturbation introduced by the removal of the barrier. The question addressed here is whether such two-dimensional simulations are useful in more complex stratifications.

The code used is an explicit, finite volume implementation of the streamfunction-vorticity equations. Vorticity and density are advected using a flux-limited third order advection scheme, while the vorticity is inverted to give the streamfunction by means of a multigrid method. Numerous tests and the use of this and similar codes in a broad variety of stratified flows have demonstrated its accuracy.

Figure 6 shows the development of the instability from these simulations for a globally neutral $A_{13} = 0$ flow. The initial conditions on the velocity field are the same two-dimensional approximation to the measurements of the barrier-induced perturbation as used by Dalziel et al. (1999). Comparison with figure 2 demonstrates that the simulated flow lacks many of the turbulent features seen clearly in the experiments. For the two-layer instability of Dalziel et al., the lack of fine scale detail and mixing in the two-dimensional simulations did not affect the longer-term growth of the instability. How-
ever, with the more complex three-layer stratification used here, these finer details and mixing are a critical part of the development and control the penetration into the lower layer. Their absence here leads to a significant over-estimate of the amount of penetration into the lower layer: the upper layer fluid is presented essentially undiluted to the lower interface, eliminating the stabilising buoyancy force found in the experiments.

7. Conclusions

This paper has demonstrated the behaviour of Rayleigh-Taylor instability in flows containing regions of local static stability as well as instability. The initial development of the Rayleigh-Taylor instability at the unstable interface proceeds identically to that in the simple two-layer flow. No substantial differences occur until the growing mixing region reaches the stable interface, at which time the flow is far from a minimum energy state. The ability for the Rayleigh-Taylor induced flow to penetrate or entrain fluid from the stable layer depends on a Richardson number which can be shown to depend primarily on the density differences between the three layers and not the depth of the layers. The results also show a self-similar behaviour of the penetration and mixing across the stable interface.

The release of potential energy by the instability drives a rearrangement of the density field and provides the kinetic energy necessary for a nominally stable interface to be eroded. Two-dimensional numerical simulations have helped to demonstrate that the degree of mixing provided by the instability is an important component in determining the penetration into or erosion of the lower layer.

References


