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Resuspension onset and crater erosion by a vortex ring interacting with a particle layer

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This paper presents results from an experimental investigation of the interaction of a vortex ring with a particle layer. The flow dynamics during the onset of particle resuspension are analysed using particle image velocimetry, while a light attenuation method provides accurate measurements of the final eroded crater shape. This work is a continuation of the research described in R. J. Munro, N. Bethke, and S. B. Dalziel, “Sediment resuspension and erosion by vortex rings,” Phys. Fluids 21, 046601 (2009), which focused on the general resuspension onset dynamics and initial crater formation. Here, we analyse the velocity induced by the vortex ring on the particle layer surface during the resuspension of particles for different particle sizes, and the shape and size of the final craters that are formed by the impact of the vortex ring. We find that the boundary condition is characterised by a quasi-slip velocity at the particle layer surface, independent of the particle size. The particle diameter, and thus bed permeability, is found to have a significant effect on the final crater characteristics.

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I. INTRODUCTION

The dispersion of particles and associated erosion of particle layers by fluid flows, whether through the process of bedload transport or resuspension, is ubiquitous in a range of environmental and industrial fluid dynamical applications. Prominent examples are the transport of sediment in rivers and estuaries, the wind-blown transport of sand and snow in deserts and mountains, respectively, as well as the presence of particles in pipeline transportation of fluids, or the settling tanks in water treatment plants. While in some applications the suspension of particles is a necessary and important feature of the fluid flow, in others it is desirable that particles remain settled, rather than being resuspended by the fluid.

In many of the applications that have been mentioned, the corresponding fluid flow is turbulent, with Reynolds numbers easily reaching $Re \sim O(10^6)$. With the understanding of particle transport hinging on an understanding of the underlying fluid flow, no generally valid macroscopic model of particle transport by fluid flows exists to date. Indeed, there are a number of semi-empirical formulations relating the rate of particle erosion to flow parameters such as the bed shear stress, which have been tested in a variety of channel flow settings using different flow rates and geometries. Studies have examined laminar channel flows over thick particle beds, as well as turbulent channel flows over particle-laden solid boundaries (where the particle layer thickness is of the order of only a couple of particles), and the interaction of wall jets with deep particle layers, just to name a few examples. While much insight has been gained through studying the bulk properties of the interaction of such flows with particles, it seems that often the results are only applicable in closely related settings.

In a range of these flow settings, individual flow structures have been identified as being efficient mechanisms for the resuspension of particles. Previous researchers have experimentally...
investigated the vertical velocity fluctuations in a turbulent channel flow, with the aim to relate these velocity fluctuations to their possible influence on a layer of sediment. This suggested that a net upward momentum flux near the lower boundary can act to keep fine particles in suspension. This net upward momentum flux appears to be connected with turbulent ejection events that propel slow fluid from near the wall further into the flow. On the other hand, it was shown that a downward flux of momentum can be caused by sweep motion: high speed fluid that rushes towards the wall.

Most studies (e.g., Refs. 2–4) have concentrated on the flow dynamics observed in the near wall regions of a fully developed turbulent open channel flow (of Reynolds number $Re \sim O(10^4)$) over a particle laden boundary. The sediment has been found to arrange itself along streaks of low velocity on the channel bottom, implying the presence of streamwise vortices (flanking the low velocity streaks) near the channel bottom. Several of these counter-rotating streamwise vortex pairs were seen to emerge and collapse quasi-periodically in time, while their position seemed to be distributed randomly. It is in these vortices that the turbulent bursts that were observed by Wei and Willmarth\textsuperscript{1} occur. Particles were flung into the flow, along with the fluid ejected by the turbulent bursts. Streamwise vortices are a commonly observed feature of turbulent boundary layers. They are found to act as sites for the production of turbulent kinetic energy, with regions of high skin friction correlating with the location of these vortices.\textsuperscript{5} This would explain the efficiency with which streamwise vortices suspend and capture particles.

In the light of the importance of these streamwise vortices in resuspending particles into the flow, it is reasonable to study the interaction of an isolated vortex with a particle layer. This approach was first pursued qualitatively by Sutherland\textsuperscript{6} who investigated the interaction of channel flows, pulsating jets and vortex rings with particle layers. The creation of vortex rings from orifices or tubes represents a robust, repeatable, and easily conducted experiment. Although this model problem differs from the interaction of naturally occurring flows with particle layers, an improved understanding of the mechanisms that govern the resuspension of particles by a vortex ring will shed light on more complicated flows. A vortex ring’s circulation and impulse are well defined and measurements of velocities and vorticity can be achieved with methods such as particle image velocimetry (PIV). Furthermore, the existence of an analytical solution for the inviscid flow field generated by a translating vortex ring in the presence of a solid boundary allows for a systematic investigation of the effects of a particle layer on the flow field.

Most of the previous work on vortex rings has focussed on their formation, with Refs. 7–9 presenting numerical studies, while experimental investigations can be found in Refs. 10–12, for example. A theoretical description of vortex ring formation is given in Ref. 13. Some research has also investigated the collision of a vortex ring with a boundary. Various experimental\textsuperscript{14,15} and numerical\textsuperscript{15,16} studies of vortex rings impinging on solid boundaries have described the dynamical evolution of a viscous vortex ring approaching a solid boundary. These works all show vortex deceleration and stretching as the ring approaches the boundary, and the formation on the boundary of a boundary layer with vorticity of opposite sign. This boundary layer was found to roll up into a secondary vortex ring, which in its interaction with the primary vortex ring separated from the wall and caused a rebound of the primary vortex off the wall. These processes were found to repeat themselves until coherent vortex motion was lost and the vortex ring disintegrated.

The impact of a vortex ring on a particle bed was first studied quantitatively by Munro, Bethke, and Dalziel,\textsuperscript{17} giving a detailed account of the key features of the resuspension of particles and the associated early erosion of the particle layer. The characteristic deceleration and stretching of the vortex ring, as well as the creation of secondary vorticity that has been observed for the impact on a solid boundary, were also present in the interaction with the particle layer. The velocity induced by the vortex ring on the particle layer was shown to have a peak underneath the vortex ring core, with the time of particle resuspension coinciding with a temporal maximum in induced velocity. The erosion pattern and dimensions were studied, indicating that the early stages of crater formation proceeded in a self-similar fashion. The dimensions of the eroded craters were found to be dependent on the particle size.

The present paper extends the analysis of Ref. 17 by studying the evolution of the bed velocity during the resuspension event for different particle sizes and comparing the findings to a theoretical model for the impact of a vortex ring on a solid boundary, paying particular attention to the effect
of slippage of particles on the layer surface and the permeability of the particle bed as a whole. The dimensions and shape of the eroded crater are investigated for a range of impact velocities and different particle sizes.

The remainder of the paper is structured as follows: after a brief overview of the experimental setup (Sec. II), a qualitative description of the impact of a vortex ring on a solid boundary is given and a theoretical model of an inviscid circular line vortex is introduced (Sec. III A). The theoretical predictions for the vortex ring trajectory and bed velocity are then compared quantitatively with the experimental data on a solid boundary and on a particle layer (Secs. III B–III D). Subsequently, the eroded craters are studied with a view to the conversion of vortex ring kinetic energy to the resuspension of particles (Sec. IV). Finally, conclusions are presented (Sec. V).

II. EXPERIMENTAL SETUP AND PROCEDURE

The bulk of the experiments were conducted in a transparent acrylic tank with internal dimensions 300 mm \(\times\) 300 mm \(\times\) 400 mm, filled with water to a height of 360 mm above a 10 mm thick layer of particles (this was deep enough for the effect of the bottom wall of the tank not to be felt). A vortex ring generator, located at the top of the tank, fired vortex rings downwards onto the particle layer. A sketch of the setup, which is essentially identical to that used by Munro, Bethke, and Dalziel,\(^{17}\) is provided in Figure 1.

The particle layer at the bottom of the tank was composed of approximately monodisperse small transparent glass spheres (ballotini). A range of particle sizes was used in different experimental runs, the smallest having a nominal diameter of 90 \(\mu\)m and the largest 1000 \(\mu\)m (all with a density of 2500 kg/m\(^3\)). Prior to the beginning of an experiment, the particle layer was smoothed to give a uniform layer depth by dragging a scraper over the surface. Two metal bars, aligned with opposite edges of the tank bottom, guided the scraper and thus set the thickness of the particle layer. The scraping of the particle surface along the metal bars provided some compacting of the particle layer. Even though the degree of compacting could not be quantified, the experimental results were found to be repeatable and very robust, suggesting a well compacted state for the particle layer.

FIG. 1. Sketch of the experimental setup showing the water tank and the apparatus for the creation of vortex rings in a quiescent fluid.
The vortex rings were created at the open end of a circular tube of internal diameter \( D_t = 39 \text{ mm} \) that was vertically aligned with the tank walls and centred on the tank. This tube was submerged to a depth of 100 mm and connected to a bicycle “track pump” via a plastic hose. The pump handle was actuated by the winding up of a cord onto a motor-driven spindle, pushing air into the tube and water out of it into the tank. The boundary layer on the inside wall of the tube rolls up and separates at the end of the tube thus producing a single vortex ring. The stroke length \( L_s \) of the pump was adjustable to yield a range of vortex rings with differing formation numbers, \( L_s/D_t \). As the internal diameter of the tube, \( D_t \), and the pump, \( D_s = 29 \text{ mm} \), were not identical, the actual length of the slug of fluid displaced from the tube is given by \( L = (D_s/D_t)^2 L_s \approx 0.55L_s \). The stroke time \( T_s \) was varied with a dial on the motor, which controlled the speed with which the cord was wound up and hence the speed with which the handle was pushed; this time was measured with an oscilloscope connected to the pump. To ensure that the speed with which the handle was moved was constant, the cord was given sufficient slack so that by the time it started pulling the handle, the spindle was rotating at a constant speed. The vortex rings translated downwards under their self-induced velocity and hit the bed, resuspending particles. A video showing the impact of a vortex ring on a 250 \( \mu \text{m} \) particle layer can be accessed in Ref. 18. Air bubbles trapped in the vortex core visualise an instability known as Kelvin waves.\(^{19,20} \) The presence of bubbles in the core significantly increases the amplitude of the waves compared to when bubbles are not present (which was confirmed through other visualisations). Note that the amplitude of the Kelvin waves decreases dramatically as the ring is stretched as it approaches the wall.

The parameters varied in the experiments were the vortex ring formation number and velocity (and thus the vortex ring Reynolds number), as well as the particle diameter. The formation number ranged between \( L_s/D_t \approx 0.55 \) and \( L_s/D_t \approx 1 \). This corresponds to pump stroke lengths of \( L_s = 40 \text{ mm} \) and \( L_s = 70 \text{ mm} \), respectively. The vortex ring propagation velocities were set by the stroke time \( T_s \), which was varied between \( T_s \approx 50 \text{ ms} \) and \( T_s \approx 100 \text{ ms} \). This resulted in vortex ring propagation velocities in the range of \( 100 \lesssim U \lesssim 700 \text{ mm}/\text{s} \) (pressure losses between the pump and ring generator made this relationship nonlinear), giving vortex ring Reynolds numbers \( 4 \times 10^3 \lesssim Re_t = UD_t/v \lesssim 30 \times 10^3 \) when based on the tube diameter.

Experiments were conducted to study the dynamics of the interaction between the vortex ring and the particle layer at its early stage, with a focus on the evolution of the vortex ring, as well as to investigate some of the properties of the particle bed after the interaction had taken place. The experiments were recorded with digital cameras and analysed using the software DigiFlow.\(^{21} \)

For the resuspension onset experiments, a side view was recorded to study the dynamics of the resuspension process. This captured the vortex ring while translating and during impact. The water was seeded with neutrally buoyant particles, and PIV was employed to yield velocity and vorticity fields during the resuspension process. As the vortex ring impact and particle resuspension occurred on a time scale of less than one second, a high speed camera (Dantec NanoSense Mk III) was used for the recording. A frame rate of 700 Hz was found to be sufficient to capture the various stages of the interaction with a resolution of \( 1024 \times 1024 \) pixels. A 300 W xenon arc lamp with a parabolic dichroic reflector was used to create the light sheet of about 3 mm thickness. The interrogation window had a size of \( 21 \times 21 \) pixels, with a spacing of 10 pixels between the window centres (the pixels had a size of \( 0.155 \times 0.155 \text{ mm}^2 \)). The light sheet intersected with the bed, allowing for unobstructed viewing of the particle layer surface to within a couple of pixels. Typically, there were about five seeding particles in every interrogation window. Eliokem Pliolite VTAC (a co-polymer resin) particles, with a nominal diameter of \( 180 \mu \text{m} \) and a density of \( 1020 \text{ kg/m}^3 \), were employed for the seeding.

In the experiments investigating the erosion capability of vortex rings of different formation number and velocity in combination with different types of particle layers, a light attenuation method was employed to determine the particle layer thickness. This method, developed by Munro and Dalziel,\(^{22} \) relies on the fact that the particles (and the tank) are transparent. A uniform light source (here an array of fluorescent lamps) illuminates the particle layer from below. Some of the light will be absorbed as it passes through the particle layer. The resulting light intensity at any point in the recorded image of the particle bed then corresponds to the layer thickness at that point. As the method’s accuracy relies on a clear image of the illuminated bed, we do not consider such top view...
recordings for the temporal evolution of the crater during resuspension, as the suspended particles obstruct the view on the bed below. The top views were recorded using a JAI CV-M4+CL camera.

III. RESUSPENSION BY VORTEX RINGS

A. Interaction with a solid boundary: Reference case

Before the interaction of a vortex ring with a particle layer is considered, it is instructive to recall the dynamics of the impact of a vortex ring on a solid boundary. The data presented here is non-dimensionalised using the (initial) ring diameter $D$ as the typical length scale, and the constant ring propagation velocity $U$ as the velocity scale. Both are measured at a height $D$ above the bed where the ring is not yet significantly affected by the presence of the boundary. Time is thus non-dimensionalised as $\tilde{t} = Ut/D$. All vortex rings discussed in this section were created with a stroke length $L_s = 70$ mm. Three realisations were recorded for each set of parameters presented below. Little variation could be observed between these realisations. For this reason, and because we are interested in the bulk properties of the resuspension process, we present ensemble averaged results.

Figure 2 shows typical ensemble averaged PIV images of a vertical plane through the centre of a vortex ring approaching a solid boundary (the solid boundary location coincides with the bottom edge of the image). The colour scheme represents vorticity, and the arrows indicate the flow velocity. (The corresponding PIV video can be accessed at URL: http://dx.doi.org/10.1063/1.4716000.1.) While the PIV was carried out between consecutive frames, the series display images spaced $\Delta \tilde{t} \approx 0.42$ apart; this corresponds to 84 frames (0.12 s in this case). The point in time where the centre of the vortex ring reaches a height $D$ above the bed is chosen to represent $\tilde{t} = 0$; the first image in Figure 2 corresponds to $\tilde{t} \approx 0.51$. The propagation velocity of the vortex ring was $U \approx 120$ mm/s, with a Reynolds number $Re_e \approx 4200$ (based on the ring diameter), which is near the low end of the range of vortex ring velocities considered for the work in this paper.

The PIV results show the distribution of vorticity in the flow. If the radius $R$ of the ring is taken as the distance from the centreline of the ring to the centroid of the vortex core, and $a$ is the radius of the core itself, the vortex ring in Figure 2 is characterised by a ratio $a/R \approx 0.35$. Here, the core radius is found by measuring the size of the vorticity patch once the ambient vorticity noise level is subtracted. This is equivalent to vorticity values higher than about 17.5% of the peak vorticity, although the results were found to be essentially the same even if a different definition had been adopted. Figure 2 documents an increase in the diameter of the ring, $D$, as it approaches the boundary, and the generation of secondary vorticity of opposite sign at the no-slip wall. This annular boundary layer of secondary vorticity separates from the surface as it is swept outwards by the primary ring, rolling up into a secondary vortical ring that is advected around the primary. Even as this happens, further opposite-signed vorticity is generated on the wall, allowing this process to be repeated. The strength of the primary ring decreases through the interaction with the secondary ring.

These experimental results were obtained in the absence of particles on the boundary, and thus one complication has been removed from the experiment. The effects of viscosity cannot be suppressed in this experimental setup, however. To investigate the dynamics of the impact of a vortex ring on a solid boundary in the absence of viscosity, an inviscid theoretical model is considered. By comparing the inviscid interaction dynamics with the viscous impact on a solid boundary and on a particle layer, we expect to be able to identify what role the no-slip condition and the presence of particles on the boundary have on the evolution of the flow.

As shown in the Appendix, we can model the trajectory of an inviscid axisymmetric vortex ring approaching a wall as

$$\frac{dZ}{dt}(R(t), Z(t)) = W(R(t)) + \frac{\gamma}{4\pi} \frac{K(\bar{k}) - E(\bar{k})}{(R^2 + Z^2)^{1/2}},$$ \hspace{1cm} (1)$$

$$\frac{dR}{dt}(R(t), Z(t)) = \frac{\gamma}{4\pi} \frac{(R^2 + 2Z^2)E(\bar{k}) - 2Z^2K(\bar{k})}{RZ(R^2 + Z^2)^{1/2}};$$ \hspace{1cm} (2)
\[ \tilde{t} \approx 0.51 \]

\[ \tilde{t} \approx 0.93 \]

\[ \tilde{t} \approx 1.35 \]

\[ \tilde{t} \approx 1.77 \]

where \( Z \) is the vortex ring height above the boundary, \( R \) is the vortex ring radius, \( \gamma \) is the strength of an equivalent circular line vortex, \( K(\bar{k}) \) and \( E(\bar{k}) \) are the complete elliptic integral of the first and second kind, respectively, and \( \bar{k} \) is given by

\[ \bar{k}^2 = \frac{4Rr}{(z + Z)^2 + (r + R)^2}. \]

Here, \( r \) and \( z \) are the radial and vertical coordinates in a cylindrical polar system. The vertical velocity of the vortex ring, \( dZ/dt \), is given by the sum of the self-induced velocity \( W \) for a ring with a finite core radius \( a \), and the velocity due to the presence of the boundary, while the sole contribution to the stretching of the ring, \( dR/dt \), comes from the boundary (i.e., the vortex ring would not stretch in its absence).

The self-induced velocity is found from a parameterisation by Norbury\(^\text{23} \) in terms of \( \alpha_c(t) = a/R \), the instantaneous relative core radius: that paper tabulates numerical solutions for the non-dimensional propagation velocity \( \tilde{W} \) for a range of values of \( \alpha_c \). The dimensional propagation

**FIG. 2.** PIV results of a vortex ring (\( D \approx 35 \text{ mm} \)) approaching a solid boundary; images are ordered from top to bottom, spaced \( \Delta \tilde{t} \approx 0.42 \) apart, starting at \( \tilde{t} \approx 0.51 \) (enhanced online) [URL: http://dx.doi.org/10.1063/1.4716000.1].
velocity relates to $W$ as

$$W = \Omega R^2 \alpha_c^2 W.$$  \hspace{1cm} (3)

The theoretical model is initialised with the measured experimental ring and core radii. In an inviscid flow, $\omega_\theta = \Omega r$, where $\Omega$ is constant in the core of the ring (and zero outside the core), but this is not the case for the experimental rings, where viscosity and the formation process inevitably play a role and produce a continuous vorticity profile. We therefore set $W$ equal to the measured propagation velocity and find the equivalent $\Omega$ from Eq. (3). With the core size, ring diameter, and propagation velocity matched as closely as possible, however, the impact of the different vorticity distribution is found to be negligible for the calculation of the vortex ring trajectory. The volume of the core is conserved in the absence of viscosity or entrainment and so $R^3 \alpha_c^2$ is a constant, allowing for the determination of $\alpha_c$ from the instantaneous ring radius $R(t)$. Equations (1) and (2) are integrated numerically using a fourth order Runge-Kutta scheme to give the vortex ring trajectory.

Figure 3 shows a comparison of the theoretical and experimental vortex ring trajectory, vertical position and diameter. The experimental trajectory of the vortex ring in Figure 3(a) is obtained from tracking the centroid of the vortex core. The normalised vortex ring diameter and the normalised vertical position of the vortex ring above the boundary are shown in Figures 3(b) and 3(c).

The plots illustrate the increase in vortex ring diameter as the ring approaches the wall, as well as the vortex ring’s deceleration. They also document the existence of a minimum distance from the wall beyond which the experimental vortex ring does not propagate. Instead, the vortex ring rebounds and gains height again. Such a rebound has been shown to be due to the creation of secondary vorticity at the wall for the case of a two-dimensional vortex pair. It seems reasonable to assume that this is the case also for a vortex ring. The ring radius more than doubles over the course of the interaction, followed by a sudden stagnation in growth and even slight decrease. This coincides with the ring’s rebound from the boundary and the “eruption” of the secondary ring from the bed, hindering the primary ring’s radial expansion. As the secondary ring is swept around the primary ring, the diameter of the latter continues to grow, albeit at a decreased rate.

The inviscid theoretical model is seen to perform very well up until the presence of the viscous boundary layer becomes important through its interaction with the primary ring. The theoretical solution illustrates how a vortex ring in an inviscid fluid would approach the wall asymptotically, stretching without bound. It is noted that the asymptotic approach towards the wall is also maintained for an inviscid ring with a finite core size and that the diameter of the core decreases faster than its distance from the wall.

B. Impact on a particle layer

The onset of resuspension of particles was studied by recording the impact of a vortex ring on a layer of particles. The experiment was conducted for five different glass ballotini sizes (90 $\mu$m, 150 $\mu$m, 250 $\mu$m, 500 $\mu$m, 1000 $\mu$m), with the initial layer thickness kept constant at 10 mm in all cases. With the pump stroke length set to $L_s = 70$ mm, the stroke time just yielding resuspension for the particular particle size was determined through trial. Particles were considered resuspended when they had actually left the particle bed, rather than being moved along it (bedload transport). While there was some degree of subjectivity in this determination of the critical stroke time and vortex ring velocity, the “error” in the latter is no larger than $\pm5\%$.

The impact at critical conditions of a vortex ring on a 150 $\mu$m glass ballotini layer is shown in the series of PIV images in Figure 4. (The corresponding PIV video can be accessed at URL: http://dx.doi.org/10.1063/1.4716000.2.) The right-hand side of the PIV sequence displays the velocity and vorticity field from the impact on the 150 $\mu$m particles, while the left-hand side shows the same vorticity field (with the sign swapped) overlaid by contours from the impact on the solid boundary (Figure 2). The vorticity contours are at 0, $\pm20$, $\pm40$, $\pm60$/s. Here, the first image corresponds to $\tilde{t} \approx 0.40$, while consecutive images are spaced $\Delta \tilde{t} \approx 0.22$ apart ($\approx 0.08$ s). The flow field in the case of the interaction of a vortex ring with a particle layer can be seen to be qualitatively very similar to the solid boundary case, with the vortex ring Reynolds number, $Re_v \approx 4500$, not differing much
FIG. 3. Comparison of the vortex ring theoretical (a) core trajectory, (b) vertical position, and (c) diameter evolution with the experimental results.

from the previous sequence ($Re_v \approx 4200$). Vortex stretching and generation of secondary vorticity are observed. The only notable difference between the PIV images here and those in Figure 2 is the comparative strength of the boundary layer. While the gradual increase in secondary vorticity can be observed distinctly on the solid boundary, it is difficult to detect the boundary layer growth in Figure 4. Only with the separation of the secondary vortex from the bed, which coincides with the resuspension of particles and their ejection into the fluid, does the boundary layer become clearly...
visible. A discussion of whether the difficulty in resolving the boundary layer is a result of the quality of the PIV recording and analysis, or a manifestation of an actual difference between the boundary layers on a solid boundary and a particle bed, is deferred until Secs. III C and III D.

A comparison of the impact trajectories for the 90 μm and 1000 μm particle layers is illustrated in Figure 5. This shows that there is little difference in the growth of the ring diameter as the particle layer is approached. One can observe a slightly more pronounced stretching for the 90 μm case, as well as a closer approach to the boundary, which is still within the bounds of experimental variation. Possible physical reasons for this will be explored in Secs. III C and III D.

Figure 6(a) shows the normalised bed velocity, $\tilde{U}_b$, as a function of time for the resuspension onset on a layer of 90 μm glass ballotini, starting at $\tilde{t} \approx 0.51$, with a spacing of $\Delta \tilde{t} \approx 0.13$ ($\approx 0.04$ s) between consecutive images. The bed velocity increases as the vortex ring approaches, while the velocity peaks move apart with the increasing vortex ring diameter. The bed velocity here, and
in all other experiments presented, was measured 0.5 mm above the nominal surface, whether this was a particle layer or a solid boundary. For a particle layer, the nominal surface was defined as a horizontal line through the maximum elevation of the bed. This somewhat arbitrary (but constant) measurement height was chosen since a measurement height based on particle diameter, for example, would not be applicable to the solid boundary. Using a measurement height related to the thickness of the boundary layer, on the other hand, requires knowledge of the structure and thickness of the boundary layer. By using the same height above the solid boundary and particle layer surfaces to measure the bed velocity the effect of differing boundary conditions is captured.

The normalised bed velocity predicted by the theoretical model (given in the Appendix) is shown in Figure 6(b) for comparison. As expected, the theoretical bed velocity increases monotonically as the vortex ring approaches. The evolution of the theoretical bed velocity was initialised with the vortex ring dimensions and propagation velocity of the 90 \( \mu \text{m} \) resuspension case and shows profiles corresponding to the same non-dimensional times, therefore the theoretical and experimental bed velocities are directly comparable. It is found that the bed velocity profile on the particle bed (Figure 6(a)) agrees well with the theoretical prediction during the bedload phase and up to the time at which particles leave the bed. Contrary to the inviscid theoretical model, the experimental bed velocity does not continue to grow indefinitely. Instead, it reaches a maximum and then decreases again. The peak in the bed velocity coincides with the moment of ejection of particles away from

FIG. 6. Bed velocity (a) induced by a vortex ring (\( D \approx 40 \text{ mm} \)) on a 90 \( \mu \text{m} \) particle layer and (b) as predicted by the inviscid circular line vortex model; consecutive profiles are \( \Delta \tilde{t} \approx 0.13 \) apart, graph times are \( \tilde{t} \approx 0.51, \tilde{t} \approx 0.64, \tilde{t} \approx 0.77, \tilde{t} \approx 0.90, \tilde{t} \approx 1.03, \tilde{t} \approx 1.16 \). The order of lines is “dotted”, “dashed”, and “solid”, with the sequence repeated.
the bed and the separation of the boundary layer. The maximum experimental bed velocity seen here is approximately three times as large as the initial propagation velocity of the vortex ring, \( U_b \approx 3U \).

C. Critical Shields parameters

A widely used measure for the ease with which particles are displaced from a particle bed is the Shields parameter

\[
\Theta = \frac{\rho u^2}{(\rho_p - \rho)gd},
\]

effectively the ratio of hydrodynamic lift to buoyancy forces. Here, \( u \) is a representative flow velocity, and \( d \) and \( \rho_p \) are the particle diameter and density, respectively. For statistically steady flows in channels or the atmosphere, the definition of \( \Theta \) is usually based on the slip velocity \( u^* \) (and hence shear stress) at the wall, and the assumption of a logarithmic boundary layer. This relates the shear stress with the buoyancy forces. However, here we do not have a steady flow and we do not know \textit{a priori} the structure of the boundary layer, so this choice is inappropriate. Moreover, the PIV resolution is not sufficient to measure the wall shear stress in the present experiments. This issue was also encountered in both Refs. 25 and 17, where the propagation speed of the ring, \( u = U \), was employed in the Shields parameter instead. Here, we have higher resolution PIV velocity measurements and so can explore the observed velocity just above the bed. Our approach is thus intermediate between the friction velocity based approach for steady flows, and the incident structure based approach of Refs. 25 and 17.

A criterion for resuspension is thus established based on a critical Shields parameter \( \Theta_c \), which is reached when the inertial lift overcomes the buoyancy force. With this definition of the Shields parameter, particles leave the bed once \( \Theta \geq \Theta_c \). The representative velocity at which this occurs is termed the critical velocity. We choose the maximum radial bed velocity (and not the vortex ring propagation velocity) as the representative velocity for the resuspension onset for each particle size and type, and define the critical velocity as

\[
U_c = \max_{r, t} U_b(r, t).
\]

In accordance with a Shields parameter based on the friction velocity, the variation of the critical Shields parameter with a Reynolds number incorporating this friction velocity is usually investigated. Here, following Refs. 25 and 17, we consider the dependence, if any, on the particle Reynolds number, \( Re_p = v_s d/\nu \), which is the Reynolds number of a single isolated particle settling at its terminal velocity in a quiescent fluid. Thus \( Re_p \) is independent of the flow and is characteristic of the particle type considered. Again, this does not correspond to the traditional use of the Shields parameter in fields such as hydraulic engineering, but we believe this to be a more useful approach in the current context, where the effect of particle type on the resuspension dynamics is investigated. The settling velocities \( v_s \) have been determined experimentally, as the Stokes settling velocity \( V_s = 1/(18\mu)gd^2(\rho_p - \rho) \) is not valid for the larger particles, considerably overestimating the velocity for the 500 \( \mu \)m and 1000 \( \mu \)m glass ballotini. Table I summarises the particle properties. It is noted here that while \( \Theta_c \) varies significantly with \( Re_p \), the variations in \( U_c \) are much smaller and not monotonic.

Figure 7(a) shows a log-log plot of \( \Theta_c \) against \( Re_p \) for the different particle sizes. The error bars associated with the measurement of the critical bed velocity and therefore the estimation of the critical Shields parameter are included.

A clear decrease of \( \Theta_c \) with increasing \( Re_p \) can be observed; here this corresponds to a decrease of \( \Theta_c \) with increasing particle size \( d \), indicating that larger particles are comparatively easier to resuspend. The line of best fit has a slope of \(-0.50 \pm 0.04\). An estimate for the scaling of the critical Shields parameter can be obtained when considering a single particle (on a solid boundary) exposed to an external flow of velocity \( u \), as outlined in Refs. 25 and 17. The flow generates a boundary layer on the surface, with the boundary layer thickness depending on the details of the external flow, and a viscous sublayer, whose thickness \( \delta \) scales with the magnitude of the flow, \( u \), such that
TABLE I. Properties of the particles used in the resuspension onset experiments. Tabulated are the nominal particle diameters \( d \), experimental settling velocities \( v_s \), Stokes settling velocities \( V_s \), critical bed velocities \( U_c \), particle Reynolds numbers \( Re_p \), and critical Shields parameters \( \Theta_c \).

<table>
<thead>
<tr>
<th>( d (\mu m) )</th>
<th>( v_s (mm/s) )</th>
<th>( V_s (mm/s) )</th>
<th>( U_c (mm/s) )</th>
<th>( Re_p )</th>
<th>( \Theta_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>7.7</td>
<td>6.6</td>
<td>348.4</td>
<td>0.69</td>
<td>91.65</td>
</tr>
<tr>
<td>150</td>
<td>15.6</td>
<td>18.4</td>
<td>307.1</td>
<td>2.34</td>
<td>42.73</td>
</tr>
<tr>
<td>250</td>
<td>37.4</td>
<td>51.0</td>
<td>326.2</td>
<td>9.31</td>
<td>28.92</td>
</tr>
<tr>
<td>500</td>
<td>64.3</td>
<td>204.0</td>
<td>331.4</td>
<td>32.01</td>
<td>14.93</td>
</tr>
<tr>
<td>1000</td>
<td>158.2</td>
<td>815.9</td>
<td>282.8</td>
<td>157.58</td>
<td>5.43</td>
</tr>
</tbody>
</table>

\( \delta \sim v/u \). Although the idea of a viscous sublayer is normally tied to a logarithmic boundary layer and associated friction velocity, on a more fundamental basis the sublayer is the region close to the boundary where molecular viscosity is more important than inertial effects. We can borrow some of the ideas of turbulent boundary layers with a turbulent eddy viscosity \( \nu_T \sim uz \), without imposing the stress continuity and scaling of \( u \) with \( z \) that leads to the logarithmic boundary layer, and still find that \( \delta \sim v/u \). It is pointed out, however, that here the fluctuations in \( u \) causing a turbulence-like transfer may be purely due to the bed roughness. Within such an approach, the velocity experienced by the particle depends on its size. Large particles that protrude from the viscous sublayer experience a velocity of the order of the free stream velocity \( u \), whereas particles that are shielded by the viscous sublayer experience a velocity of the order \( ud/\delta \). For a large particle, the criterion of lift overcoming buoyancy thus takes the form \( \rho u^2 d^2 > (\rho_p - \rho)gd^3 \), which on re-arranging for the velocity and combining with the expression for the Shields parameter, Eq. (4), gives

\[
\Theta_c \sim 1.
\]  

Hence, one does not expect a variation of critical Shields parameter with particle Reynolds number for particles large compared to the viscous sublayer thickness. Following the same argument for the small particles, but now with an effective velocity \( u^2d/v_s \), yields

\[
\Theta_c \sim \left( \frac{\rho}{\rho_p - \rho} \right) \frac{v_s^2}{gd^3} \sim Re_s^{-1/2},
\]  

where the particle Reynolds number based on the Stokes settling velocity, \( V_s \), is denoted by \( Re_s \), to distinguish it from \( Re_p \), which is obtained using the experimental settling velocity \( v_s \). Note that \( Re_s \) can be calculated even for particles where \( Re_p \gg 1 \), but in such cases \( Re_s \) does not represent the Reynolds number of a settling particle. When \( Re_p \ll 1 \), however, we expect \( Re_s \approx Re_p \).

![FIG. 7. Critical Shields parameters as a function of particle Reynolds numbers (a) \( Re_p \) and (b) \( Re_s \). The solid line indicates the best fit to the data, the dashed line has a slope of \(-0.5\) and is a guide to the eye.](image-url)
To assess the performance of the proposed scaling, the critical Shields parameters are plotted against $Re_s$ in Figure 7(b). The best fit now results in a more shallow slope of approximately $-0.37 \pm 0.02$. The variation of $\Theta_c$ with $Re_s$ is applicable for particles sufficiently small to not protrude from the viscous sublayer, and therefore the scaling would be expected to perform better for smaller particles, given the viscous sublayer thickness remains similar on the different particle layers. (For such particles $Re_s$ is also closer to $Re_p$, for which we have seen $\Theta_c \sim Re_p^{-1/2}$.) To verify this, however, more experiments are necessary to increase the amount of data covering a wide range of particle sizes.

As has been mentioned by Munro, Bethke, and Dalziel,17 base $\Theta_c$ on the vortex ring propagation velocity, rather than the peak bed velocity. For the experiments presented here, these two velocities differ by a factor of two to three (with the bed velocity $U_c$ being the larger one), leading to a value of $\Theta_c$ that is smaller by a factor of around ten in the data presented in Ref. 17. However, the dependence on Reynolds number is unaltered by this scaling, and thus the approach in Ref. 17 is not pursued here.

There are various complications that arise in the case of the flow over a particle layer, which limit the validity of the scaling presented for the flow over a single particle to predict the variation of the critical Shields parameter $\Theta_c$ with particle Reynolds number $Re_p$. A key issue is the ambiguity of what constitutes the boundary layer in the flow over a granular bed. On a solid boundary, the boundary layer is created by a uniformly valid no-slip condition. A laminar boundary layer on a solid boundary beneath an impulsively started flow will grow initially as $\delta \sim (\nu t)^{1/2}$. For a turbulent flow there exists a viscous sublayer of thickness $\delta \sim \nu/\nu$; this was used by Eames and Dalziel25 to estimate the velocity experienced by the particle. In the case of turbulent flows over rough boundaries the fluid viscosity is replaced by a turbulent eddy viscosity, $\nu T \sim ud$, if the roughness lengthscale (here the particle diameter $d$) is larger than $\delta \sim \nu/\nu$.

Obviously the scale of a growing laminar boundary layer and viscous sublayer are not the same, with the laminar boundary layer becoming the thicker (and hence less important for the particles) in a time of $O(\nu/\nu^2) \ll D/U$. However, as here the laminar boundary layer is driven by an accelerating flow, we cannot simply expect it to extend to the height at which we are measuring the bed velocity in a time of the order $O(1/s)$.

In the case of the impact of a vortex ring on a particle layer, a number of issues affect the growth of the boundary layer. Aside from the difficulty in comparing a developing boundary layer to that in a statistically steady flow, and the issue of whether the interaction between the vortex ring and the particle layer is laminar or turbulent, the ability of surface particles to move with the flow may modify the boundary condition. While the no-slip condition is valid locally on the surface of an individual particle, the motion of particles in more-or-less coherent groups could result in the introduction of a global “slip” effect. This raises the question of the height at which a macroscopic no-slip boundary condition can be applied. At this point all that is known about the boundary layer on the particle layers, and the boundary layer on the solid surface, is the vorticity evolution as documented by the PIV images. These seem to indicate that the boundary layer on the solid surface and the 150 $\mu$m particle layer are not of the same thickness. This difference is also suggested by the PIV images for the resuspension onset of the other particle sizes.

Another difference between the flow over an isolated particle and over a monodisperse particle bed is the exposure of any one particle to the flow. Closely packed monodisperse particles tend to shield each other from the flow, resulting in a lower effective velocity that can provide lift. The presence of cohesive forces can also increase the velocity needed to displace particles from the bed. For very small particles with diameters $d \lesssim 30 \mu$m, cohesion effects can become important.26 Here, $d \geq 90 \mu$m and so these are probably not significant. Both these effects (shielding and cohesion) are not easily quantifiable, and thus will not be discussed here, but it is noted that their combined influence will play a role in the resuspension dynamics up to the point where particles leave the proximity of their neighbours on the bed. Finally, it is noted that the particle layer is porous (with considerable but finite thickness) and this may lead to different interaction dynamics at the particle layer surface than a particle laden solid boundary (effectively a particle layer with a maximum thickness of a few particle diameters). This is true even in the absence of mobility effects, such as would be the case for the flow over a porous but solid material. This issue will be discussed in Sec. III D.
In the analysis of the critical Shields parameters, the viscous sublayer scaling was used to estimate the velocity experienced by a particle on the particle layer surface. The type of boundary layer in this experiment is not known, however. The added complications of an unsteady flow, and a mobile, porous, and rough boundary, effectively preclude an accurate estimate of the growth and thickness of the boundary layer. A basic model is therefore employed to assess what boundary layer thickness is to be expected in the analysis of the bed velocities that follows in Sec. III D. Essentially, the laminar boundary layer scaling, \( \delta \sim (\nu t)^{1/2} \), is assumed to hold initially as the flow develops with the approach of the vortex ring. Taking the vortex ring turnover time, \( D/U \), as a representative time scale for the growth of the boundary layer then yields \( \delta \sim (\nu D/U)^{1/2} \). If the flow transitions to turbulence, or at least a velocity field that is not essentially horizontal, the viscous sublayer scaling, \( \delta \sim \nu/u_b \), where the relevant velocity scale is the bed velocity \( U_b \), determines the thickness of the boundary layer shielding the particles and thus the velocity experienced on the particle layer surface. As the laminar boundary layer grows in time, and thus with the turnover scale \( (D/U) \), this laminar estimate represents an upper bound on the boundary layer thickness, while the viscous sublayer becomes thinner as the bed velocity increases with the approach of the vortex ring, and thus will act as a lower bound.

### D. A closer look at the bed velocity

The maximum bed velocity observed in the resuspension onset experiments is used in the calculation of the critical Shields parameter for resuspension of particles of different size. To assess whether the flow induced on the different particle layers remains qualitatively and quantitatively similar, the time evolution of the bed velocity is considered. Rather than inspecting the bed velocity profile at different stages of the interaction, its normalised absolute maximum is calculated for each time frame,

\[
\hat{U}_m(t) = \max \hat{U}_b(r, t).
\]  

The experimental bed velocity is compared to the inviscid theoretical prediction obtained from the streamfunction-Norbury formulation described in Sec. III A.

Figure 8 shows how the experimental measurements of the maximal bed velocity, \( \hat{U}_m \), evolve for both a solid boundary (Figure 8(a)) and for a layer of 90 \( \mu \)m particles (Figure 8(b)), where the vortex ring dimensions and velocity were nominally the same. In both cases, the corresponding maximal bed velocity computed for an inviscid vortex ring, \( \hat{U}_m \), is shown as a solid line. Because there is no inviscid mechanism that could result in a loss of circulation of the ring, \( \hat{U}_m \) monotonically increases. It is immediately evident that initially there is a closer agreement between the experimental and theoretical curves for the particle layer. Both on the solid boundary and the 90 \( \mu \)m particle layer \( \hat{U}_m \) increases slowly at first, and then more rapidly until it reaches a peak, after which a decrease in \( \hat{U}_m \).

---

**FIG. 8.** Experimental and theoretical maximum bed velocity, \( \hat{U}_m \) and \( \hat{U}_m \), plotted against non-dimensional time, \( \hat{t} \), for (a) a solid boundary and (b) a 90 \( \mu \)m particle layer. Vortex ring travels at the critical velocity for resuspension on a 90 \( \mu \)m particle layer.
is observed. The maximum bed velocity at its peak is approximately two to three times as large as the vortex ring speed, and its decrease is primarily caused by the interaction of the primary ring with the separated boundary layer and the subsequent cancellation of vorticity.

While, for the impact on the solid boundary, the experimental bed velocity is consistently lower than the inviscid prediction (which is to be expected due to the no-slip condition and associated boundary layer), for the particle layer $\tilde{U}_m$ and $\tilde{U}_m$ agree very well up until $\tilde{t} \approx 1.05$. Only as the peak is reached, beyond which $\tilde{U}_m$ decreases monotonically, do the theoretical and experimental velocity diverge. The match between the bed velocity on the particle layer and the inviscid prediction is a striking result, as it implies that viscous effects do not notably influence the maximum of the induced radial bed velocity prior to resuspension. Given the existence of a boundary layer is documented by the PIV images showing its separation, the phenomenon of a seemingly inviscid bed velocity is even more unexpected. It is important to note, however, that the separation always occurs in a region where the flow is decelerating and so is at a slightly larger radius than the location of the maximum in the radial bed velocity.

The question arises whether the boundary layer is too thin to affect the bed velocity as measured approximately 0.5 mm above the bed. As the measurement height was the same for both cases, the macroscopic boundary condition on the particle layer must be quite different to result in such a significant and sustained modification in the boundary layer thickness. As discussed previously, the thickness of a laminar boundary layer on a solid boundary scales as $\delta \sim (vD(U)/\nu)^{1/2}$ when the vortex ring turnover time is used as a representative time scale, giving an estimate of $\delta \sim O(1 \text{ mm})$ for a typical ring diameter $D = 40 \text{ mm}$ and propagation velocity $U = 100 \text{ mm/s}$. This suggests a boundary layer thickness comparable with the height of the measurement location. The viscous sublayer scaling, on the other hand, suggests $\delta \sim O(1 \mu \text{m})$, three orders of magnitude smaller than the laminar estimate; while if the particle layer diameter is used as roughness lengthscale in an eddy viscosity, this necessarily gives $\delta \sim O(0.1 \text{ mm})$ (as $\delta \sim \nu/(\nu/U = Ud/\nu = d)$). To resolve the boundary layer structure on the particle layer requires further research.

One possible explanation for the apparent free-slip velocity could lie in the movement of particles on the surface (bedload transport, which can be observed prior to resuspension on high speed movies of the impact). With a certain degree of slip at the surface, a boundary condition is created that is macroscopically neither a no-slip, nor a free-slip condition, with the boundary layer strength just above the layer of particles weakened. Even in the case of a true free-slip boundary, a velocity gradient is expected to be established above the particle layer due to the momentum transfer between the fluid and the initially resting particles.

To assess how robust the “slip” effect is, the bed velocity on a 1000 $\mu \text{m}$ particle layer is considered. It is noted that, in the high speed recording of the 1000 $\mu \text{m}$ impact, it is difficult to discern any motion of the particles prior to resuspension, suggesting the critical velocities for bedload and resuspension are much closer for the 1000 $\mu \text{m}$ particles than for the 90 $\mu \text{m}$ particles. Therefore, the slip effect for the former should be less pronounced. Nevertheless, as documented in Figure 9, the bed velocity for the 1000 $\mu \text{m}$ particle layer collapses onto the inviscid model prediction up to the point of resuspension, suggesting that the boundary condition is similar on particle layers of such differing particle size. On the 1000 $\mu \text{m}$ particle layer, the bed velocity peak occurs earlier, however, and is therefore associated with a lower peak velocity compared to the 90 $\mu \text{m}$ particle case. The comparison between the bed velocity on the 90 $\mu \text{m}$ and the 1000 $\mu \text{m}$ particle layers raises the questions why a slip condition is satisfied macroscopically on both particle layers, despite their different mobility prior to resuspension, and why the peak maximum normalised bed velocity is attained earlier (with the ring further from the bed) and thus has a lower relative magnitude in the 1000 $\mu \text{m}$ case.

The bed velocity measurement height only varied minimally (by the order of the pixel size, i.e., 0.15 mm) between different experiments and is thus ruled out as cause for the observed difference in the peak bed velocity for the 90 $\mu \text{m}$ and 1000 $\mu \text{m}$ particle layer. While the primary reason for the observed decrease in bed velocity is the interaction of the vortex ring with the boundary layer, a secondary effect could be due to the limitations of the PIV method once resuspension is initiated. Both the introduction of glass particles and tracer particles that have settled out of suspension into the flow affect the PIV analysis and reduce the accuracy in the velocity measurement.
The importance of the vortex ring circulation in determining the bed velocity should not be neglected, however. With the vortex ring position above the bed at the onset of resuspension still clear of the resuspended material, the vortex ring circulation can be estimated accurately, as the magnitude of the circulation is large and the vorticity is confined to a well-defined core. The circulation of the vortex ring for the impacts on the 90 $\mu$m and 1000 $\mu$m particle layers is found to be constant up until the maximum bed velocity is reached in both cases, after which point the circulation decreases, suggesting that the boundary layer has detached and started interacting with the vortex ring. This supports the earlier and lower peak in bed velocity on the 1000 $\mu$m particle layer, as it is linked to the earlier detachment of the boundary layer.

So far we have discussed the flow at the interface between the fluid and the particle layer. The particle layer, however, constitutes a porous medium, which could admit a flow through it when a horizontal pressure gradient is applied at the surface of the bed. The flow over and through porous media has been studied both in the context of laminar and turbulent flows. An early experimental investigation of the boundary condition at a permeable wall by Beavers and Joseph\textsuperscript{27} found that, for a laminar flow above a porous medium, the boundary layer extended well into the porous material, resulting in a macroscopic slip velocity at the material surface. These findings have since been confirmed experimentally and theoretically, for example in Refs.\textsuperscript{28} and \textsuperscript{29}. Studies of turbulent channel flows have found that viscous effects near a permeable wall are of little importance, leading to a thinning of the viscous sublayer, see Refs.\textsuperscript{30} and \textsuperscript{31}. These observations support the suggestion of a slip velocity and thin boundary layer as an explanation for the present measurements of the impact of a vortex ring on a particle layer.

While the vortex ring is not directly comparable to a shear flow, the induced bed velocity could lead to a similar flow locally. As the particle bed is nominally homogeneous and isotropic, the flux of fluid volume per unit area, $q$, as a function of the applied pressure gradient, $\nabla p$, is given by Darcy’s law, $q = -\kappa \nabla(p + \rho g z)/\mu$, where $\kappa$ is the permeability of the porous medium, and $\mu$ is the dynamic viscosity of the fluid. The interstitial flow velocity $U_p$ is obtained by dividing the flux $q$ by the porosity $\phi_p$ of the medium, as the fluid travels through the interstitial spaces. The porosity is defined as the fraction of the void space in the medium, $\phi_p = V_v/V_t$. While $\phi_p$ is determined by the particle shape, the permeability is proportional to the square of the particle size. For the glass ballotini used in the present experiments we found $\phi_p \approx 0.4$, and the permeability ranged from $\kappa \approx 9.4 \times 10^{-12}$ m$^2$ for $d = 90$ $\mu$m to $\kappa \approx 9.4 \times 10^{-10}$ m$^2$ for $d = 1000$ $\mu$m.

The magnitude of a flow through the bed is estimated from the bed velocity induced by the vortex ring. Just outside the boundary layer the pressure is composed of a hydrostatic and a dynamic component. Assuming the pressure is constant across the thickness of the boundary layer, this gives the pressure distribution just above the particle bed. As seen in Figure 6, the bed velocity peak moves with the vortex ring position. This peak is quite narrow, resulting in a radial pressure gradient in
the bed that is localised underneath the vortex ring core. Because of continuity, the flow velocities inside the particle layer satisfy

$$\frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{\partial w}{\partial z} = 0.$$ 

The radial flow, which changes over a length scale comparable with the core radius $a$, is driven by the pressure minimum applied by the vortex ring induced bed velocity, giving

$$u_r \sim \frac{\partial p}{\partial r} \sim \frac{\partial U_b^2}{\partial r} \sim \frac{U_c^2}{a}.$$ 

Substituting for the radial velocity $u_r$ in the continuity equation and re-arranging for the vertical velocity $w$ results in

$$w \sim \frac{U_c^2}{a^2 h},$$

where the bed thickness $h$ has been chosen as a representative length scale in the vertical spatial derivative. The interstitial velocities have to take into account the bed permeability and porosity resulting in a vertical velocity

$$w_p \sim \frac{\rho \kappa U_c^2}{\mu \phi_p a^2 h}. \tag{9}$$

Substituting the measured values for $\kappa$ and $\phi_p$, and setting $U_c \approx 2 \times 10^{-1}$ m/s (as measured on the 1000 $\mu$m particle layer) gives a pore velocity $w_p = O(10^{-2})$ m/s. This is a tenth of the bed velocity itself and could therefore influence the resuspension in regions where the flow is directed out from the porous layer as it effectively results in a drag force acting on the particle from below. Since $w_p d/\nu \approx 0.1 \ll 1$, this drag force scales as $F_D \sim 3\pi \mu d w_p \sim d^3 h U_c^2 / a^2$, which is – albeit smaller – proportional to the lift force, $F_L \sim d^2 U_c^2$, experienced by the particles. This estimate presents a lower bound for the drag force, which will increase as the associated Reynolds number, $w_p d/\nu$, increases. This outflow could contribute to the monotonic decrease of the critical Shields parameters with particle Reynolds number that has been observed experimentally.

The permeability of the 1000 $\mu$m particle layer could also account for the earlier and decreased peak bed velocity. The interstitial flow scales with the square of the maximum bed velocity. As $U_m$ initially grows slowly, the vertical interstitial flow velocity $w_p$ is expected to become significant only at the late stages of the interaction, just before and during the onset of resuspension. The induced flow inside the particle layer would be accompanied by a flow of fluid from outside the particle layer into its interstitial pores over part of the boundary, and a flow out of the particle layer in other parts. Less fluid would be forced to pass through the gap between the vortex ring and the particle layer, reducing the velocity of the fluid just above the particle bed. In terms of inviscid dynamics, the vertical velocity at $z = 0$ is not identically zero, but instead is reduced relative to a ring with the same circulation in the absence of such a boundary. The resulting flow is that of a ring that is effectively further from its image, and so the horizontal velocity at $z = 0$ is decreased. The flow out of the particle layer at the site of resuspension, where the pressure minimum is located, could also thicken the boundary layer at that location and cause it to separate earlier, leading to the drop in circulation mentioned before.

Although a porous flow is required to account for the apparent slip velocity on the 1000 $\mu$m particle layer surface (as a slipping of particles is not observed), this same flow should be accompanied by a decrease in the bed velocity through continuity considerations not only at the moment of resuspension, but also leading up to it. The bed velocity measured here matches the inviscid prediction up to the moment of resuspension, however. It can only be suggested that the associated mass flux through the porous medium is quite low, at least until resuspension is initiated, while the velocities inside the particle layer remain non-negligible. This could then ensure the no-slip effect without resulting in a reduced mass flux between the vortex ring and the particle layer. Clearly more work is required to resolve this issue.
In summary, we note that two factors characterise the interaction of the vortex ring with the particle bed: the surface mobility and the layer permeability. Bed velocity measurements on monodisperse particle layers have clearly shown an apparent slip velocity for a range of different particle sizes. While the apparent slip velocity on the 90 μm particle layer is thought to have been caused (at least in part) by the mobility of a few surface layers prior to resuspension, on the 1000 μm particle layer it was more likely a consequence of the bed permeability. Furthermore, it was shown that the associated seepage velocity through a permeable particle layer could become significant for sufficiently large bed velocities, thereby aiding the resuspension process. The permeability of a particle bed can therefore directly affect the critical Shields parameters for the onset of resuspension.

IV. EROSION OF PARTICLE LAYERS BY AN IMPACTING VORTEX RING

The results presented so far have shed some light on the dynamics of the interaction between the vortex ring and particle layer near the onset of resuspension. To establish how effectively vortex rings displace material from the particle layer once the critical Shields parameter has been exceeded, we study the craters eroded by the impact of the vortex ring. The term “erosion” will be used to refer to net changes in the bed profile caused by the combined processes of bedload transport and resuspension.

A. Crater characteristics on a 250 μm particle layer

Experiments were conducted for rings of varying formation number and velocity, by using different stroke lengths \( L_s = [40, 50, 60, 70] \) mm, which resulted in different relative core sizes (compared to the ring diameter), and varying the stroke time for each set stroke length. The bulk of the experiments were conducted with a particle layer composed of 250 μm glass ballotini, where the critical vortex ring propagation velocity for the onset of resuspension was found to be \( U_r = 97 \) mm/s. The stroke length did not change the critical vortex ring velocity for the parameter range investigated.

Typical light attenuation images of the craters created by an impacting vortex ring are shown in Figure 10, where the colour scheme represents the layer thickness. The images show an eroded crater surrounded by an approximately circular region of net deposit, where the term “deposit” is used in the sense of a positive change in particle layer thickness. The azimuthal wave-like structure of the crater is caused by Kelvin waves displacing the core of the vortex ring. These, in turn, are a result of an instability related to the self-induced velocity field in proximity of the vortex core. For an account of this family of instabilities the reader is referred to Refs. 19 and 20. For the quantitative analysis presented here, we will concentrate on the leading order (azimuthally averaged) features of the crater profiles rather than the second order features resulting from the instability.

Figure 10 shows that the crater shape remains qualitatively similar, while its depth and width vary with the ring parameters. The net eroded volume of such a crater is easily obtained by integrating the measured change in layer thickness \( \Delta h = h - h_0 \), where \( h_0 \) is the thickness of the initial, undeformed particle layer, over the eroded area \( A_e \) of the crater where \( \Delta h < 0 \). It was found that the error associated with measuring the deposit (where \( \Delta h > 0 \)) was higher than in the case of the eroded volume, as the deposit was usually spread over a larger area and thus typical \( \Delta h \) values were lower. Hence, the net eroded volume is used as an estimate for the displaced material. It is noted that the eroded volume of the final crater, represents a lower bound on the displaced material (that is particles displaced through resuspension and bedload transport), as some particles settle out in the final net erosion region of the particle layer (and in principle some particles will be eroded in the final net deposit region, although this is believed to be far less pronounced).

Figure 11(a) shows the net eroded volumes as a function of vortex ring velocity for rings created with four different stroke lengths. In all cases the volume increases with vortex ring velocity, which is what would be expected intuitively. It can also be seen that the net eroded volume increases with stroke length. This can be explained by the fact that rings created with a larger stroke length carry more fluid and thus are larger and more energetic than those created with a smaller stroke.
An alternative measure to characterise a vortex ring, which incorporates changes in ring geometry due to differing formation numbers, is its kinetic energy. Both the vortex ring velocity and diameter are incorporated into the scaling for the ring kinetic energy, as proposed, for example, in Ref. 32,

\[ E_k \propto \rho D^3 U^2. \]

To obtain the actual kinetic energy of the vortex rings, the asymptotic results of Ref. 23 are used. In much the same way as the propagation velocity was found, the kinetic energy is thus

\[ E_k = f(a/D)\rho D^3 U^2, \tag{10} \]

where the dimensionless function \( f(a/D) \) depends on the relative core size of the vortex ring, and is tabulated along with other ring parameters in Ref. 23. To investigate what proportion of the kinetic energy of the ring is stored in the final crater shape, we consider, instead of the eroded volume, the potential energy of the eroded bed, which is given by

\[ E_p = \frac{1}{2}(1 - \phi_p)(\rho_p - \rho)g \int_{A_c} (h - h_0)^2 \, dA. \tag{11} \]

FIG. 10. Light attenuation images of craters on a 250 \( \mu \)m particle layer. The velocity of the vortex rings (\( L_s = 70 \) mm) was (a) \( U \approx 709 \) mm/s \( \approx 7.3U_r \), (b) \( U \approx 518 \) mm/s \( \approx 5.3U_r \) and (c) \( U \approx 280 \) mm/s \( \approx 2.9U_r \).

FIG. 11. (a) Crater eroded volume as a function of vortex ring velocity, and (b) crater potential energy as a function of vortex ring kinetic energy, on a 250 \( \mu \)m particle layer. Different symbols indicate different stroke lengths: triangles pointing up, left, right, and down correspond to stroke lengths 40 mm, 50 mm, 60 mm, and 70 mm, respectively. The line in (b) indicates the best fit.
The integral is taken over the eroded region only, as we focus on the energy converted into resuspension of particles, and this is thought to be the best estimate. Here we will non-dimensionalise the potential energy by

\[ P = (1 - \phi_p) \rho_p g' D^4, \]

where \[ g' = (1 - \rho/\rho_p)g \] is the reduced gravitational acceleration. Similarly, we re-scale the kinetic energy using

\[ K = \rho D^3 U^2, \]

which is a ring energy based on the critical vortex ring velocity for the onset of resuspension, \( U_r \). Because of the dependence of the ring energy on the core radius as outlined in Eq. (10), \( K \) is close to (but not exactly equal to) the critical ring energy for resuspension onset. The non-dimensional kinetic and potential energies are thus \( \tilde{E}_k = E_k/K \) and \( \tilde{E}_p = E_p/P \). Although no distinction is made between rings of different stroke length for the analysis that is to follow, this information is retained in different symbols (stroke lengths 40 mm, 50 mm, 60 mm, and 70 mm are represented by triangles pointing up, left, right, and down, respectively) for data obtained with different stroke lengths.

To establish the scaling of the crater potential energy with the ring kinetic energy, the data is plotted on log-log axes in Figure 11(b), showing that the conversion of kinetic to potential energy can be described by a power law. As was indicated by Figure 11(a), the presence of a critical bed velocity for the onset of resuspension below which the bed is not deformed leads to a critical kinetic energy \( E_c \). The vortex ring kinetic energy above the critical energy \( E_c \) is considered in this plot to account for the fact that there is a threshold below which no motion is induced on the particle layer. As the ring kinetic energy approaches \( E_c \) from above, the value of \( \tilde{E}_p \) should approach zero. The associated functional relationship

\[ \tilde{E}_p \propto (\tilde{E}_k - \tilde{E}_c)^\beta \tag{12} \]

satisfies this requirement, ensuring that for impact energies just above critical, an arbitrarily small amount of material is eroded. The line of best fit to the data has a slope of \( \beta = 1.31 \pm 0.02 \). Another possible functional relationship would be

\[ \tilde{E}_p \propto \tilde{E}_k - \tilde{E}_c, \tag{13} \]

which has a different physical meaning than Eq. (12). Whereas Eq. (12) implies a zero gradient for \( \tilde{E}_p \) at the critical transition point, leading to an infinitely small amount of eroded material just above the critical kinetic energy, Eq. (13) results in a finite gradient at the critical kinetic energy, and thus a finite amount of eroded material just above critical conditions. Equivalently, the two cases could be thought of as converting the available energy in different ways. While Eq. (12) states that only the energy above the critical value is available for the resuspension process, Eq. (13) also takes into account the portion of kinetic energy below the critical value, so that once that value is reached, some of the sub-critical energy is available for the resuspension process, too. Unfortunately, we are not able to determine which of these models is superior from the current experimental data. It is noted that neither of the equations gives a physical result below the critical kinetic energy, specifically a non-zero value of \( \tilde{E}_p \), for kinetic energies below the critical value \( \tilde{E}_c \).

The eroded volume (and the potential energy) as a bulk property of a crater provides important but limited information on the process by which it was formed. The azimuthally averaged crater profiles are therefore considered next. Some typical crater profiles are shown in Figure 12, where the particle layer height is plotted against radial position. Bedload transport, resuspension, and settling of particles together form the final crater shape. The bedload component (albeit comparatively small) affects areas where a net increase and a net decrease in particle layer thickness is observed, as it implies a shifting of particles away from one region and a subsequent piling up in another. The resuspension and settling processes, however, do not contribute equally to the net eroded and net deposited region. Resuspension will be most prominent where the bed velocities are largest, that is just underneath the vortex ring core. As the crater is eroded by the vortex ring, the ring descends into the continuously deforming crater. Thus the velocities induced on the particle bed are effectively
bounded by the region of net erosion, resulting in little, if no erosion in the net deposit region of the crater. It is not entirely obvious on the other hand, how the settling of particles is distributed, as this is a continuous process while particles are being flung outwards and advected around the vortex ring, leading to a much less localised effect. There is also the possibility of a density current being established by a cloud of resuspended material, carrying particles radially outwards away from the impact site.25

Figure 12 illustrates the typical features of craters left by impacting vortex rings on a 250 μm particle layer. The profiles correspond to impacts of vortex rings of different velocity, but identical formation number ($L_s = 60$ mm). All profiles show a distinctive region of eroded material, surrounded by a region of deposit, where the absolute value of the deposit height is smaller than that of the erosion depth. The length scales of the bed deformation in the radial direction are more than an order of magnitude larger than those in the vertical. The maximum depth of eroded material and the maximum deposit, as well as the radial extent of the crater, vary with vortex ring properties. Clearly both the erosion depth and width are increased with increasing vortex ring velocity.

The radial extent of the deposit when compared to the eroded region is representative of the settling velocity of the particles – the longer particles remain in suspension the further they are expected to be carried radially outwards by the flow induced by the impacting vortex ring. The 250 μm particles have a settling velocity of 0.035 m/s, taking about 0.5 s to settle out (in a quiescent fluid) from a suspension height of approximately half a vortex ring diameter (high speed images suggest that particles are lifted up to heights of $\approx 20$ mm). With interaction times well below 1 s, the particles remain in suspension for a reasonable part of the interaction before the vortex ring is dissipated, to be advected radially outwards by the vortex ring – induced flow. This radially outward redistribution of particles is due to the negative buoyancy and associated settling velocity of the particles. When such an inertial particle leaves the rotational fluid of the vortex ring along a trajectory that is initially approximately tangential to the circular vortex core, it can be re-entrained into the vortex ring. This is much more likely for particles that are flung out of the vortex core at a height above the vortex ring. Particles leaving the vortex core at a height comparable to the location of the vortex centre, on the other hand, do not settle back into the vortex ring. This in effect leads to particles predominantly being propelled radially outwards. The interplay between the removal of particles by resuspension, and their deposition by settling, can be expressed as a ratio between the vortex ring turnover time $U/D$ and the settling time, which can be written as $v_s/D$. This gives the non-dimensional parameter $U/v_s$, indicating whether advection or settling will be more dominant in the crater formation process. For the 250 μm particles and the range of impact speeds considered $3 \lesssim U/v_s \lesssim 20$.

The maximum erosion depth $h_e$ and deposit height $h_d$, as well as the maximum radial extent of the eroded and deposited region, $R_e$ and $R_d$, are non-dimensionalised by the initial ring diameter, $D$, and plotted in Figures 13(a) and 13(b). The data corresponds to the range of experiments considered.
in Figure 11. The vertical dimensions \( h_e \) and \( h_d \) are easily obtained as the minima and maxima in profiles such as those shown in Figure 12; the radial dimension \( R_e \) is found from the transition point between net erosion and net deposition, \( h = 0 \), where \( r(h_e) < R_e < r(h_d) \), and \( R_d \) is defined as the radius for which \( h = 0.1 \max (h_d) \) such that \( r(h_d) < R_d \).

The crater depth and deposit height are seen to increase monotonically with the vortex ring energy. It is not expected that \( h_e \) and \( h_d \) approach a value of exactly zero at \( \tilde{E}_k = \tilde{E}_c \), as bedload transport contributes to \( h_e \) and \( h_d \) before \( \tilde{E}_c \) is reached. In the case of the radial extent, the limiting values for very weak impacts are of course distinctly non-zero, as any vortex ring that exceeds the critical energy required to move particles will leave a crater at least as wide as itself. In fact, it can be seen that the minimum observed eroded crater radius is \( \tilde{R}_e \approx 0.75 \), indicating that the vortex ring has stretched by a factor of \( \sim 1.2 \) before it is fully destroyed by its interaction with the particle layer. This estimate takes into account that the ring radius is measured between the vortex core centroids, and therefore does not fully include the rotational region of the cores. With a core diameter of approximately \( D/4 \), the crater radius would thus become \( \tilde{R}_c \approx 0.625 \) simply through contact with the vortex ring, even in the absence of any stretching. The crater radii also grow monotonically with the ring kinetic energy, albeit more weakly than the vertical dimensions. Both in the case of the crater vertical dimensions and the crater radii, the functional dependence on the kinetic energy seems to be very similar, whether the eroded or the deposited region is considered.

The conversion of ring kinetic energy to potential energy of the final crater was shown to be described well by a power law of the form \( \tilde{E}_p \propto (\tilde{E}_k - \tilde{E}_c)^\beta \). As the maximum erosion depth and width are representative of the crater size, the potential energy is expected to scale with the eroded crater dimensions as

\[
\tilde{E}_p \propto \tilde{h}_e^2 \tilde{R}_0 (\tilde{R}_e - \tilde{R}_0),
\]

where \( \tilde{R}_0 \) represents the crater radius obtained for an impact infinitesimally close to critical conditions. From the arguments presented in the discussion of the crater dimensions, it is estimated that \( \tilde{R}_0 \approx 0.625 \). While the crater depth and height are close to zero for kinetic energies close to the critical value \( \tilde{E}_c \), the crater radius therefore tends to \( R_0 \). This results in an annular erosion pattern, resembling a “ring-shape” when viewed from above, giving the scaling in Eq. (14). Equation (14) in turn implies that

\[
\tilde{h}_e \propto (\tilde{E}_k - \tilde{E}_c)^{\xi}
\]

and

\[
(\tilde{R}_e - \tilde{R}_0) \propto (\tilde{E}_k - \tilde{E}_c)^{\zeta}.
\]

Fitting these to the experimental data shown in Figure 14 results in exponents of \( \xi = 0.51 \pm 0.02 \) and \( \zeta = 0.25 \pm 0.02 \), suggesting \( \beta = 1.27 \pm 0.06 \) (as we expect \( \beta = 2\xi + \zeta \)), which is consistent
with the directly observed value of $\beta = 1.31 \pm 0.02$. The same approach suggests the eroded volume, and hence mass, should also scale as $V \propto (E_k - E_c)^\epsilon$. Plotting the volume data shown in Figure 11(a) against $E_k - E_c$ indeed reveals a power law relationship. The corresponding exponent $\epsilon = 0.83 \pm 0.01$ is consistent with what we would predict from $E_p$ and $h_e$, namely, $\beta - \xi = 0.80 \pm 0.04$. Although related only indirectly, previous work (e.g., Ref. 33) has revealed a similar relationship between the energy of a saltating particle and the mass ejected through its collision with a layer of dust.

It has been shown that the crater size grows monotonically with the strength of the impacting vortex ring while retaining similar features throughout. To assess to what extent the crater features are conserved over the range that has been investigated, the crater profiles for different impacts are plotted on normalised axes. The radial coordinate is scaled by the extent of the eroded region, $R_e$, and the vertical coordinate is normalised by the maximum depth of the crater, $h_e$. Albeit somewhat arbitrary, these re-scalings are chosen since the crater radius and maximum depth are believed to be robust features of any crater.

Figure 15 shows the re-scaled crater profiles for the 250 $\mu$m particle layer over the entire range of vortex ring speeds and stroke lengths employed. The profiles exhibit great similarity, indicating that the governing physical mechanisms responsible for the deformation of the particle layer, bedload, resuspension, and settling, remain similar over the range of vortex ring speeds. There is good agreement over most of the crater region, with more scatter near the crater centre and on the outer slopes of the deposit. The self-similarity of the profiles also justifies the scaling of potential energy with the maximum crater dimensions as given by Eq. (14).

One reason for the scatter around the self-similar profile is the amplification of noise in the profiles obtained from the very weak vortex ring impacts. As the magnitude of the maximum bed deformation approaches the noise level in the light attenuation data for the smallest craters, the re-scaling enhances small deviations resulting in the perceived scatter in the profiles.

At the other end of the range, deviations are also observed for very energetic impacts, with a tendency towards net erosion near the crater centre. It should be noted here that, in the light attenuation top views of the craters such as those in Figure 10, there always exists a region in the crater centre that is uneroded. This uneroded region becomes smaller the more energetic the impacting ring is. Higher vortex ring kinetic energies lead to higher levels of erosion, while the radial extent of the crater grows more slowly. This leads to a steepening of the inner crater side walls (this is illustrated by the profiles in Figure 12). We denote the slope of the crater wall at the point of inflection between $r(h_e) < r < r(h_d)$ as $\delta_s$. In the dynamics of granular media, the characteristic angle of stability of such a medium is the maximum angle such a slope can support before it collapses. The angle of repose is the characteristic slope the granular medium relaxes to, once the motion of particles has subsided. Usually the two angles are only a few degrees apart. The angle of repose for
FIG. 15. Normalised crater profiles for a wide range of vortex ring velocities and stroke lengths on a 250 μm particle layer.

the 250 μm particles was measured as η = 24°. The maximum crater slope observed for the profiles on the 250 μm particle layer was δ_s ≈ 17°, and thus below the critical angle of repose.

When considering the flow field induced by the vortex ring, it is clear that there is no radial component of velocity along the central axis of the vortex ring, but there is an axial flow velocity, which is directly related to the vortex ring velocity. This axial velocity and the radial velocity near the centreline increase with the velocity of the vortex ring. The increase in radial velocity near the crater centre results in higher erosion levels, and an effective decrease in the size of the uneroded crater centre, as it is eroded from all azimuthal directions (this is also illustrated by the profiles in Figure 12). The local stagnation pressure associated with increasing axial velocity can trigger a collapse of this narrow region of uneroded material, even before the critical angle of stability (which is a static measure) is reached, leading to a net reduction of the particle layer thickness at the crater centre. While a collapse of the central uneroded region can therefore contribute to the shape of the final crater profile, the self-similarity of the crater profiles and the consistently sub-critical slope angle δ_s, indicate that collapse dynamics are not very significant for the impacts on a 250 μm particle layer.

Overall the results in Figure 15 show that the collapse of the normalised crater profiles is good, if it is kept in mind that the resuspension, advection, and settling of particles are complex mechanisms in their own right.

So far the discussion of the erosion of particle layers by an impacting vortex ring has not taken into account the effect of particle size on the interaction. To investigate whether the particle size has an influence on the erosion characteristics other than through the modification of the bed velocity as indicated by the resuspension onset experiments, final crater volumes were measured for impacts on the 90 μm and 1000 μm particles, for which the critical vortex ring velocity for resuspension were obtained as U_r = 115 mm/s and U_r = 135 mm/s, respectively. The stroke lengths employed were L_s = 40 mm and L_s = 70 mm, with the same range of vortex ring speeds as in the 250 μm experiments.

B. Very fine particles: 90 μm ballotini

Figure 16 shows typical craters created by the impact on a 90 μm particle layer. The crater structure is qualitatively different from the one observed on the 250 μm particle layer. The craters are more filamental and less deep, with a maximum crater depth well below 1 mm even for the strongest impacts. Furthermore, the maximum extent is increased considerably, indicating that the ring does not penetrate the bed much but stretches significantly and, while doing so, scrapes the uppermost layers of the particle bed.

Figure 17(a) shows $\tilde{E}_p$ as a function of ($\tilde{E}_k - \tilde{E}_c$) for the 90 μm particle case. It should be noted that the potential energies are decreased by almost an order of magnitude when compared to the 250 μm particle layer. The energy conversion is well described by a power law, with a slope $\beta$
FIG. 16. Light attenuation images of craters on a 90 μm particle layer. The velocity of the vortex rings ($L_s = 70$ mm) was
(a) $U \approx 656 \text{ mm/s} \approx 5.7 U_r$, (b) $U \approx 408 \text{ mm/s} \approx 3.5 U_r$, (c) $U \approx 251 \text{ mm/s} \approx 2.2 U_r$.

$= 1.15 \pm 0.03$. This implies that the overall eroded volumes are not only lower for the finer particles,
but also that the efficiency of the conversion of kinetic to potential energy does not increase as much
with the imparted kinetic energy as it did for the 250 μm particles. Again the growth of the
crater depth and width are considered, and it is found that $\xi = 0.42 \pm 0.04$ and $\zeta = 0.32 \pm 0.03$,
suggesting a consistent value of $\beta = 1.16 \pm 0.11$.

It should be noted that the scaling exponents indicate that the craters on the 90 μm particle layer
show a more pronounced growth in the radial direction with ring kinetic energy, when compared to the
250 μm case. This supports the observations made in the discussion of the images in Figure 16. The
crater depth, on the other hand, is seen to increase more slowly than on the 250 μm particle layer. In
absolute terms, the radial crater extent on the 90 μm particle layer varies between $0.75 \lesssim \hat{R}_e \lesssim 1.75$,
with the crater depths ranging between $0.001 \lesssim \hat{h}_e \lesssim 0.014$, compared to $0.75 \lesssim \hat{R}_e \lesssim 1.1$ and
$0.004 \lesssim \hat{h}_e \lesssim 0.06$ for the 250 μm particle layer.

Figure 17(b) shows the normalised final crater profiles on the 90 μm particle layer. In comparison
to the 250 μm case, the crater profiles here are more noisy. This is in part a consequence of the
greater variation of the 90 μm crater profiles in the azimuthal direction, but the main cause is
the reduced depth of the craters, and thus the lower signal to noise ratio in the light attenuation
images. The collapse of the profiles is quite good, nevertheless, so that a distinct change in the crater

![image](a)

![image](b)

FIG. 17. Craters in 90 μm particle layers. (a) The potential energy of the final crater as a function of vortex ring kinetic
energy; line indicates the best fit. (b) Normalised crater profiles for a wide range of vortex ring velocities.
characteristics from the 250 μm particle layer can be discerned. The main difference between the 90 μm and the 250 μm normalised profiles is the maximum height and the distribution of the deposit. While the 250 μm craters displayed a rather localised ring of deposit around the eroded crater region, the deposit here is spread much further and therefore reduced in height. This is a result of the reduced settling velocity of the 90 μm ballotini, which therefore remain in suspension longer and in the meantime are advected radially outwards by the flow. Here, the formation of a gravity current (see Ref. 25) is a plausible mechanism for the transport of particles away from the impact site. The ratio between the vortex ring turnover time and the settling time is in the range $10 \lesssim U/v_s \lesssim 70$, illustrating the dominance of advection in forming the deposit. The location of the maximum eroded crater depth seems to be shifted inwards slightly to a position $\hat{r} \approx 0.5$, compared to its location of $\hat{r} \approx 0.7$ on the 250 μm particle layer. This may be due to the jagged boundary and therefore less well defined edge of the eroded region, the extent of which sets the normalisation length scale in the radial direction.

C. Coarse particles: 1000 μm ballotini

The analysis of the PIV experiments has suggested that the higher permeability of a particle bed consisting of 1000 μm particles could lead to a relaxation of the condition of no flow through the boundary and possibly a seepage flow within the particle bed. This and the increased settling velocity of the 1000 μm glass ballotini are expected to affect the impact craters.

Figure 18 shows images of the craters formed by the impact of vortex rings on a 1000 μm particle layer. The typical crater shape is again quite different from that observed on the 250 μm and 90 μm layers. All three craters are approximately circular with little variation in the azimuthal direction. The absence of the wave-like structure that has been found on the other particle layers is a combination of a change in the interaction dynamics and a direct consequence of the particle size. As the order of magnitude of the particle diameter here is comparable to the typical wavelength of the instability, the wave-like structure can simply not be reproduced as easily with such coarse particles. It should also be noted that the eroded region is quite even, with little variation in height, resembling a “depressed plateau”; only the weakest impact crater displays an uneroded centre. The ring of deposit around the crater is relatively localised here, which is expected in the light of the large particle settling velocity of the 1000 μm ballotini.

Individual particles can be clearly discerned in the images in Figure 18. Because of the relatively large size of the particles, the light intensity transmitted through the particle layer is no longer homogeneous at the resolution of the camera, even on a nominally even particle surface. Individual points within the eroded region are actually perceived as if they were a deposit. This implies that the

![Images of craters](image_url)

FIG. 18. Light attenuation images of craters on a 1000 μm particle layer. The velocity of the vortex rings ($L_s = 70$ mm) was (a) $U \approx 684$ mm/s $\approx 5.1U_r$, (b) $U \approx 491$ mm/s $\approx 3.6U_r$, (c) $U \approx 221$ mm/s $\approx 1.6U_r$. 
light attenuation method cannot be applied to individual pixels, but instead the azimuthally averaged crater profile is used to obtain the crater depth or height at any point. For the smaller particles the application of the light attenuation to the individual pixels gives practically the same result as if the height is calculated from the averaged profile.

The potential energies of the craters on the 1000 μm particle layer are plotted in Figure 19(a). This shows an increase by almost an order of magnitude when compared to the 250 μm particle layer, and consequently an increase by two orders of magnitude in comparison to the 90 μm particle layers. The collapse of the data is less satisfactory than for the other particle sizes, especially for kinetic energies $(\tilde{E}_k - \tilde{E}_c) \gtrsim 20$, where the values of $\tilde{E}_p$ are seen to reach a nearly constant level.

The first issue that is addressed is the drop-off of the potential energies for kinetic energies $(\tilde{E}_k - \tilde{E}_c) \gtrsim 20$. Considering the slope angles on the crater walls reveals that $\delta_s$ steadily increases to reach a maximum of $\delta_s \approx 21^\circ$ (the measured angle of repose is $\eta \approx 24^\circ$) at $(\tilde{E}_k - \tilde{E}_c) \approx 20$, and then decreases drastically to a value of $\delta_s \approx 10^\circ$ for the most energetic impacts. This is a clear indicator of the collapse of the crater, mechanisms for which were discussed in Sec. IV A. Why the collapse is observed here and not for the 250 μm particle layers cannot be said with certainty. It should be noted that the increased crater depth as indicated by the images in Figure 18, while maintaining roughly the same radial extent, results in steeper slopes and thus the possibility of the crater walls collapsing, whether this occurs “spontaneously” because the critical slope angle has been reached, is induced by the stagnation pressure of the vortex ring at the impact, or is triggered by radial flow out of the porous bed.

The scaling of $\tilde{E}_p$ with $(\tilde{E}_k - \tilde{E}_c)$ for ring kinetic energies that do not result in the collapse of the crater has a slope of $\beta = 0.86 \pm 0.05$, presenting a significant reduction compared to $\beta = 1.31 \pm 0.02$ for the 250 μm particle layer, and $\beta = 1.15 \pm 0.03$ for the 90 μm particle layer. Here, the conversion of kinetic energy to potential energy becomes less efficient with increasing ring kinetic energy. There are a number of possible causes that could lead to this reduced exponent.

A mechanism that certainly influences the final crater volume, and thus potential energy, is the settling velocity of the particles. The settling velocity for the 1000 μm ballotini is four times as high as that for the 250 μm particles, and more than an order of magnitude larger than the settling velocity of the 90 μm particles. This means that resuspended particles will not be carried as far before settling, hence more settle in the net eroded crater region. This results in an underestimate of the potential energy of the net eroded region (it is noted that any partial crater collapse has an analogous effect). Another factor that becomes important with increasing particle size is the coupling of particles to the flow. Large particles have considerable inertia and are capable of decelerating the vortex ring as it interacts with the particle layer, which may lead to the vortex ring disintegrating sooner, affecting the amount of eroded material. As has been indicated in Sec. III D, the particle size also directly affects the particle layer permeability. For a considerable flow through the particle

FIG. 19. Craters in 1000 μm particle layers. (a) The potential energy of the final crater as a function of vortex ring kinetic energy; line indicates the best fit. (b) Normalised crater profiles for a wide range of vortex ring velocities.
layer, energy would be dissipated inside the porous medium, reducing the energy available for the crater formation.

The absolute radial and vertical crater extent for the 1000 μm particle layer is found to be bounded by $0.75 \lesssim \hat{R}_e \lesssim 1.0$ and $0.02 \lesssim \hat{h}_e \lesssim 0.07$, which illustrates a reduced radial crater growth when compared to the other particle types, while the crater depth is increased noticeably. The crater depth and radius are found to scale with exponents $\xi = 0.21 \pm 0.05$ and $\zeta = 0.15 \pm 0.05$ (for the non-collapsed craters). The low value of $\zeta$ indicates that the crater width barely grows with the energy of the impacting vortex ring, at least when compared to impacts on layers with smaller particles, which points to the high permeability having an effect on the interaction dynamics. When $\xi = 0.21 \pm 0.05$ is considered, it would seem that the crater depth increase with vortex ring kinetic energy is much reduced in comparison to the 90 μm and 250 μm particle layer. This is misleading, however, as the high particle settling velocity here has a much more pronounced impact on the final crater shape than in the other cases, as has been pointed out above. The values of $\xi$ and $\zeta$ suggest a value of $\beta = 0.57 \pm 0.15$. The discrepancy between the scaling exponent obtained from the energies and those found from the crater dimensions is thus greatly increased for the 1000 μm particles. This difference can be explained by examining the crater profiles on the 1000 μm particle layer.

A plot of the normalised crater profiles is shown in Figure 19(b). Although the azimuthal averaging has removed most of the noise due to the inhomogeneity of the light attenuation images, some further averaging between individual neighbouring points has been applied to the profiles to obtain a clearer image of the profile characteristics. This has not affected the general profile height at any radial position, but has smoothed the appearance of the latter. Both at the crater centre and the outside wall of the deposit the profiles vary considerably, confirming what was suggested by the light attenuation images in Figure 18, namely, that the uneroded crater centre disappears for high energy impacts. We believe that the flatter and wider circular deposit for some profiles could also be due to a collapse of a previously more concentrated circular deposit, as for radial positions $\hat{r} \gtrsim 1.75$ it is found that $\hat{h} \approx 0$ rather abruptly. The narrow region of deposit is, of course, also a consequence of the high settling velocity of the particles, which results in $0.6 \lesssim U/\nu_s \lesssim 5$, indicating that the settling velocity here is more dominant in the formation of the deposit. The fact that the profiles do not exhibit self-similarity accounts for the mismatch between the scaling obtained for the crater potential energy and the crater dimensions.

V. CONCLUSION

This paper has presented experimental results of the interaction of a vortex ring with a particle layer, complemented by theoretical considerations of a circular line vortex ring impacting on a solid boundary. The analysis of the experimental PIV data revealed interesting features in the flow field induced by the vortex ring above a particle bed. While the bed velocity just above a solid boundary clearly displayed the effects of viscosity and the no-slip condition through a significant reduction of the inviscid velocity prediction, the corresponding velocity induced above the particle layers was found to match the inviscid velocity obtained from the line vortex model. It is thought that a combination of particle mobility and layer permeability contributes to the apparent slip velocity, with the former being dominant in establishing the slip velocity on the 90 μm (effectively impermeable) particle layer, while the latter is thought to cause the slip effect on the 1000 μm (effectively immobile) particle layer. The experimental results also suggest that the particle layer permeability influences the induced bed velocity as it admits a flow through the interface and therefore within the bed, which may affect the growth and detachment of the boundary layer as well as the magnitude of the bed velocity. The critical Shields parameters were found to decrease monotonically with particle size. It is suggested that the porous flow through the particle layer may have actively aided resuspension by providing an upward drag force from fluid leaving the particle bed. Further experiments will be necessary to confirm this.

Our experiments also suggest an effect of bed permeability on the interaction of the vortex ring with the particle layer. This is supported by measurements of the crater dimensions for impacts with differing vortex ring velocities and for a range of particle sizes. For increasing particle size, the crater depth was observed to increase, accompanied by a decreased crater radius, implying that
the vortex ring penetrates the layer further and stretches less. Conversely, for the 90 \( \mu \text{m} \) particle layer, the vortex ring barely eroded the layer in the vertical, but stretched noticeably more. This is thought to be a result of the differing critical Shields parameters associated and combined with the permeability of the layers. While in the case of a truly solid boundary the approaching vortex ring is forced to stretch as it moves closer, the relaxation of the condition of no flow through the boundary reduces the effective “solidity” of the boundary. This then results in a diminished stretching of the vortex ring. Equivalently, the reduced stretching can be thought of as a consequence of the decreasing critical Shields parameter with increasing particle size. For a vanishingly small critical Shields parameter, the particles would be resuspended readily. An approaching vortex ring would therefore move particles out of its way without significant stretching, but instead penetrate deeply into the particle layer.

By considering the scaling of the potential energy of the final crater with the kinetic energy of the impacting vortex ring, it was shown that a power law provides a good description of the energy conversion, at least for a range of kinetic energies far from critical conditions. This was the case for the 90 \( \mu \text{m} \) and 250 \( \mu \text{m} \) particle layers, for which porous flow effects can be neglected. The collapse of the potential energy of the craters with the kinetic energy of the impacting vortex ring indicated that for these particle layers the mechanisms involved in resuspending material remain the same over a range of vortex ring energies. This points to a robust resuspension process in which the kinetic energy of the vortex ring is more or less directly converted into raising the potential energy of the lifted particles. In the case of the permeable 1000 \( \mu \text{m} \) particle layer, however, the potential energy of the final craters did not exhibit such a simple energy scaling. In the light of the existence of a flow through the particle layer this is not surprising, as it effectively shifts the mechanisms involved away from the lifting of particles by an external flow to the complex interaction of a vortex ring with an array of moving spheres of a size not negligibly small when compared to the vortex ring core \((d/(2a) \approx 0.1)\).

The azimuthally averaged and normalised profiles obtained from the craters on the different particle layers all show some degree of self-similarity. The best collapse is obtained for the 250 \( \mu \text{m} \) particle layer, resulting in a characteristic crater profile which is maintained for impacts over a range of ring kinetic energies. Similarly, the crater profiles on the 90 \( \mu \text{m} \) particle layer collapse to give a typical crater shape. On the 1000 \( \mu \text{m} \) particle layer some systematic deviation from a self-similar profile is observed at the crater centre. For weak impacts the crater centre is seen to be left uneroded, whereas for stronger impacts this uneroded pile of material disappears. This is a consequence of the collapse of the granular pile as well as result of the changed interaction dynamics.

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APPENDIX: THEORETICAL MODEL OF AN INVISCID CIRCULAR LINE VORTEX

The approach of a vortex ring towards a solid wall can be modelled by considering the axisymmetric streamfunction \( \psi(r, z) \) of a circular line vortex of strength \( \gamma \) (e.g., Ref. 35):

\[
\psi(r, z) = \frac{\gamma Rr}{4\pi} \int_0^{2\pi} \frac{\cos \theta \, d\theta}{(z^2 + r^2 + R^2 - 2rR \cos \theta)^{1/2}}.
\]  

(A1)

Here \( r, \theta, \) and \( z \) are cylindrical polar coordinates, and \( R \) is the vortex ring radius. The corresponding radial and vertical velocities in polar cylindrical coordinates are obtained from \( \psi \) as

\[
u_r = \frac{1}{r} \frac{\partial \psi}{\partial z}, \quad \nu_z = -\frac{1}{r} \frac{\partial \psi}{\partial r}.
\]
By setting

$$k^2 = \frac{4Rr}{(z-Z)^2 + (r+R)^2},$$

this equation can be re-written as

$$\psi(z, r) = \frac{\gamma (Rr)^{1/2}}{4\pi} \int_0^{\pi} \left\{ \left( \frac{2}{k} - k \right) \left( 1 - k^2 \cos^2 \frac{1}{2} \theta \right)^{-1/2} - \frac{2}{k} \left( 1 - k^2 \cos^2 \frac{1}{2} \theta \right)^{1/2} \right\} \ d\theta$$

$$= \frac{\gamma (Rr)^{1/2}}{2\pi} \left\{ \left( \frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right\}, \quad \text{(A2)}$$

where $K$ and $E$ are the complete elliptic integral of the first and second kind, respectively,

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 x)^{-1/2} \, dx,$$

$$E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 x)^{1/2} \, dx.$$

As demonstrated by Batchelor,\textsuperscript{35} this streamfunction cannot be used to obtain the self-induced velocity of the vortex ring due to the singularity at the position of the vortex line. This singularity leads to a theoretical infinite propagation velocity of a circular vortex line. In contrast to the idealised circular vortex, the core of a real vortex ring has a finite size that is determined by the creation mechanism. A semi-analytical solution for a family of vortex rings described by one geometric parameter $\alpha_c = a/R$, the mean core radius, is presented by Ref. 23, giving ring properties such as circulation and kinetic energy. The non-dimensional propagation velocity $\bar{W}$ is also determined for a range of values of $\alpha_c$, and the dimensional propagation velocity relates to $\bar{W}$ as

$$W = \Omega R^2 \alpha_c^2 \bar{W}, \quad \text{(A3)}$$

where, in an inviscid flow, $\Omega = \omega_0/r$ is constant in the core of the ring.

In the present work we concentrate on vortex rings with relatively thin cores. For distances far from the cores, Eq. (A2) provides a good approximation to the flow and allows us to make further progress analytically as the ring approaches the boundary where the normal component of velocity must vanish. Here we follow previous researchers (e.g., Ref. 15) and employ the method of images to model the presence of the wall. The addition of a streamfunction of equal and opposite strength to the original expression can be thought of as placing an image vortex ring on the “other side” of the wall at the same distance, giving

$$\psi(z, r) = \frac{\gamma (Rr)^{1/2}}{2\pi} \left\{ \left( \frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) - \left( \frac{2}{\bar{k}} - \bar{k} \right) K(\bar{k}) + \frac{2}{\bar{k}} E(\bar{k}) \right\}, \quad \text{(A4)}$$

where $\bar{k}$ takes the form

$$\bar{k}^2 = \frac{4Rr}{(z-Z)^2 + (r+R)^2}.$$

This far-field streamfunction may be used not only to determine the radial velocity at the boundary $z = 0$ as

$$u_r |_{wall} (R, Z) = \frac{\gamma Z}{2\pi r [Z^2 + (r+R)^2]^{1/2}} \left\{ 1 + \frac{(Z^2 + (r+R)^2)}{(Z^2 + (r-R)^2)} \right\} E(k) - 2K(k), \quad \text{(A5)}$$

but also to provide a correction to the velocity at the core ($r = R$ and $z = Z$) as the sum of the self-induced finite-core velocity, and the velocity due to the image vortex ring,

$$\frac{dZ}{dt} (R(t), Z(t)) = W(R(t)) + u_z |_{ring} (R(t), Z(t)) = W(R(t)) + \frac{\gamma}{4\pi} \left( \frac{K(\bar{k}) - E(\bar{k})}{(R^2 + Z^2)^{1/2}} \right), \quad \text{(A6)}$$
\[ \frac{dR}{dt}(R(t), Z(t)) = u_r|_{ring}(R, Z) = \frac{y}{4\pi} \frac{(R^2 + 2Z^2)E(\hat{k}) - 2Z^2K(\hat{k})}{RZ(R^2 + Z^2)^{1/2}}. \]  