CLEANING OF VISCOUS DROPS ON A FLAT INCLINED SURFACE USING GRAVITY-DRIVEN FILM FLOWS

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ABSTRACT

We investigate the fluid mechanics of cleaning viscous drops attached to a flat inclined surface using thin gravity-driven film flows. We focus on the case where the drop cannot be detached from the surface by the mechanical forces exerted by the cleaning fluid on the drop surface. The fluid in the drop dissolves into the cleaning film flow, which then transports it away. To assess the impact of the drop on the velocity of the cleaning fluid, we have developed a novel experimental technique based on particle image velocimetry. We show the velocity distribution at the film surface in the situations both where the film is flowing over a smooth surface, and where it is perturbed by a solid obstacle representing a very viscous drop. We find that at intermediate Reynolds number the acceleration of the starting film is overestimated by a plane model using lubrication approximation. In the perturbed case, the streamwise velocity is strongly affected by the presence of the obstacle. The upstream propagation of the disturbance is limited, but the disturbance extends downstream for distances larger than 10 obstacle diameters. Laterally, we observe small disturbances in both the streamwise and lateral velocity, owing to stationary capillary waves. Finally, the flow exhibits a complex three-dimensional converging pattern immediately below the obstacle.

INTRODUCTION

Cleaning of fouling deposits using film flows is a common problem in many industrial processes, particularly in the food industry (e.g. Wilson, 2004). The shearing action of a film flow is often used to clean fouled surfaces in industrial processes as well as in our daily life (Yeckel and Middleman, 1987), such as in a household dishwasher. In a full dishwasher, a jet of water impinges on the surface of some of the plates while others are simply covered by a thin draining film. The ability of the film to clean the drops of grease attached onto the plate surface is critical. Moreover, minimizing the water consumption and the energy of such automatic cleaning devices can have an important environmental and sustainable impact. In this study, we investigate the case where shear forces cannot overcome adherence, and thus the drop remains attached onto the surface until it dissolves completely in the film.

We are interested in the case of cleaning a single drop of viscous liquid lying on an inclined planar surface using a gravity driven falling film (see figure 1). Blount (2010) developed a mathematical model for the dissolution and transport of the fluid from the drop into the film flow. The streamwise velocity in the film is obtained assuming a viscous-gravity balance and lubrication approximation,

\[ u_\infty(y) = \frac{\sin \alpha}{2\nu} y(2h_\infty - y), \]  

where \( y \) is the spatial coordinate in the direction orthogonal to the substrate (\( x \) the streamwise direction and \( z \) the lateral or spanwise direction), \( \alpha \) is the inclination angle of the substrate from horizontal, \( \nu \) is the film kinematic viscosity and \( h_\infty \) is the far-field film thickness.
The drop fluid, considered as a passive tracer, is described using the advection-diffusion equation in the film phase

$$\partial_t A + \mathbf{u} \cdot \nabla A = D \nabla^2 A,$$

where $\partial_t$ is the partial differentiation with respect to time, $A$ is the local concentration of the drop fluid, $\mathbf{u} = (u, v, w)$ is the local film velocity and $D$ is the constant diffusion coefficient of the drop fluid in the film phase. Assuming that just outside the drop interface $A$ is fixed, and equal to the maximum solubility, $A_s$, of the drop fluid in the film phase, and that the film fluid forms a boundary layer such that $u \propto y$, Blount (2010) solved equation (2) to obtain a prediction for the total flux of drop fluid, integrated along the drop surface, into the film flow

$$F = 0.808 A_s \left( \frac{3g^2 D^3 r d^6 \sin^2 \alpha}{v^2} \right)^{1/9},$$

where $\Gamma$ is the two-dimensional flow rate and $d$ is the drop length.

Our objective is to test the validity of the model developed by Blount (2010) and compare its prediction with experimental measurements. For simplicity, we focus on the case of a non-deformable drop, which corresponds to the very viscous limit. One of the main assumptions in Blount’s (2010) model is to consider that the film velocity is not affected by the drop and the velocity in the diffusive boundary layer remains linear with distance away from the boundary. To test this assumption we measure the velocity field of the film flow in the vicinity of a solid obstacle, representing a non-deformable drop.

The flow of a gravity-driven film falling along a rigid surface has been studied extensively (e.g. Craster and Matar, 2009). Since the pioneering work of Nusselt in 1916, measuring the velocity field in film flows has always remained a technical challenge (see e.g. Moran et al. 2002, for a review of various existing experimental techniques). Indeed, film flows are typically less than 1 mm thick and their velocity can reach 1 m s$^{-1}$ at the free surface (Lan et al. 2010). Only recently, with the photochromic dye activation technique presented by Moran et al. (2002), the measurement of the instantaneous velocity field with a non-intrusive technique became possible. However, the measurements were still limited to only one location in the $(x,z)$ plane. Moreover, Moran et al.’s (2002) technique required the ability to see through the sides of the film, by confining the film inside a channel.
We have designed a new technique based on particle image velocimetry, which allows measurements of the two-dimensional instantaneous surface velocity of the film: \( i.e. u(x, H, z, t) \) and \( w(x, H, z, t) \) over an almost arbitrarily large area. This technique is non-intrusive, neglecting the impact of the tracer particles on the flow. It can achieve large temporal and spatial resolution, depending on the capabilities of the high-speed camera and the strength of the light source. It also enables us to measure the film surface velocity in the vicinity of obstacles, such as drops. The deformation of a liquid film flowing down an inclined plane due to obstacles has been investigated theoretically (\textit{e.g.} Pozrikidis and Thoroddsen, 1991), numerically (\textit{e.g.} Blyth and Pozrikidis, 2006) and experimentally in the past 20 years (\textit{e.g.} Decré & Baret, 2003). However, owing to the difficulties mentioned above, there has been much less experimental work. To the best of our knowledge, there has not been any study reporting measurements of the velocity field in the vicinity of a film flowing over an obstacle. Experimentalists have usually focused on the measurement of the film thickness.

Our goal in this work is to measure experimentally the influence of a non-deformable viscous drop on the velocity field of a film flowing over the drop. Ultimately, we want to characterize the impact on the dissolution and cleaning of drops lying over an inclined planar surface.

**EXPERIMENTAL PROCEDURES**

We produced gravity-driven thin film flows in the experimental apparatus shown schematically in figure 2. A liquid film flows from a constant-head reservoir through a thin gap (thickness \( h_0 = 0.4 \text{ mm} \), width \( L_0 = 200 \text{ mm} \), length \( l_0 = 15 \text{ mm} \)) on a flat solid substrate inclined at an angle \( \alpha \) to the horizontal. We measure the angle of inclination using an electronic inclinometer. At the gap outlet \( (x = 0) \), the flow can be well approximated by a plane Poiseuille flow. Then, the film flows freely over a one millimetre thick sheet of polished stainless steel mounted on a 20 mm thick piece of PVC, which ensures the rigidity of the experimental apparatus. The stainless steel substrate is cleaned before each experiment with some water and soap, vinegar-based de-scaler and finally isopropanol. The film flows on the substrate for a distance of approximately 300 mm from the outlet of the reservoir gap to the bottom-end of the substrate, and then falls freely into a large collecting tank. The flow rate of the film is obtained by measuring the flow rate of the pipe which supplies the reservoir and maintains the water level at a constant height. Using a precision balance (to measure the mass of fluid) and a stopwatch for each experiment, the flow rate is found to be consistent through repeated measurements with an accuracy of approximately 1%. The fluid can recirculate in the experimental apparatus using a submersible pump located in the collecting tank. The fluid is pumped into a primary reservoir located upstream. The fluid turbulence in the primary reservoir is dampened as it penetrates through a piece of foam (reticulated polyether foam with 57 to 70 pores per inch and a pore size of 0.5 mm) and a 5 mm gap into the main U-shaped reservoir. Using some pearlescence (Iriodin 120 pigment, Merck) we observed that the fluid in the main U-shaped reservoir was free of turbulence.

Once the flow is stable, we start recording the experiment with a high-speed grey-scale camera, Photron–Fastcam SA1.1, mounted with a 60 mm focal-length lens (AF Micro-Nikkor) and a UV/IR blocking filter. We filter out the infrared part of the spectrum using a cold mirror because the camera is sensitive to this part of the spectrum. The lens aperture is f/4.0D or f/5.6D, depending on the shutter speed. The lens aperture is adjusted to prevent over-illumination. We record the experiments as 8-bit sequences with the high-speed camera for a duration of approximately 1 s. We perform the PIV experiments in a dark room. Two
300 Watts arc lamps and two 250 Watts halogen lamps produced a uniform illumination on the film with minimal shadows or reflections at the crests and troughs of surface waves.

Figure 2. Schematic diagram of the experimental apparatus. (a) Side view; (b) top view.

The details of the control parameters for each experiment are presented in Table 1. For Exp. 1-4, the camera view is centred on the film mid-width and with the top of the image just above the outlet, so as to see the film immediately after flowing through the gap. The images are analysed using DigiFlow (Sveen & Dalziel, 2005). The spatial velocity resolution is 2.7 mm based on interrogation areas of $17 \times 17$ pixels with 75% overlapping. The film Reynolds number is defined as $\text{Re} = 4\Gamma/\nu$ with $\Gamma(x) = Q/L(x)$ the local two-dimensional flow rate, $Q$ the three-dimensional constant flow rate, $L$ the local film width along the spanwise direction, and a viscosity $\nu = 10^{-6} \text{ m}^2\text{s}^{-1}$. In Exp. 5, the camera view is centred on the obstacle in the flow. The resolution is increased by approximately five times in order to obtain a very detailed measurement of the flow in the vicinity of the obstacle. The new spatial velocity resolution is $0.7 \text{ mm}$ based on interrogation areas of $21 \times 21$ pixels with 5% overlapping.

Table 1. Summary of the control parameters for all the experiments.

<table>
<thead>
<tr>
<th>Exp</th>
<th>Angle (°)</th>
<th>$Q$ (cm$^3$ s$^{-1}$)</th>
<th>Resolution (pixel × pixel)</th>
<th>View (cm × cm)</th>
<th>Frame rate (Hz)</th>
<th>Shutter time (s)</th>
<th>$\text{Re} = 4\Gamma/\nu$</th>
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<tr>
<td>1</td>
<td>44</td>
<td>50</td>
<td>1024 × 1024</td>
<td>17 × 17</td>
<td>2000</td>
<td>1/3000</td>
<td>1000-1200</td>
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<tr>
<td>2</td>
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<td>71</td>
<td>1024 × 1024</td>
<td>17 × 17</td>
<td>2000</td>
<td>1/2000</td>
<td>1400-1700</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>41</td>
<td>1024 × 1024</td>
<td>17 × 17</td>
<td>2000</td>
<td>1/2000</td>
<td>800-1000</td>
</tr>
<tr>
<td>4</td>
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<td>65</td>
<td>1024 × 1024</td>
<td>15 × 15</td>
<td>2000</td>
<td>1/2000</td>
<td>1300-1600</td>
</tr>
<tr>
<td>5</td>
<td>44</td>
<td>50</td>
<td>1024 × 896</td>
<td>3.4 × 3.0</td>
<td>6250</td>
<td>1/9000</td>
<td>1100</td>
</tr>
</tbody>
</table>

The film liquid used for the PIV experiments is a mixture of approximately 4 litres of cold water with 40 g (i.e. 1% wt) of methylene blue and 20 g (i.e. 0.5% wt) of artificial pearlescence, which is made of titanium-dioxide coated mica particles (size: 5 to 25 microns; density: 3 g cm$^{-3}$). The purpose of this very dark mixture of dye is to render the film opaque for the camera so that only the surface of the film can be seen. The artificial pearlescence comprises small plates acting as tracers. Aligning with the shear, the plates predominantly orientate themselves parallel to the film surface. These tracers produce a non-uniform reflecting texture of light intensity at the surface of the film, from which the surface velocity can be computed using a PIV algorithm. Since the particles of pearlescence are denser than water, they can sediment in the film flow. Assuming Stokes law for the settling velocity of the particles, we estimate that the smallest particles sink through less than 20% of the film depth. The sinking depth of the particle is of the order of the layer depth seen by the camera.
The impact of a solid obstacle on the film flow is studied. We made a small obstacle by sticking a piece of Blu-Tack (Bostik) on the substrate located at a distance of approximately 91 mm downstream of the outlet and approximately 10 mm to the right of the centreline. The size of the obstacle is 0.5 to 0.8 mm in thickness and 2.8 mm in diameter. The shape is a rough flattened hemisphere, which models the shape of a very viscous sessile drop. We can note that the obstacle is located sufficiently far away from the outlet that the film flow approaching the obstacle is fully developed. The obstacle is fully submerged by the film.

RESULTS AND DISCUSSION

In figure 3 we plot the distribution of the local time-averaged surface velocity $\bar{u}$ of the film and its standard deviation (dashed curves) along the streamwise ($x$) direction. These results correspond to Exp. 1 (see Table 1). The velocity is non-dimensionalised by the depth-averaged velocity as $x \to +\infty$, i.e. $< u_\infty >$ computed from equation (1). We show the velocity profile at two lateral ($z$) locations: with black pluses, the flow is undisturbed; with blue dots, the flow is disturbed by a solid obstacle located between the two dotted vertical lines. The film Reynolds number varies from 1200 to 1000, owing to the change of film width with streamwise distance. We also plot with a thin red solid line and a thick green solid line theoretical curves for two values of $\beta = h_0/h_\infty$. Cerro and Whitaker (1971) modelled the laminar, steady, plane flow of a film falling on an inclined plane as it develops from a Poiseuille profile at the outlet $x = 0$ (thickness $h_0$) to the viscous-gravity profile described in (1) as $x \to +\infty$ (thickness $h_\infty$). They make use of the boundary layer approximation to simplify the Navier-Stokes equation, and then use the von Mises transformation to account for the kinematic condition at the free surface and the continuity equation. We solve the resulting non-linear partial differential equation using a finite-difference implicit numerical scheme.

The thick green curve in figure 3 corresponds to the theoretical value for $\beta = 0.85 \pm 0.15$. This curve does not fit the experimental data well in the transition region for $0 \leq \xi = 4x/(h_\infty Re) \leq 0.5$: the model predicts a much more rapid growth and even an overshoot, since $\beta < 1$ (Cerro and Whitaker, 1971). Instead, the experimental data are better fitted with the value $\beta = 1.2$ (thin red curve), which predicts a slower growth of the surface velocity. We observed a similar discrepancy between the theoretical value of $\beta$ and the best fit value for experiments at various flow rates or inclination angles. An ensemble average of Exp. 1-4, shows a best fit for $\beta$ 35% larger than the theoretical value. We believe that this mismatch could in part be explained by the experimental error on the gap height of the slit $h_0$ (owing to technical imperfection). Secondly, the film is not constrained laterally within a channel but flows freely on a planar surface. Surface tension across the film tends to pull the film inwards, narrowing it. The adjustment is particularly rapid at the beginning for the first 3 or 4 cm, with the edges of the film forming an angle of up to 30° to the $x$-axis. However, one should expect the film flow to accelerate as the film width reduces. Thirdly, surface tension effects, owing to surfactants in the film, could reduce the velocity at the free surface. Surface tension is not included in the theoretical model, which assumes no shear at the free surface. This could also explain the mismatch between the experimental results and the theoretical model of Cerro and Whitaker (1971), which consistently predicts a larger velocity in the acceleration region.

The profile plotted at the location of the obstacle (blue dots) shows a clear and strong disturbance of the time-averaged surface velocity both upstream and downstream of the obstacle. The propagation of the disturbance propagates approximately one obstacle diameter upstream. At $\xi \approx 0.6$ we can note first a very small decrease of the velocity followed by a slight increase. Then the velocity drops sharply over the obstacle, by approximately 20 to
50%, compared with the undisturbed velocity. The decrease is found consistently throughout the different experiments. The velocity increases again after the flow passes the centre of the obstacle. However, we can see that in the wake of the obstacle the surface velocity remains 5% lower than the undisturbed velocity. Comparing the different experiments, the velocity recovers its undisturbed value after 5 or more obstacle diameters downstream. The recovery distance tends to increase with Reynolds number. The profile of the disturbed surface velocity presented in figure 3 is typical across all the experiments. Only the magnitude of the velocity reduction and the recovery distance vary between the experiments. We believe that the velocity reduction is strongly related to the film thickness at the obstacle.

![Figure 3](image)

Figure 3. Non-dimensional distribution of the time-averaged surface velocity of the film and its standard deviation (dashed curves) along the streamwise direction at two lateral locations: with black pluses, the flow is undisturbed; with blue dots, the flow is disturbed by a solid obstacle located between the two dotted lines. We also plot with a thin red solid line and a thick green solid line theoretical fits for two values of $\beta = h_0/h_\infty$ (Cerro and Whitaker, 1971). The film Reynolds number varies from 1200 to 1000, owing to the change of film width with streamwise distance.

In figure 4, we show the spatial distribution of the surface velocity for the non-dimensional time-averaged streamwise velocity $\bar{u}/\langle u_\infty \rangle$ (figure 4a), and the non-dimensional time-averaged lateral velocity $\bar{w}/\langle u_\infty \rangle$ (figure 4b). The obstacle is located at $(x, z) = (x_0, 0)$ on the right-hand-side and top axis. Upstream of the obstacle, we can see that the amplitude of the time-averaged surface velocities $\bar{u}$ and $\bar{w}$ are fairly uniform. In figure 4(a), we can notice a small and gradual increase, by 6 to 7% compared with the maximum amplitude, in the amplitude of the streamwise velocity from left to right. This variation could be due to a small misalignment between the camera and the substrate. Moreover, we can observe, in figure 4(b), vertical white bands in the lateral velocity field, denoting small variations of approximately 5% compared with the maximum amplitude. These bands could be due to a non-uniformity of the tracers, which we have observed to segregate into streaks downstream of very small defects. The defects could be lying on the substrate or at the gap outlet, such as: variations in surface roughness, or some solid particles or micro bubbles stuck on the surface.

In figure 4(a), we can see that the impact of the obstacle on the streamwise velocity is very limited upstream, but spreads laterally due to the formation of stationary capillary waves. These capillary waves, or ‘bow waves’, have a characteristic V shape similar to the wave
front in the wake of ships (Pozrikidis and Thoroddsen, 1991). The reduction in the velocity is concentrated on the obstacle and also immediately downstream of the obstacle. As we observed in figure 3, the magnitude of the velocity does not recover its upstream value in the wake of the obstacle, for a band ranging the full width of the obstacle.

Figure 4. Spatial distribution of the surface velocity of a film flowing over a fully submerged obstacle (located at \((x, z) = (x_0, 0)\) on the right-hand-side and top axis) at an angle of 44°, a flow rate of 50 cm^3 s^-1 (Exp. 5 in Table 1). (a) Non-dimensional time-averaged streamwise velocity; (b) non-dimensional time-averaged lateral velocity (with negative values, in darker grey, pointing to the left).

In figure 4(b), we should first note that the magnitude of the lateral velocity is at most 3% of the magnitude of the undisturbed streamwise velocity. The diverging flow on the obstacle is clearly visible in the velocity field, starting exactly at the top edge of the obstacle. Then, immediately downstream of the obstacle, \(\vec{w}\) points inwards revealing flow convergence in a narrow region extending more than five obstacle diameters downstream. At the bottom edge of the obstacle, the flow is quite complex and three-dimensional. We find that the standard deviation is rather large in this region. It is possible that the tracers segregate away from this region owing to the divergence of the flow immediately upstream. Therefore, there might be less information for the computation of the velocity field, and the velocity field is slightly less accurate in a narrow band downstream of the obstacle. The V-shape pattern of the stationary capillary waves is also clearly revealed by the distribution of the lateral velocity.

CONCLUSION

We investigate the problem of cleaning a very viscous drop attached to an inclined surface by a gravity-driven falling film flowing over the drop. We are interested in the case where the film cannot detach the drop from the substrate. Instead, the drop fluid diffuses slowly into the cleaning film before being transported away by the bulk flow. This problem was modelled theoretically by Blount (2010) using an advection-diffusion equation. One of the key assumptions in the model is to consider that the drop does not impact the velocity in the diffusive boundary layer at the interface. To test this assumption, we have developed a new experimental technique, based on particle image velocimetry, to measure the velocity field at the surface of a liquid film. We report in this study the first measurements of the two-dimensional distribution of the film surface velocity in the vicinity of an obstacle. The film Reynolds number is in the intermediate range: 800 to 1700. The flow is laminar, but inertial effect can be important.
First, we investigated the undisturbed streamwise velocity profile, developing from a plane Poiseuille flow at the gap outlet, to the viscous-gravity regime in the far field. We found that the surface velocity reaches asymptotically the viscous-gravity regime. However, the model developed by Cerro and Whitaker (1971) consistently predicted a faster increase of the velocity. This discrepancy could be due to three-dimensional effects in the flow, or surface tension might have had a strong impact at the free surface. Second, we studied the impact of an obstacle on the film velocity. We observed a large drop in the magnitude of the streamwise velocity starting one obstacle diameter upstream of the obstacle. The recovery of the streamwise velocity downstream of the obstacle can be larger than 10 obstacle diameters. Laterally, characteristic V-shaped capillary waves perturb the velocity field. The magnitude of the disturbance due to the waves is small compared with the disturbance at the obstacle. We can also observe a complex three-dimensional converging flow just below the obstacle.

In conclusion, the drop has a strong impact on the film velocity. Therefore, the diffusion and advection of the drop fluid can be significantly affected as the film velocity decreases in the vicinity of the drop. The return flow immediately downstream of the drop could also have an effect of the cleaning process.

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REFERENCES