Scattering of internal gravity waves

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Motivation

Internal waves reflecting from boundaries with geometric features comparable with wavelength of incident waves.
Apparatus

Tank: 1666×173×490mm (fill to ~400mm)
Salt stratification: \( N \sim 1.96 \) rad/s
Cylinder: 49 mm
Bottom: flat and 90° saw-tooth (10mm and 20mm amplitudes)
Synthetic schlieren
\[
\frac{d^2 \xi}{dy^2} = \left[ 1 + \left( \frac{d\xi}{dy} \right)^2 + \left( \frac{d\zeta}{dy} \right)^2 \right] \frac{1}{n} \frac{\partial n}{\partial x}
\]

\[
\frac{d^2 \zeta}{dy^2} = \left[ 1 + \left( \frac{d\xi}{dy} \right)^2 + \left( \frac{d\zeta}{dy} \right)^2 \right] \frac{1}{n} \frac{\partial n}{\partial z}
\]

\[
\Delta \xi = \frac{1}{2} W(W + 2B) \frac{1}{n_0} \frac{\partial n'}{\partial x}
\]

\[
\Delta \zeta = \frac{1}{2} W(W + 2B) \frac{1}{n_0} \frac{\partial n'}{\partial z}
\]

In salt water

\[
\beta = \frac{\rho_0}{n_0} \frac{dn}{d\rho} \approx 0.184
\]
Qualitative mode

Look for differences between current image and reference image: \[ \text{diff} = \gamma |P_{ij}(t) - Q_{ij}|, \]
Quantitative mode

Interpolation

Dot tracking

Pattern matching
interpolation

- Lines or 2D features
- Pixel gives integral of light over area
- Mid-point rule $\Leftrightarrow$ quadratic interpolation

$$P = Q_0 + \frac{1}{2}(Q_1 - Q_{-1})\Delta_\zeta + \frac{1}{2}(Q_1 - 2Q_0 + Q_{-1})\Delta_\zeta^2 = 0$$

- Solve for $\Delta_\zeta$
- Binomial expansion with consistent order

$$\Delta_\zeta = \left[ \frac{(P - Q_0)(P - Q_{-1})}{(Q_1 - Q_0)(Q_1 - Q_{-1})} - \frac{(P - Q_0)(P - Q_1)}{(Q_{-1} - Q_0)(Q_{-1} - Q_1)} \right] s$$

- Shift image if out of $\Delta_\zeta$ exceeds one pixel
Interpolation

\[ x \text{ gradient} \]

\[ z \text{ gradient} \]
pattern matching

Similar to PIV

Difference between window contents

\[ F(P_{ij}, Q_{ij}, \delta) \]
Many choices

\[ F(P_{ij}, Q_{ij}, \delta) = \sum_{k=i-w}^{i+w} \sum_{l=j-w}^{j+w} |P(k + \delta_i, l + \delta_j) - Q(k, l)| \]

PIV normally uses cross-correlation function.

Find \( \delta = \Delta \) (integer) that minimises \( F(\cdot) \).

Fit surface near \( \delta = \Delta \) to improve estimate.

Noisey, especially if \( \Delta \ll \text{pixel} \).

\textit{A problem in nonlinear optimisation.}

Aim: to calculate the distortion to the image pair.

\[ F(\delta; x) = \int_W W(x' - x, \delta) f(P(x'), Q(x'), \delta) \, dx' \]

\[ \Delta(x) = \{ \delta : \min_\delta F(\delta; x) \} \]

\[ f(P, Q, \delta) = \text{diff}(P(x + \alpha \delta), Q(x - (1 - \alpha) \delta)) \]

The measure of the difference, \textit{diff}, could take many forms.
BUT, we do not know $P(x + \alpha \delta)$, etc., except at integer spacing. We also do not know $\delta$ except at discrete locations.

Solution:

◊ Interpolate $\delta$

◊ Distort $P$, $Q$ or both

◊ Use interpolation when calculating $\text{diff}(..)$ for non-integer $\delta$
Waves from oscillating cylinder
Cross-beam structure at $r/R = 8$
Flat bottom

Movie
Large amplitude saw-tooth

Movie

Linear reflection possibilities
Nonlinear reflection

Also effect of boundary layers and shedding from crests
As superposition of waves
RMS amplitude

Flat bottom

20mm saw-tooth
Cross-beam structure

Flat bottom 20mm saw-tooth

Difference movie

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Spectral development

Expt 22: Flat bottom

Expt 21: Saw tooth (20mm)
Amplitude dependence

Spectra

Expt 22: Flat bottom

Expt 18: Saw tooth (10mm)

Key:
- Incident x gradient
- Reflected x gradient
- Incident y gradient
- Reflected y gradient

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Expt 22: Flat bottom

Expt 21: Saw tooth (20mm)

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Conclusions

Preliminary experiments
◊ Increased energy at high wavenumbers
◊ Reflection back along incident wave beam
◊ Enhanced dissipation & diffusion
◊ Back reflection → steepening
◊ Other nonlinear effects → frequency doubling, mixing

Synthetic schlieren
◊ Benefit from higher resolution digital video

http://www.damtp.cam.ac.uk/lab/digiflow/