Gravity Currents on Slopes

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A dissertation submitted for the degree of
Doctor of Philosophy
in the
University of Cambridge

September 2000

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Preface

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration. No part of it has been submitted for a degree or any similar qualification at any other university.

Financial support for this work has been through an EPSRC studentship CASE award in conjunction with the Health and Safety Executive. Churchill College also provided a travel grant towards conference expenses.

I would like to give special thanks to my supervisor, Paul Linden, and my acting supervisor, Stuart Dalziel, for their invaluable suggestions, assistance and encouragement during the last three years. I would also like to thank Robin Hankin, David Webber and Steve Porter at the HSE for their input.

Many people in the laboratory in DAMTP have helped me in one way or another and made my time here so enjoyable. Thank you all. I would particularly like to thank the technicians, David Page-Croft, Brian Dean, David Lipman and Chris Mortimer, for their great technical assistance in building the experimental apparatus used in this thesis.

Finally, a big thank you to Alison for her help and support, for proof reading and for keeping me going while I was writing up.
Chapter 1

Introduction

1.1 What is a gravity current?

The spreading of one fluid into another fluid of a lower density due to the horizontal density gradient is important for many geophysical and industrial applications. Such flows are known as “gravity currents” or “density currents”. The flow is predominantly horizontal and consists of a head region at the front that is deeper than the following flow. Often this head region is characterised by billows and strong mixing. Despite the mixing, the clear distinction between the denser gravity current and the lighter ambient fluid typically remains.

1.2 Motivation

There are many practical problems which can be thought of as gravity currents. The primary motivation behind the work in this thesis is the spreading of a dense gas cloud. This will be discussed in detail in the following section. Some other examples will be given in §1.2.2 to demonstrate the range of applications of this work.

1.2.1 Dense gas dispersion

Many poisonous, explosive or suffocating gases are routinely manufactured and used in the chemical industry. Such gases are usually denser than air. If a leak occurs, the dense gas can spread and form a gravity current. Such a spillage could be an instantaneous release of gas from the catastrophic failure of a storage vessel, or a continuous release from a leaking tank or pipeline.

The consequences of such an accident can be disastrous. Undoubtedly the worst accident of this nature was the leak from the Union Carbide plant, Bhopal, India in December 1984.
At least 4000 people lost their lives when 40 tonnes of the poisonous gas, methyl isocyanate, leaked from a tank and spread across the shanty town next to the plant. The full extent of the tragedy may never be known since the long term effects of exposure to the chemical are hard to measure. The worst incident in the UK was the 1974 Flixborough disaster. An explosion, following a leak of cyclohexane, killed 28 people and caused devastation to the rest of the plant and the nearby area. While these are extreme examples, smaller scale leaks occur on a more regular basis and can still have serious consequences.

The chemical industry, regulatory bodies (e.g. the UK Health and Safety Executive (HSE)) and local planning authorities all need to understand the dynamics of an accidental spillage of dense gas in order to make informed decisions about the risk to people and property. The safety of the site and the risk to others in the event of a leak form an important part of any planning decision. In order to assess the consequences of an accident such as Bhopal, and the risk to those living nearby, it is necessary to be able to predict the spread of the gas cloud. Some substances, such as liquefied natural gas (LNG) and liquefied propylene gas (LPG), are explosive above a certain concentration. In these cases it is important to know how rapidly the cloud will dilute. For many toxic gases (e.g. chlorine) it is the cumulative dose which causes danger to human health. In this case it is not only the concentration at a point which is important, but also how fast the cloud moves past the point.

Previous research has often concentrated on the problem of a dense gas cloud spreading on a horizontal surface. In practice, many industrial factories are situated on a hill or in a valley. In such cases the local topography can make a huge difference to the spread of a gas cloud. An understanding of the effects of a slope is necessary in order to ascertain how far from the industrial site it is safe to situate houses, schools and shops.

Earlier work has often neglected the effects of the wind on the flow. The research that has been done shows that the presence of wind can make a large difference to the direction and dilution of the flow. Assessing the impact of the wind on a dense gas release is made more difficult due to the variability of the wind direction and speed. An assessment of the risk can be made by studying the effects of different wind conditions and the probability of such conditions occurring.

In order to predict the motion and dilution of a cloud of dense gas it is important to understand the effects of the topography and the background flow, and how they can interact. The simplest case of topography is a uniform slope. The purpose of this thesis is to study aspects of the motion of gravity currents on such a slope, including some of the effects of wind.

Understanding the release of a dense gas is complicated by the thermodynamics of the flow. In the chemical industry gases such as LNG and LPG are stored at high pressures in
a liquefied form. When the liquid is released it rapidly depressurises and some of it boils to form a gas. The gas cools as a result of the rapid expansion and then warms up to adjust to the atmospheric conditions. During this initial expansion phase the density of the gas can change rapidly. This makes the gravity current more difficult to model. Initial thermodynamic effects will be neglected in this thesis. Instead we shall concentrate on problems where the density of the gases involved does not change, although the ratio in which they are mixed may. It is a reasonable assumption to neglect thermodynamic effects, except near the source of the release. A thorough understanding of the density driven problem is necessary in order to appreciate the additional effects of the thermodynamics.

Atmospheric conditions other than the mean wind speed can also affect the dispersion of a gas cloud. Atmospheric turbulence can act to enhance the mixing of the dense gas with the air. The stability of the atmospheric boundary layer can also play a role. In particular, unstable conditions will result in large scale convection in the atmosphere which can cause large amounts of mixing. In extreme cases this background turbulence and mixing can lead to the destruction of the gravity current head. The effects of turbulence and background stability have been studied elsewhere (see Linden & Simpson, 1986, 1989) and will be neglected in this thesis.

### 1.2.2 Other industrial, geophysical and environmental applications

There are many other examples of gravity currents besides dense gas releases. In his book, Simpson (1997) describes and illustrates a whole range of these density driven flows. Other industrial problems driven by horizontal density differences include the movement of smoke and heat in buildings and the spreading of sewage or cooling water from an outlet pipe into the sea. Gravity currents also occur widely in nature and include such diverse examples as turbidity currents in the ocean, sea breeze fronts propagating inland from the coast, salt water wedges in river estuaries, and snow avalanches.

The density difference driving a gravity current may be caused by chemical differences, such as dense gas (e.g. chlorine) being released in the atmosphere or fresh water from a river flowing into salty sea water. Thermal differences can also lead to density differences, for example in the ventilation of buildings, in sea breezes or in a spillage of LNG (where the methane is only denser than air because it is cold). Turbidity currents in the ocean, dust storms and pyroclastic flows from volcanic eruptions are all driven by suspended particles increasing the bulk density of the surrounding fluid. In these particulate flows the density can change through sedimentation of the dense particles as well as by entrainment of the lighter ambient fluid.

In many of these examples the flow can be affected by the surrounding topography. Snow
avalanches and pyroclastic currents are extreme examples of gravity currents flowing down a mountain or volcano. The background flow is also a significant feature in many of the examples. In the atmosphere dust and ash clouds from volcanoes are greatly influenced by the wind, while the tide can play a huge role in the formation of estuarine salt wedges. Turbidity currents are another example where both topography and background flow can be influential. These flows are formed on the continental shelves of the oceans which slope down from the coast to the deep ocean and are also subject to any ocean circulation which may exist.

These examples show the variety of flows which can be classified as gravity currents and the importance of studying such flows.

1.3 Methodology

In many examples of gravity currents, such as the spreading of a cloud of dense gas, the flow within the current is turbulent. Small scale mixing processes between the ambient fluid and the dense fluid, due to the turbulence, are important to the dynamics of the flow. As a result of the complexity of the flow in a gravity current, it is difficult or impossible to obtain simple analytic, or even numerical, solutions for the problem, even in the case of simple geometries. There are several approaches which can be taken to this problem. These range from simple scaling arguments and integral models to more complicated models and direct numerical simulations (DNS). Experiments also play a valuable role.

The use of simple scaling arguments and integral models for the bulk motion of the fluid gives some insight into the physical processes involved in the flow, but cannot capture the details. They can help to clarify the underlying physics of the problem, and often give surprisingly good answers, despite their simplicity.

More complicated models can incorporate greater detail, but are often restricted to two dimensions and require numerical solution. Such models include those based on the shallow water equations or the Euler equations. In general these models still fail to capture both the small scale turbulence and the large scale bulk motion of the fluid. Mixing is generally included through some form of parameterisation. With the ever increasing power of computers it is now becoming possible to perform DNS calculations in certain situations to solve the full Navier–Stokes equations directly. Such DNS calculations can model all scales of the flow, but are currently limited to moderate Reynolds numbers by the computational requirements.

The third approach to studying problems involving gravity currents, which complements the other two methods, is the use of experiments. These can be small scale experiments in the laboratory or measurements from larger scale field trials or natural flows. The experiments, of course, include all the details of the flow and can be used to obtain values for experimental
parameters in the models and check that the models give reasonable predictions. Experiments still provide the only means to study fully high Reynolds number flows.

1.4 Outline of work

1.4.1 Overview

There are two main classes of release which are considered in this thesis. The first is an instantaneous release of a fixed volume of dense fluid. This is representative of a catastrophic failure of a storage tank of dense gas, leading to a sudden release of a large amount of the gas. The second is a continuous release of fluid with a fixed flow rate. This situation is representative of what could happen if a storage tank ruptured, leading to the release of dense gas from the tank over some period of time. The tank may be pressurised, or there may be a large hydrostatic head, in which case the gas would initially be released in the form of a jet.

The aim of this work is to understand and predict the motion and dilution of such gravity currents. The problem of an instantaneous release of dense fluid on a uniform slope will be investigated. The related problem of a continuous release, in the form of a dense plume on a uniform slope, will also be studied. Neither of these problems has been fully investigated in previous research, yet the presence of a slope is important for many of the applications of gravity currents discussed above. A uniform slope is a good model for situations where the slope only varies slowly but other sloping geometries may also occur. In particular, an instantaneous release on a cone will be considered here. This provides a model for the release of dense gas on top of a hill or in a sloping valley. In order to extend the work to incorporate the effects of both a slope and the wind then the simplest possible geometry of a sloping channel will be used. The interaction of wind and slope has not been studied before, despite its importance for many applications.

In this thesis gravity currents are studied using a combination of simple integral models, numerical simulations and laboratory experiments. The experimental work provides important data to validate existing or new models for dense gas dispersion. It also provides insight into the detailed structure of the flow and the relative importance of different physical processes.

1.4.2 Structure

Chapter 2 provides a review of the previous literature on gravity currents, with a particular emphasis on the effects of wind and slope. Important experimental and theoretical results are described, and their relevance to the problems considered in this thesis is discussed. In chapter 3 the use and applicability of laboratory experiments to model larger scale atmospheric
flows is discussed. The various experimental apparatus and techniques used in this thesis are described in detail.

In chapter 4 the problem of an instantaneous release of dense fluid on a uniform slope with no barriers or constraints will be considered. It is common for the site of a chemical factory to be on a gentle slope. Often the angle of the slope will only vary slowly, so modelling the site as a uniform slope is a reasonable approximation. An understanding of simple geometries is necessary in order to appreciate the effects of more complicated arrangements of hills and valleys. While the problem of a gravity current in a sloping channel has been studied by several workers (e.g. Ellison & Turner, 1959; Beghin et al., 1981; Britter & Linden, 1980; Montgomery & Moodie, 1999), gravity currents on an unconstrained slope have received less attention. There has been some theoretical work by Webber et al. (1993) and Tickle (1996), but little accompanying experimental evidence. The work in this thesis is intended to fill this gap. A series of laboratory experiments are described which provide detailed measurements of gravity currents on a slope. The results are compared with previous results and with a new integral model developed here.

Chapter 5 looks at the case of a continuous release of dense fluid on a slope. Attention is focused on a jet or forced plume initially directed up the slope. A series of laboratory experiments studying this problem are described and the results presented. A plume model is developed, based on the ideas of Morton et al. (1956) and Lane-Serff et al. (1993), and the predictions of the model are compared with the experimental results and with simple scaling arguments. The model is extended to consider a plume initially tilted at an angle to the up slope direction. Further experiments on these tilted plumes are described and the results are again compared with the model.

In chapter 6 a numerical model for the flow of gravity currents in a channel and axisymmetric gravity currents is developed. The model, based on the vorticity–streamfunction formulation of the Euler equations, includes the effects of both slope and wind. The model is validated by comparing the predictions with previous experimental and theoretical results. Two new problems are investigated in chapter 7 using the model. Firstly, an instantaneous release of dense fluid on a cone, such as a hill, is studied. Some new laboratory experiments and an integral model are compared with the predictions of the numerical model. Secondly, the interaction of slope and wind in a channel is studied. This includes a study of the effects of the wind profile, wind speed and slope. The question of whether the current can be prevented from spreading downslope under the correct wind conditions is considered.

In the final chapter conclusions are drawn from the work presented here and its relevance to modelling the spread of a dense-gas cloud is discussed. Some ideas for future extensions to this work are suggested.
Chapter 2

Background

2.1 Overview

The aim of this chapter is to provide a review of some of the literature on gravity currents that is relevant to this thesis.

The study of gravity currents dates back over many years. Much of the work has concentrated on currents flowing over a flat bottom, with no motion in the ambient. Often currents in a channel have been considered, as this is a two-dimensional problem and easier to study both theoretically and in the experimentally. Many of the results can be generalised to axisymmetric flows as well. Both instantaneous and continuous releases of dense fluid have been considered. Some of the existing literature is reviewed here, concentrating on the results and observation from experiments and on some of the simplified models used to describe these flow. These results provide a basis for the study of gravity currents in more complicated situations and also provide a useful set of test cases for the numerical model developed in chapter 6. There has been limited work focusing on the various effects of wind or slope on a gravity current. This work will be discussed in order to provide a basis for the new research in this thesis. There is a brief presentation of some results for gravity currents where the Reynolds number is small and viscous effects are important. This can occur in the laboratory due to the smaller scale of the experiments. Finally some of the literature on buoyant and dense plumes is reviewed in connection with the continuous releases on a slope discussed in chapter 5.
CHAPTER 2. BACKGROUND

2.2 Gravity currents on a flat surface

2.2.1 Early work

The work of von Kármán (1940) is some of the earliest in this field. He used an inviscid analysis to predict the front speed and interface shape of a gravity current flowing under an infinite ambient. The angle of the nose with the ground is predicted to be 60°. Nearly thirty years later, Benjamin (1968) used a similar approach to model the flow of an air cavity into a fluid-filled channel, including the effects of dissipation. The inviscid model of Benjamin (1968) makes use of Bernoulli’s theorem along the boundaries and the interfacial streamline, in addition to the balances for mass and flow force. While this is an idealised situation with no dissipation or mixing, it provides fair agreement with experiments. It has also been the basis for several later analyses, including the studies of wind shear by Xu (1992) and Xue (2000) described in §2.4, so is worth considering in some detail. The current is assumed to be in a steady state so we can choose a frame of reference with the gravity current head at rest. The problem is illustrated in figure 2.1.

Firstly, continuity in the liquid layer gives

\[ c_1 d = c_2 h. \]  \hfill (2.1)

Applying Bernoulli’s theorem along the streamline OB, with the pressure zero everywhere in the cavity, gives

\[ c_2^2 = 2g(d - h). \]  \hfill (2.2)

Similarly, applying Bernoulli’s theorem on the streamline OA gives

\[ p_0 = -\frac{1}{2}\rho c_1^2. \]  \hfill (2.3)

Assuming the pressure far upstream at A and downstream at B is hydrostatic, the total flow
force balance between A and B gives

\[ \frac{1}{2} \rho c_1^2 d + \frac{1}{2} \rho g d^2 = \rho c_2^2 h + \frac{1}{2} \rho g h^2. \]  \hspace{1cm} (2.4)

Combining these four equations gives \( h = d/2 \), \( c_1 = (gd)^{1/2} / 2 \) and \( c_2 = (2gh)^{1/2} \). This says that for the flow to be dissipationless the current must be half the depth of the channel. If the gravity current is not half the channel height then there must be some dissipation in the current. Benjamin (1968) considered dissipation in the form of a constant head loss along OB and showed that the current height must be less than half the channel height for a positive dissipation.

Benjamin (1968) modelled this dissipation in the flow by assuming a constant head loss along the interfacial streamline. This is an assumption that the dissipation is evenly distributed over the fluid layer. In practice the dissipation is not likely to be evenly distributed over the fluid layer, but to be located in the vicinity of the interface. Klemp et al. (1994) modified the work of Benjamin (1968) to localise the dissipation to the interface and discovered that the predictions were almost identical. The presence of dissipation reduces the height and speed of the current as might be expected, but the results do not appear to be too sensitive to the precise form of the dissipation.

Instead of considering an air cavity flowing into a liquid, we can invert the problem and consider the flow of a dense fluid into a channel filled with a lighter fluids. In this case the pressure inside the gravity current cannot be neglected. Assuming the pressure in the current is hydrostatic far away from the head, we can apply Bernoulli along the lower boundary of the dense fluid, starting from the stagnation point. The flow in the head relative to the front is assumed to be negligible. This leads to exactly the same equations for the gravity current, but with \( g \) replaced by \( g' = g(\rho - \rho_a)/\rho_a \), where \( \rho \) is the density of the gravity current and \( \rho_a \) the density of the ambient fluid. The quantity \( g' \) is known as the “reduced gravity” or “effective gravity”.

Hoult (1972) approached the problem of gravity currents from a different perspective. He was considering the spreading of oil on the sea. He analysed both channel and axisymmetric gravity currents. By considering the relative importance of terms in the momentum equation, he divided the motion of a gravity current into various stages depending on whether the inertial, viscous or surface tension terms were more important. Inertial effects are more important than viscous effects provided the Reynolds number, \( \text{Re} = Uh/\nu \), of the flow is large. Here \( U \) is the speed of the gravity current, \( h \) is the height of the current and \( \nu \) is the kinematic viscosity of the fluid. This thesis concentrates on high Reynolds number flows where the effects of viscosity are small.
2.2.2 Laboratory experiments and integral models

As discussed in chapter 1, laboratory experiments can provide important information on gravity currents and provide a check on the accuracy of proposed theories or numerical models. In a series of saline laboratory experiments, Simpson (1972); Britter & Simpson (1978); Simpson & Britter (1979) studied in detail the motion of a lock-release gravity current in a horizontal channel. They looked at various aspects of the problem including the rate of spreading, and the shape and height of the current head. One of the most important findings was the observation that, after the initial adjustment, the head of the gravity current develops in such a way that the Froude number of the head, defined as

\[
Fr = \frac{u_f}{(g' h_f)^{1/2}},
\]

is a constant. Here \( u_f \) and \( h_f \) are the front speed and height of the head, respectively. The constant Froude number front condition has proved important in the modelling of gravity currents.

Simpson & Britter (1979) also performed some experiments in a sector tank to simulate a gravity current spreading axisymmetrically: the same constant Froude number behaviour was observed to occur. For both geometries the Froude number was 1.19, averaged over the experiments where the fractional depth of the current head was small. In comparison, the model of Benjamin (1968) described in §2.2.1 predicts a value of \( \sqrt{2} \) for the Froude number which is slightly higher than the measured value, presumably as a result of dissipation and mixing in the experiments.

Experiments by Barr (1967) showed that there was some dependence of the Froude number on the Reynolds number. The Froude number increases with Reynolds number up to a Reynolds number of about 1000. Above that the Froude number is essentially constant. Atmospheric measurements of sea-breeze fronts have been made at Reynolds numbers of about \( 10^8 \) and these are consistent with laboratory experiments at Reynolds number of about 1000 (see Simpson & Britter, 1979).

Barr (1967) also carried out some experiments with buoyant surface gravity currents. Again the front Froude number was found to be a constant, although the value of the constant was slightly higher. This difference could be due to the different boundary condition at the surface.

The saline laboratory experiments provide a good way to observe in more detail some of the structure in the head of the gravity current. It is observed that, at the Reynolds numbers of order \( 10^3 \) used in the experiments, there are turbulent billows on the head. Mixing occurs between the dense fluid in the head of the current and the lighter fluid behind. There are two...
mechanisms, described below, by which this occurs.

There is observed to be a shear layer at the top of the head, between the moving dense fluid in the head and the stationary ambient fluid. This interface is unstable to a Kelvin–Helmholtz instability, with its characteristic billows being seen on the interface. The instability can lead to the mixing of dense fluid with ambient fluid behind the head of the gravity current. This mixed fluid is left behind as a layer of intermediate density.

The second mechanism applies if the gravity current is travelling on a surface with a no-slip boundary condition, such as a rigid surface. Ambient fluid at the surface cannot be pushed along ahead of the current as there is no horizontal motion on the boundary. Since there is no vertical motion at the surface either, the ambient fluid cannot be lifted over the head. As a result the dense fluid in the head overruns this lighter ambient fluid. This situation is hydrostatically unstable and the lighter fluid rises up through the head of the current in a fingering pattern, mixing in with the dense fluid in the head as it goes. Viewed from above, the front of the current appears to consist of “lobes” of dense fluid and “clefts” where the light fluid rises up between. This “lobe and cleft” structure makes the fluid motion fully three-dimensional and provides one possible mechanism for the breakdown of the two-dimensional Kelvin–Helmholtz billows. Another effect of the no-slip condition is to raise the nose of the gravity current up from the floor. Recent DNS calculations by Härtel et al. (2000a) suggest that the cause of the instability may be slightly different. The simulations indicate that nose of the gravity current is raised above the ground and the stagnation point of the flow is slightly behind and below the nose. This means there is a small region of unstably stratified fluid near the nose which does not contain overrun fluid. They suggest that the front instability occurs in this region. The exact mechanism by which the “lobes and clefts” form is still open to debate.

In his experiments with a moving floor, Simpson (1972) was able to simulate a situation similar to the free-slip boundary condition by ensuring the floor moved at the same speed as the current. He observed that the “lobes and clefts” and the raised nose only appeared in experiments where the floor was not moving so there was a no-slip boundary condition. In another experiment with a stationary floor a thin layer of dense fluid was laid ahead of the current to ensure the overrun fluid was not lighter than the fluid in the head. Again the ”lobe and cleft” instability was suppressed, showing that the overrunning of less dense fluid by the denser head is responsible for the instability.

Simpson & Britter (1979) also present some simple integral models (sometimes known as box models) for the development of the current. These simple models can often capture the physics of the problem and provide a surprisingly good estimate of the spread of a gravity current. In later chapters of this thesis several integral models will be considered so it is useful to look first at the simplest case of a gravity current in a horizontal channel. Suppose the current
is assumed to be rectangular in shape with a length \( l \) and a height \( h \). Conservation of mass implies that \( V = lh \) is a constant and the front speed is given by \( u_f = \frac{dl}{dt} \). Substituting these two equations into the constant Froude number condition (2.5) gives a differential equation

\[
\frac{dl}{dt} = Fr \left( \frac{g' V}{l} \right)^{1/2}, \tag{2.6}
\]

which can be integrated to give

\[
l = l_0 \left( 1 + \frac{3}{2} Fr \left( \frac{g' V}{l_0^{3/2}} \right)^{1/2} t \right)^{2/3}. \tag{2.7}
\]

This predicts that for large time \( l \propto t^{2/3} \). This contrast with the model of Benjamin (1968), which assumes a steady gravity current, so the speed of the current is a constant and the front position increases linearly with time. The steady current requires fluid to be fed into the current from behind to maintain its thickness. In instantaneous release experiments there is no continued feeding of fluid into the current, so it slows down. In the experiments of the transition to the self-similar regime was observed to occur after the current has travelled approximate 10 lock lengths. Before this time the speed of the current remained nearly constant.

An entirely analogous integral model can be developed for an axisymmetric current, giving

\[
r = r_0 \left( 1 + 2 Fr \left( \frac{g' V}{r_0^{3/2}} \right)^{1/2} t \right)^{1/2}. \tag{2.8}
\]

For the axisymmetric case, \( r \propto t^{1/2} \) for large times. This is in agreement with the scaling of Hoult (1972). These models, despite their simplicity, provide a good estimate for the spread of the current. The transition to the self-similar phase is found to occur after only about 3 lock lengths in the axisymmetric case because the current thins more rapidly with the radial spreading.

Huppert & Simpson (1980) studied more closely the initial stages of the flow in which the current slumps down from the lock. Initially, the constant Froude number condition and the shallow water approximation are not valid. From laboratory experiments Huppert & Simpson (1980) observed that the initial stages of the motion could be described by the empirical formula

\[
Fr = \frac{1}{2} \phi^{-1/3} (0.075 \leq \phi \leq 1), \tag{2.9}
\]

where \( \phi = h/H \) is the fractional depth of the current. This relation between the front depth and speed in the initial stages can be substituted into (2.6). Integrating gives the front position
as

\[ l = l_0 \left( 1 + \frac{7}{12} \left( \frac{g^3 V H^2}{l_0^3} \right)^{1/6} \right)^{6/7} \quad (l_0 \leq l \leq l_s \equiv V/(0.075H)) \]  

(2.10)

The solution for the axisymmetric problem is

\[ r = r_0 \left( 1 + \frac{2}{3} \left( \frac{g^3 V H^2}{\pi r_0^5} \right)^{1/6} \right)^{3/4} \quad (r_0 \leq r \leq r_s \equiv V/(0.075H)) \]  

(2.11)

Once the current has reached a fractional depth of 0.075, the top boundary no longer considered to be important and the constant Froude number condition \( Fr = 1.19 \) is taken.

### 2.2.3 Shallow water models

As a gravity current spreads, the length of the current increases and, in order to conserve volume, the height decreases. Even for an initially tall release, the aspect ratio of the current will eventually become low. This suggests that the shallow water equations may provide a useful approximation to the flow. The shallow water equations are derived by assuming that the horizontal length scale of the flow is much longer than the vertical length scale. This means that the vertical velocity and vertical variations in the horizontal velocity can be ignored. This gives a depth averaged set of equations for the height and horizontal velocity of the flow as a function of horizontal position and time.

This approach has been taken by various people, including Hoult (1972) and Grundy & Rottman (1985, 1986). Shallow water models have become an important way of modelling gravity currents and are worthy of further discussion, particularly to allow comparison with the shallow water model for a gravity current on a sloping boundary described in §2.3.

For a current which has spread sufficiently that the aspect ratio is low, the shallow water approximation is valid over most of the flow. However, at the front of the gravity current it is not satisfied and some other condition is needed to close the problem. The constant Froude number condition is usually used to provide a front condition.

The simplest model for a gravity current in a channel is discussed here. The model is one layer, so flow in the ambient layer is ignored. This is a reasonable assumption provided the ambient layer is much deeper than the current. The channel is taken to have a solid end wall and the release is at one end of the tank. There is no explicit drag in this model, although the constant Froude number condition effectively parameterises the drag of the ambient fluid on the current. In this case the shallow water equations are

\[ \frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = 0 \]  

(2.12)
for the depth averaged continuity equation and

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = 0$$  \hspace{1cm} (2.13)$$

for the horizontal momentum equation. The Froude number condition,

$$u_f = Fr (g' h_f)^{1/2},$$  \hspace{1cm} (2.14)$$
is applied at the front, $x_f$, of the current and the no-flow condition,

$$u = 0, \frac{\partial h}{\partial x} = 0,$$  \hspace{1cm} (2.15)$$
at the end wall of the channel, $x = 0$.

Hoult (1972) first demonstrated that there exists a similarity solution for this problem. The appropriate similarity variable is

$$\eta = (g' V)^{1/3} x r^{-2/3},$$  \hspace{1cm} (2.16)$$

with

$$\eta = \eta_f = \left( \frac{27 Fr^2}{12 - 2 Fr^2} \right)^{1/3}$$  \hspace{1cm} (2.17)$$
at the front.

The front position is given by

$$x_f = \eta_f (g' V)^{1/3} r^{2/3}.$$  \hspace{1cm} (2.18)$$

This can be compare with the long-time approximation to the integral model prediction (2.7) which gives

$$l = \left( \frac{3}{2} Fr \right)^{2/3} (g' V)^{1/3} r^{2/3}$$  \hspace{1cm} (2.19)$$

which has exactly the same time dependence and dimensional form, but a slightly different coefficient in front. The height of the current is given by

$$h = \frac{V}{x} \eta \left( \frac{1}{9} \eta^2 + \eta_f^{-1} - \frac{1}{27} \eta_f^2 \right)$$  \hspace{1cm} (2.20)$$

and the velocity by

$$u = \frac{2}{3} x r^{-1}.$$  \hspace{1cm} (2.21)$$
2.2. GRAVITY CURRENTS ON A FLAT SURFACE

Numerical solutions of the shallow water equations tend towards this similarity solution in the long-time limit, as demonstrated by Grundy & Rottman [1985]. For a full depth release in a channel the self-similar phase is reached after approximately 10 lock lengths. This corresponds to the time at which the bore initially generated at the release travels back to the end wall, reflects and catches up with the head of the current once more. This distance is in agreement with the experimental observations of Rottman & Simpson [1983] for the onset of the self-similar phase.

The same idea can be applied to an axisymmetric gravity current. In this case the similarity variable is

\[ \eta = \left( g'V \right)^{1/4} r t^{-1/2} \] (2.22)

and

\[ \eta_f = \left( \frac{16Fr^2}{\pi(4 - Fr^2)} \right)^{1/4}. \] (2.23)

The front position is

\[ r_f = \eta_f \left( g'V \right)^{1/4} t^{1/2}. \] (2.24)

The height and speed of the current are given by

\[ h = \frac{V}{8r^2} \eta^2 \left( \eta^2 + \eta_f^2 \frac{2 - Fr^2}{Fr^2} \right) \] (2.25)

and

\[ u = \frac{1}{2} r t^{-1}. \] (2.26)

For both the channel flow and the axisymmetric flow the height of the current increases with distance from the origin until attaining its maximum at the front. Substituting for \( \eta \) into (2.20) and (2.25) shows that, at a fixed time, the height increases from the back wall to the front of the current. At a fixed distance, \( r \), the height decreases with time. Similarly, at the front where \( \eta \) is a constant, the front height decreases with time like \( t^{-1} \). From (2.21) and (2.26) it can be seen that the speed also increases with distance from the origin and decreases with time. The front speed decreases like \( t^{-1/2} \).

The front position predicted by the similarity solutions compare well with experimental results. The similarity solutions also make more realistic predictions for the height of the current than those made by the integral models. The agreement is qualitative rather than quantitative, since the shallow water equations do not accurately model the head of the current. The deeper head with a thinner layer behind the head is observed in both the experiments and the similarity solution.

More sophisticated shallow water models can be formulated. The model of Montgomery
& Moodie (1999) has two layers in order to model the ambient flow as well as the gravity current, and includes the effects of topography. Instead of a constant Froude number condition, a frictional forcing term is applied in the vicinity of the front. Comparison with a constant Froude number condition showed that this gave similar prediction for the development of the gravity current. These more complicated models require numerical integration to obtain a solution.

2.2.4 Field studies

In addition to laboratory experiments, experimental studies of large scale gravity currents have been carried out. The driving motivation behind the studies has often been the desire to understand and predict the spread of spillages of dense gases. The Thorney Island trials, as discussed by McQuaid (1985); McQuaid & Roebuck (1985), were one of the most extensive studies of this kind. The trials consisted of large releases of dense gas on an airfield. The release was of 2000m$^3$ of a mixture of nitrogen, Refrigerant 12 and smoke. The initial density of the mixture was about twice that of the ambient air. The smoke was used for visualisation. The spread and dilution of the cloud of dense gas was measured with an array of gas sensors and with video recordings and photos of the cloud. The trials were conducted in a variety of different atmospheric conditions and with different wind speeds. These data have proved valuable for validating dense gas dispersion models and for understanding the effects of the atmosphere on the spreading of a dense gas. The information is particularly valuable since more stringent environmental regulations mean that gases such as Refrigerant 12 can no longer be released. As part of the Thorney Island project wind tunnel experiments were also conducted to try and model the conditions in which the full scale releases occurred. Some of this work is described by Hall & Waters (1985) and Spicer & Havens (1985).

2.2.5 More detailed experiments on flow and mixing

The observations of Simpson & Britter (1979) on mixing were mainly qualitative. More recently experiments have been carried out by Hacker et al. (1996) and Hallworth et al. (1996) to make quantitative measurements of the mixing. The experiments of Hacker et al. (1996) utilised an optical technique, using the absorption of light by a dye to measure the dilution of the current. Hallworth et al. (1996) used a complementary technique where acid / alkali mixtures with pH indicator were used to measure the point at which the fluid diluted to a certain concentration. It was found that while the head of the current remains relatively undiluted in the constant velocity of the flow, there is significant mixing behind the head leading to a stratification of the gravity current tail. The Kelvin–Helmholtz waves on the head are seen to
break, detraining fluid from the head into the tail behind. During the slumping phase this fluid is replaced with dense fluid from the lower layer in the tail of the current. This leads to a recirculation of fluid through the head and a gradual dilution of the dense fluid as ambient fluid is mixed in. The integral models in §2.2.2 can be extended to include the effects of entrainment of ambient fluid into the head of the current. Such a model was proposed by Hallworth et al. (1996) and it was shown to provide a fair estimate of the entrainment rate in the current.

A more detailed study of the flow within the head of a gravity current in a flat channel has recently been carried out by Thomas & Dalziel (2000). This study used particle tracking techniques to measure the velocities in the head of the current. The results clearly illustrated the recirculation within the head and a reverse flow section near the bottom of the head. The speeds in the head are small compared to the speed of the front, of the order of 10% in the experiments of Thomas & Dalziel (2000). The low speed relative to the head partly explains the success of the analysis of Benjamin (1968) in modelling gravity currents as well as cavity flows. The flow speed behind the head can be significantly larger than the front speed. In laboratory experiments, Britter & Simpson (1978) observed the speed of the following flow to be 10-30% greater than the front speed. Most of the flow in from behind is mixed back through the interface rather than penetrating into the head. These detailed measurements of the flow help to explain the observed mixing patterns.

2.2.6 Direct numerical simulations

A more recent method of studying gravity currents has been to utilise direct numerical simulation (DNS). This involves numerically solving the full Navier–Stokes equations, including resolving the turbulence in the flow. DNS involves large amounts of computer power so the initial simulations were two-dimensional. Examples include Härtel et al. (1997, 1999, 2000b). These simulations suffered some of the same problems as other two-dimensional models in that they could not capture all the three-dimensional features of the flow. More recently it has become feasible to perform fully three dimensional DNS of gravity currents, at least at moderate Reynolds numbers. The work of Härtel et al. (2000b) has produced results which agree well with other experimental and theoretical work. The DNS simulations also capture the finer details of the flow including the Kelvin–Helmholtz billows and the flow within the head measured by Thomas & Dalziel (2000). The difference in speed between gravity currents with free-slip and no-slip boundary conditions was investigated, and the dependence of the Froude number on the Reynolds number was also measured. The findings agreed with the early experiments of Barr (1967). These DNS calculations are still extremely expensive in terms of computer time and memory size. Experiments and more theoretical work are still needed to provide a full explanation of gravity current phenomena and to validate the DNS
CHAPTER 2. BACKGROUND

2.3 Gravity currents on a slope

While much of the existing work has concentrated on the problem of a gravity current on a horizontal surface it is often the case that the effects of topography are important in practical applications. This section outlines the previous work which has looked at aspects of this problem.

2.3.1 Instantaneous releases on a slope

The motion of an instantaneous release of dense fluid in a sloping channel was studied by Beghin et al. (1981). By considering the gravity current as a “thermal” they developed an integral model for the motion of the head. Suppose that the head of the current has height $H$ and length $L$. Suppose also that the shape is self-similar so that the cross sectional area of the head can be written as $A = S_1HL$ and the perimeter as $S_2(HL)^{1/2}$, for some shape constants $S_1$ and $S_2$. The mass conservation equation reduces to

$$\frac{d}{dt}(S_1HL) = S_2(HL)^{1/2}\alpha U,$$  \hspace{1cm} (2.27)

where $\alpha$ is the entrainment coefficient, which may be a function of slope. The momentum equation becomes

$$\frac{d}{dt}(\alpha U) = B \sin \theta,$$  \hspace{1cm} (2.28)

where $k_v$ is an added mass coefficient and $B = S_1g'HL$ is the buoyancy of the thermal, which is a constant.

Integrating these equations gives $H$ and $L$ as being proportional to $x$, the position of the head, for a suitably defined origin. The current speed becomes

$$U^2 = U_0^2 \left( \frac{x_0}{x} \right)^4 + \frac{2}{3} C \frac{1}{x} \left( 1 - \left( \frac{x_0}{x} \right)^3 \right),$$  \hspace{1cm} (2.29)

where $x_0$ is the initial position, $U_0$ is the initial velocity and

$$C = \frac{4}{(1+k_v)} \frac{S_1}{S_2^2 \rho_a} B \sin \theta.$$  \hspace{1cm} (2.30)

Beghin et al. (1981) made comparisons with saline laboratory experiments to determine the entrainment rate, $\alpha$. It was found that $\alpha$ was an approximately linear function of $\theta$, varying
from 0.08 for a 5° slope to 0.59 for a 90° slope. A long-time solution for the speed of the gravity current can be found by taking the large \( x \) limit of (2.29). This gives

\[
U = K(B \sin \theta)^{1/2} x^{-1/2},
\]

where the scale term

\[
K \sin^{1/2} \theta = \frac{1}{S_2 \alpha} \left( \frac{8S_1}{3(1+k_v)} \right)^{1/2} \sin^{1/2} \theta
\]

was found to vary from a maximum of about 2.6 for slopes of 10°-20° to about 1.5 at 90°. The integral model gave fair agreement for the size and dilution of a gravity current, but for slopes of less than 45° the predicted speed was between 10% and 20% less than the speed measured in the experiments.

Dade et al. (1994) extended the model of Beghin et al. (1981) for particle-laden gravity currents where the density difference is generated by small suspended particles which slowly sediment out of the current, reducing the density difference.

Some experiments of an instantaneous release of dense gas on a slope were carried out by Müller & Fanneløp (1999). These experiments gave some insight into the problem but they were only for small slope of less than 15° and the slope was not long enough to allow the current to develop very far. There was also only limited visual and concentration data available from the experiments to provide a comparison with theory.

There is little or no field data available for dense gas releases on a slope. Most large-scale experiments have been interested in releases on horizontal surfaces (often the sea), and possible the effects of the wind. The site used for the Thorny Island field trials (see McQuaid & Roebuck, 1985) was essentially flat. The change in height over approximately 1km was no more than 2m, with a steepest gradient of about 1:200.

### 2.3.2 Shallow water solutions

As discussed in §2.2.3 the shallow water equations can provide a useful means of modelling gravity currents on a horizontal surface. Webber et al. (1993) used the shallow water equations to model the instantaneous release of a gravity current in a sloping channel. For the shallow water equations to be valid it is still necessary for the horizontal lengthscale of the flow to be much greater than the vertical lengthscale. In addition the angle, \( \theta \), of the slope must be small enough that \( \tan \theta << 1 \).

Equation (2.12) is unaltered, except that \( h \) now measures the vertical height of the current
above the slope. Equation (2.13) becomes

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} - g' \tan \theta = 0, \]  

(2.33)

where the extra term takes into account the added acceleration as a result of the slope.

By analogy with the flow on a horizontal surface, Webber et al. (1993) chose to close the problem by applying a constant Froude number condition at the front of the current. At the rear of the current the height of the flow was set to zero.

After some numerical simulations, they observed that there was a similarity solution to their problem, with a constant shaped wedge of dense fluid moving down the channel at a constant speed. The wedge is illustrated in figure 2.2

Conservation of volume gives the wedge length as \( l = (V/\tan \theta)^{1/2} \) and the wedge height as \( h = (V \tan \theta)^{1/2} \), where \( V \) is the volume of the wedge per unit width. Using the constant Froude number front condition and expressing \( h \) in terms of \( V \) and \( \theta \) gives the front speed of the wedge as

\[ u_f = Fr \left( 2g^2 V \tan \theta \right)^{1/4}. \]

(2.34)

Webber et al. (1993) also showed that a solution exists for an instantaneous release on a uniform, unconfined slope. In this case the shallow water equations become

\[ \frac{\partial h}{\partial t} + \nabla \cdot (uh) = 0 \]

(2.35)

and

\[ \frac{\partial u}{\partial t} + (u \nabla) u + g' \nabla (h + \tan \theta) = 0. \]

(2.36)

The constant Froude number is applied all around the front, with \( u_f = u_f(x, y, t) \) being the velocity normal to the front at each point. At the rear of the current the height is zero.

Equations (2.35) and (2.36) and the boundary conditions on \( u_f \) admit a solution with a
2.3. GRAVITY CURRENTS ON A SLOPE

wedge moving at uniform speed down the slope. The shape of the wedge is given by the parametric equation

\[
\begin{align*}
  x &= x_f + l(\cos^2 \omega - 1) \left\{ 1 - \frac{\pi}{2} \leq \omega \leq \frac{\pi}{2}, \right. \\
  y &= l(\omega + \cos \omega \sin \omega)
\end{align*}
\]

(2.37)

where \(x\) and \(y\) are the down-slope and across-slope coordinates and \(l\) is the length of the wedge. The distance downslope of the front-most point of the wedge is \(x_f\). The overall width and height of the wedge are given by \(b = \pi l\) and \(h = l \tan \theta\). The constant Froude number boundary condition gives \(u_f = Fr(g'l \cos^2 \omega \tan \theta)^{1/2}\).

Entrainment was added to this shallow water solution in an ad hoc manner by Tickle (1996). He assumed that the shape of the wedge was still given by (2.37), but that the current entrained ambient fluid at a rate proportional to the speed of the current. The entrainment leads to an increase in the size of the current and a decrease in the density. Applying the Froude number condition at the front gives the front speed as

\[
u_f(x,0) = u_{f0} \left( 1 + \frac{t}{T_0} \right)^{-1/2},
\]

(2.38)

where

\[
T_0 = \left( \frac{5}{2^8 \pi} \right)^{1/6} \left( \frac{\tan^{1/3} \theta}{\alpha Fr} \right) \left( \frac{V_0^{1/6}}{g'^{1/2}} \right),
\]

(2.39)

\(u_{f0}\) is the speed of the foremost point at \(t = 0\) and \(\alpha\) is the entrainment coefficient. The effective gravity of the current is

\[
g' = g'_0 \left( 1 + \frac{t}{T_0} \right)^{-3/2},
\]

(2.40)

where \(g'_0\) is the effective gravity at \(t = 0\). The wedge shape grows in size and slows down as it entrains fluid.

2.3.3 Continuous releases

The case of a continuous release of dense fluid in a sloping channel, forming a steady-state dense layer, was studied by Ellison & Turner (1959). They carried out laboratory experiments and found that the entrainment was a function of the layer Richardson number. They also presented a steady state inclined plume model for the flow. The volume flux in the dense layer changes as a result of entrainment of ambient fluid so

\[
\frac{d(uh)}{dx} = \alpha u,
\]

(2.41)
where $h$ is the layer thickness, $u$ is the mean layer speed, $x$ is the distance down the slope and $\alpha$ is the entrainment coefficient. Note that the notation has been changed from the paper of Ellison & Turner (1959) to make it consistent with that used in this thesis. The momentum equation can be written as

$$
\frac{d(hu^2)}{dx} = -C_Du^2 - \frac{1}{2} \frac{d(S_1 g' h^2 \cos \theta)}{dx} + S_2 g' h \sin \theta, \quad (2.42)
$$

where $S_1$ and $S_2$ are shape parameters. The shape parameters take into account the density profile of the layer. They can be calculated from an assumed profile or measured from experiments. The effective gravity is $g'$, the angle of the slope is given by $\theta$ and $C_D$ is a drag coefficient. Equations (2.41) and (2.42) can be rearranged and expressed in terms of the layer height, $h$, and the layer Richardson number, $R_i = g' h \cos \theta / u^2$, to give

$$
\frac{dh}{dx} = \frac{(2 - \frac{1}{2}) S_1 R_i \alpha - S_2 R_i \tan \theta + C_D}{1 - S_1 R_i} \quad (2.43)
$$

and

$$
\frac{dR_i}{dx} = \frac{(1 + \frac{1}{2}) S_1 R_i \alpha - S_2 R_i \tan \theta + C_D}{1 - S_1 R_i}. \quad (2.44)
$$

In their experiments Ellison & Turner (1959) found that the flow rapidly adjusted to a state where $R_i$ reached a constant value, $R_{i_n}$, the “normal” Richardson number. In that case

$$
\frac{dR_i}{dx} = 0 \quad (2.45)
$$

and

$$
\frac{dh}{dx} = \alpha(R_{i_n}). \quad (2.46)
$$

Britter & Linden (1980) considered the start of such a flow by looking at the head of a continuous release gravity current in a sloping channel. From experiments they observed that the non-dimensional front speed was nearly constant and did not depend strongly on the slope, with a value of about $1.5 \pm 0.2$. The head increases in size through direct entrainment of ambient fluid and by flow in from the dense layer behind. Which factor is more important depends on the angle of the slope.

The front velocity was related to the velocity in the following layer by supposing the energy loss along the lower boundary streamline from the stagnation point is equal to the change in potential energy. The mean velocity, $u$, in the following layer was specified using the analysis of Ellison & Turner (1959). It was assumed that the velocity on the stagnation streamline along the lower boundary is given by $\beta u$, for some constant $\beta$. This gives a front
2.3. Gravity Currents on a Slope

Figure 2.3: Diagram illustrating a continuous release on a slope and showing the dimensions.

The velocity

\[ u_f = \left( g_0^1 Q \right)^{1/3} S \left( \frac{\cos \theta}{\beta} + \frac{\beta \sin \theta}{2(\alpha + C_D)} \right) \left( \frac{\sin \theta}{\alpha + C_D} \right)^{-2/3}, \quad (2.47) \]

where \( g_0^1 \) is the effective gravity at the source and \( Q \) is the volume flow rate per unit width. This predicts that for small slopes (up to about 30°) the front velocity will increase with slope as the component of gravity downslope increases. However, for larger slopes the front speed decreased again as a result of increased entrainment into the head reducing the driving density difference. The agreement of their model with experimental results was fair, although the scatter in the data was large.

Tsihrintzis & Alavian (1996) have looked at continuous releases of dense fluid from a point mass and momentum source on a slope. They restricted their attention to a horizontal jet pointing in the downslope direction. Further, they did not consider the flow near the source. By balancing the gravity, buoyancy and inertial forces in the downslope and across slope direction, Tsihrintzis & Alavian (1996) split the flow into regimes. Typical scales for the plume length, width and height are \( L, b \) and \( h \) respectively. These scales are illustrated in figure 2.3. The density of the plume is \( \rho \), and \( \Delta \rho \) is the difference between the densities of the plume and the ambient. Estimates can be made for the magnitude of the forces acting on the jet in both the down slope and across slope directions. These estimates are given in table 2.1. Tsihrintzis & Alavian (1996) assume laminar flow and that bottom friction is always greater than interfacial friction. The buoyancy flux \( B_0 \sim g^1 h b L / t \) is a constant.

For mild slopes the friction force becomes important before the gravity force so there is only one regime of interest here, in which there is a balance between the buoyancy and the inertia forces. This gives

\[ L \sim B_0^{1/4} (\cos \theta)^{1/4} t^{3/4}, \quad (2.48) \]
<table>
<thead>
<tr>
<th></th>
<th>Downslope Across-slope (per unit length)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gravity</strong></td>
<td>$\Delta \rho ghL \sin \theta$</td>
</tr>
<tr>
<td><strong>Buoyancy</strong></td>
<td>$\Delta \rho gh^2 b \cos \theta$</td>
</tr>
<tr>
<td><strong>Inertia</strong></td>
<td>$\rho bhL^2 / t^2$</td>
</tr>
<tr>
<td><strong>Friction</strong></td>
<td>$\rho \nu b L / (ht)$</td>
</tr>
</tbody>
</table>

Table 2.1: Estimates for the forces acting on a three dimensional plume on a slope.

\[
b \sim B_0^{1/4} (\cos \theta)^{1/4} t^{3/4}, \quad (2.49)
\]

\[
g' \sim \frac{B_0^{1/2}}{h (\cos \theta)^{1/2}} t^{-1/2}, \quad (2.50)
\]

The effect of the slope is merely to reduce the effective gravity by a factor of $\cos \theta$, which is close to 1 for small angles. The scaling is the same as for a comparable release on the flat.

For steeper slopes, the initial balance is between buoyancy and inertia as for a mild slope. At some stage in the flow, before the frictional force is significant, the gravity force becomes more important than the buoyancy force. This leads to a new force balance and

\[
L \sim \frac{B_0^{2/5} (\sin \theta)^{3/5}}{h^{3/5} (\cos \theta)^{1/5} t^{4/5}}, \quad (2.51)
\]

\[
b \sim \frac{B_0^{1/5} \cos \theta)^{2/5} h^{1/5}}{(\sin \theta)^{1/5} t^{3/5}}, \quad (2.52)
\]

\[
g' \sim \frac{B_0^{2/5}}{h^{3/5} (\sin \theta)^{3/5} (\cos \theta)^{1/5} t^{-6/5}}. \quad (2.53)
\]

This leads to a different shape for the developing plume as the length, width and density scale differently with time.

These scalings do not uniquely determine the motion of the plume as the height, $h$, is not determined as a function of the time $t$. The product $g' h$ depends on the entrainment into the plume, which is not specified by this scaling. This in turn affects the transition time between the different regimes and whether the slope is classified as mild or steep i.e. whether viscous drag or gravity forces become important first.

Bonnecaze & Lister (1999) used the shallow water equations to model steady continuous releases of particle-laden fluid on a slope, leading to the formation of particle-driven gravity currents. The use of the shallow water equations is only valid provided that the thickness of the current varies slowly across the slope. The mean down-slope and cross-slope velocities are assumed to dominate, with very small vertical velocities. The rate of entrainment is assumed to be proportional to the speed of the flow and any turbulent frictional drag is taken to be
proportional to the speed squared. Neglecting the terms for particle-settling gives the model for the saline gravity currents considered here. In this case the shallow water equations are

$$\nabla .(uh) = \alpha |u|$$  \hspace{1cm} (2.54)

and

$$\nabla .(uuh) + \frac{1}{2} \nabla (g' \cos \theta h^2) - g' \sin \theta h e_x = -\frac{1}{2} C_D |u|u,$$  \hspace{1cm} (2.55)

where $e_x$ is a unit vector in the downslope direction. If entrainment is neglected then the primary balance in the momentum equation is found to be between the buoyancy and drag terms. This reduces the momentum equation down to

$$u = \left(\frac{2g' \sin \theta h}{C_D}\right)^{1/2},$$  \hspace{1cm} (2.56)

$$v = -\cot \theta \left(\frac{2g' \sin \theta h}{C_D}\right)^{1/2} \frac{\partial h}{\partial y}.$$  \hspace{1cm} (2.57)

Substituting these into the mass equation gives

$$\frac{\partial h^{3/2}}{\partial x} = \cot \theta \frac{\partial}{\partial y} \left( h^{3/2} \frac{\partial h}{\partial y} \right).$$  \hspace{1cm} (2.58)

The boundary conditions used are that the height, $h$, goes to zero at the edge $y_e(x)$ of the current. The global conservation of mass constraint gives

$$2 \int_0^{y_e(x)} uh \, dy = Q,$$  \hspace{1cm} (2.59)

where $Q$ is the source flow rate. There is a similarity solution for this problem with

$$y_e(x) = \left(\frac{16}{3\pi^{3/5}}\right)^{5/8} \left(\frac{C_D Q^2 \cos^3 \theta}{8g' \sin^4 \theta}\right)^{1/8} x^{3/8},$$  \hspace{1cm} (2.60)

and

$$h(x, y) = \left(\frac{16}{3\pi^{3/5}}\right)^{5/4} \left(\frac{C_D Q^2}{8g' \cos \theta}\right)^{1/4} x^{-1/4} \frac{3}{16} \left(1 - \frac{y^2}{y_e(x)^2}\right).$$  \hspace{1cm} (2.61)

Including the effects of entrainment makes a fundamental difference to the solution. The across-slope extent of the current is now infinite with

$$h(x, y) = \frac{3}{5} \alpha x$$  \hspace{1cm} (2.62)
and

\[
g'(x,y) = \frac{5}{3} \left( \frac{5g'C_D Q^2}{4\pi\alpha^4 \cos \theta} \right)^{1/3} x^{-5/3} e^{-\eta^2} \tag{2.63}
\]

where

\[
\eta = \left( \frac{5 \tan \theta}{3\alpha} \right)^{1/2} \frac{y}{x} \tag{2.64}
\]

Although the current has infinite extent, the density decays exponentially with across slope distance, effectively localising the current about the \( y = 0 \) axis. Taking the edge of the current to be where \( g' \) is a fixed percentage of the centreline value gives \( y_e(x) \propto x \).

Other work by Emms (1997, 1998) has lead to several streamtube models for gravity currents on a slope in the ocean. The effect of the Earth’s rotation is important in these models to set the scale of the motion. For smaller scale phenomena rotation does not play a significant part, so these streamtube models will not be discussed further.

### 2.4 Gravity currents in a moving ambient

Different studies have looked at the effect of the wind on a gravity current. Experimental work by Simpson & Britter (1980) on saline currents in a water flume with a moving floor showed that an opposing wind slowed the current, while a following wind sped up the current. The change in speed was about 3/5 of the imposed wind speed. A semi-empirical theory based on Bernoulli’s equation was presented which gave fair agreement with the experiments and with some atmospheric measurements of meso-scale fronts (such as thunderstorm outflows and sea breezes). The shape of the gravity current head was observed to be affected by the ambient flow. With a head wind the gravity current became lower with a more pointed nose. For a tail wind the head of the current was much higher and more blunt.

A more detailed study of the effects of a shear flow on a gravity current has been carried out, based on the analysis of Benjamin (1968). Xu (1992) considered the case of a uniform shear in a channel while the later work of Xu et al. (1997); Xue (2000) included the effects of two layers in the ambient, with different shears in each layer.

For a uniform shear, as used by Xu (1992), the vorticity in the ambient is constant far upstream of the gravity current. Since vorticity is conserved along streamlines, the vorticity (and shear) far downstream of the current is the same as upstream of the current. Figure 2.4 illustrates the problem. The frame of reference is one in which the head is at rest. This is the same problem described in §2.2.1, apart from the shear flow in the ambient. The simplest case to consider would be a uniform flow in the ambient, but this is not particularly interesting. Since the model of Benjamin (1968) is inviscid, a uniform flow just acts to translate the
solution at a speed given by the ambient flow. The case of a shear flow is the next simplest flow, but the presence of the shear introduces some new dynamics. The analysis in Xu (1992) and described below is for two immiscible fluids rather than the analysis of Benjamin (1968) for air and water. It also assumes no flow in the head. Assuming the wind speed at the ground is zero, a positive shear, $\omega$, corresponds to a tail wind and a negative shear corresponds to a head wind.

The Bernoulli equation can be applied along the streamline OA to give

$$p_A = p_0 - \frac{1}{2}\rho c_1^2. \tag{2.65}$$

The pressure at O and C is the same by applying Bernoulli along the streamline OC. Far upstream and downstream from the head of the gravity current the pressure is assumed hydrostatic so applying Bernoulli on the streamline OB gives

$$c_2^2 = 2g(d - h). \tag{2.66}$$

Equating the two flow forces at A and B gives

$$\frac{1}{2}c_1^2d - c_2^2h + \omega(c_1d^2 - c_2h^2) + \frac{1}{2}g'(d^2 - h^2) + \frac{\omega^3}{3}(d^3 - h^3) = 0. \tag{2.67}$$

Finally continuity gives

$$c_1 = c_2\frac{h}{d} + \frac{1}{2}\omega d\left(1 - \frac{h^2}{d^2}\right). \tag{2.68}$$

Combining these and writing $\Omega = \omega \sqrt{d/g}$ for the non-dimensional vorticity and $H = h/d$
for the fractional depth gives

\[
\Omega^2 \left( \frac{1}{8} (1 - H^2)^2 + \frac{1}{3} (1 - H^3) - \frac{1}{2} (1 - H^2) \right) - \frac{\Omega}{\sqrt{2}} H (1 - H)^2 + (1 - H)^2 \left( \frac{1}{2} - H \right) = 0. \tag{2.69}
\]

In the case \(\Omega = 0\), where there is no shear, (2.69) reduces to a quadratic equation in \(H\), with the physically realisable solution \(H = \frac{1}{2}\). For non-zero shear the equation is more complicated, requiring the roots to be found numerically. For \(\Omega << 0\) there are two roots in the interval \((0,1)\) as well as the root at \(H = 1\). The one plotted in figure 2.5 is that corresponding to \(H = 0.5\) for \(\Omega = 0\) rather than the one splitting from the root at \(H = 1\). It might be expected that the solution would vary smoothly as the wind speed was increased. The solution with the smaller height, \(H\), is supercritical while the larger one is subcritical. In practice a hydraulic jump may occur taking the flow from the lower to the higher solution. In this case there will be dissipation though.

It can be seen from figure 2.5 that the speed of the current predicted by the model of Xu (1992) exhibits the same qualitative behaviour as was observed in the experiments of Simpson & Britter (1980). The current travels faster with a tail wind and is slowed by a head wind. The curve marked “Experimental best fit” is a straight line corresponding to \(3/5\) of the mean applied wind speed. The change in speed is approximately \(3/5\) of the applied wind speed over a range of values of the shear, with the comparison being particularly good for a tail wind. The experiments carried out by Simpson & Britter (1980) were for \(-0.4 < \Omega < 0.4\) and the factor of \(3/5\) was a best fit to the experimental data in that range. The increase in height with the tail wind and the flattening of the current with a head wind are also observed in both the theory and experiments.

Rottman et al. (1985), as part of the analysis of the Thorney Island field trials (see McQuaid, 1985; McQuaid & Roebuck, 1985), compared the field trial results with the earlier work of Simpson & Britter (1979, 1980). They found that, for the initially axisymmetric atmospheric releases, assuming the change in the velocity was \(0.62\) of the mean applied wind speed provided a reasonable fit to the data, much better than the simplistic model that the effect of the wind is just to advect the gravity current at the same speed as the wind. In addition, they presented an extension of the analysis of von Kármán (1940) to predict the head shape of a gravity current in a head wind or a tail wind. This gives the head steepening near the front for a tail wind and becoming flatter for a head wind, as would be expected from the experimental observations.
2.5 Viscous effects on a gravity current

All the work reviewed so far has concentrated on the regime where the Reynolds number is large enough that viscous forces can be neglected. However in some circumstances viscous effects can be important. This may be because the fluid is very viscous (for example lava), because the motion is very slow or because the scale of the problem is small. Often in laboratory experiments the flow may initially be at a high Reynolds number and turbulent, but viscous forces play a larger role as the flow slows down. A full explanation of the observations will have to account for this.

The instantaneous and continuous release of a viscous gravity current on a horizontal surface has been studied by various workers including Didden & Maxworthy (1982) and Huppert (1982). Except possibly at the time of release, the horizontal extent of the current is much larger than the vertical extent allowing the standard lubrication theory equations to be applied. Under this assumption the pressure in the fluid is hydrostatic. The dominant balance in the momentum equation is between the pressure gradient and viscous forces so

\[ g \frac{\partial h}{\partial x} = \nu \frac{\partial^2 u}{\partial z^2}. \]  

(2.70)
CHAPTER 2. BACKGROUND

The no-slip boundary condition at the bottom of the current gives \( u = 0 \) on \( z = 0 \), and the zero stress condition at the top gives \( \nu \partial u / \partial z = 0 \) on \( z = h \). This leads to a solution

\[
    u = -\frac{1}{2} \frac{g'}{\nu} \frac{\partial h}{\partial x} (2h - z). \tag{2.71}
\]

The depth integrated continuity equation is

\[
    \frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left( \int_0^h u \, dz \right) = 0. \tag{2.72}
\]

Combining this with (2.71) gives

\[
    \frac{\partial h}{\partial t} - \frac{1}{3} \frac{g'}{\nu} \frac{\partial}{\partial x} \left( h^3 \frac{\partial h}{\partial x} \right) = 0. \tag{2.73}
\]

This differential equation has a similarity solution of the form

\[
    h(x, t) = \eta_N^{2/3} \left( 3q^2 \nu / g' \right)^{1/5} t^{(3\zeta - 1)/5} \phi(\eta/\eta_N), \tag{2.74}
\]

where \( \eta = \left( g' q^3 / (3\nu) \right)^{-1/5} t^{(3\zeta - 1)/5} \) is the similarity variable and \( \eta_N \) is the value of \( \eta \) at the nose of the current. The volume of the current is \( q \zeta \), so \( \zeta = 0 \) for an instantaneous release and \( \zeta = 1 \) for a continuous release with a constant rate. If \( \zeta = 0 \) then an analytic solution exists for \( \phi \) with

\[
    \phi(y) = \left( \frac{3}{10} \right)^{1/3} (1 - y^2)^{1/3} \tag{2.75}
\]

and

\[
    \eta_N = \left[ \frac{1}{5} \left( \frac{3}{10} \right)^{1/3} \pi^{1/2} \Gamma(1/3) / \Gamma(5/6) \right]^{-3/5}. \tag{2.76}
\]

The theoretical predictions agree well with laboratory experiments using silicone oil (Huppert, 1982) and salt water (Britter, 1979).

The flow of a continuous release on a slope, where viscous forces play a dominant role has been studied by various workers. The flow can be modelled using the standard equations of lubrication theory. A similarity solution for the steady problem was first presented by Smith (1973), along with some experimental validation. Later work by Lister (1992) extended the model to describe point and line sources with flow rates proportional to some power of time. Numerical solutions for the start up and the near field were given, along with further experimental results.

For a point source with flow rate \( Q \), the similarity solution for the edge of the region of
2.6 DENSE PLUMES AND JETS

viscous fluid is
\[ y = \left( \frac{12005 Q \nu \cos^3 \theta}{36 g \sin \theta} \right)^{1/7} x^{3/7}, \]  
(2.77)
as described by Smith (1973). This solution complements the high Reynolds number solution of Bonnecaze & Lister (1999) described in §2.3.3.

2.6 Dense plumes and jets

When studying continuous releases of dense fluid on a slope there are two extremes to consider. The first is the limit as the slope tends to zero. This is the gravity current limit described above. The second limit is as the slope becomes vertical. In this case the fluid will behave like a plume or jet against the wall. It is therefore relevant to this thesis to review some of the literature on plumes and jets.

The large body of literature on buoyant and dense plumes and jets all, more or less, draws on the work of Morton et al. (1956). In their famous paper, Morton et al. (1956) described how a buoyant plume from a point source can be modelled by writing down equations for the horizontally integrated buoyancy, volume and momentum fluxes in the plume. Essential to the model is the entrainment assumption, that the rate at which fluid is entrained in at a given height is proportional to the speed and perimeter of the plume at that height. The constant of proportionality, \( \alpha \), is known as the entrainment coefficient. The volume flux, \( Q = \pi b^2 U \) changes as a result of this entrainment so
\[ \frac{dQ}{dx} = 2\pi \alpha b U, \]  
(2.78)
where \( b \) is the radius of the plume and \( U \) is the average speed in the plume. The momentum flux, \( M = \pi b^2 U^2 \), is governed by the momentum equation
\[ \frac{dM}{dx} = \pi b^2 g', \]  
(2.79)
and the buoyancy \( F = \pi b^2 g' U \) is conserved. These fluxes are written in terms of \( b, U \) and \( g' \) assuming a top-hat profile for the density and velocity inside the plume. For a Gaussian profile, or any other profile, the equations are essentially the same, but with suitable shape factors.

This model predicts the shape and density of the plume as a function of height and has proved remarkably successful when compared with both laboratory experiments and field observations.

This type of model can be applied to a series of other related problems. List (1982) in a
review article gives a good overview of some of the problems involving jets and plumes.

The forced plume, where there is initially an excess of momentum compared to the pure plume described by Morton et al. (1956), is studied by Morton (1959). For a dense forced plume, with the initial momentum directed upwards, the buoyancy will act against the flow. The maximum rise height for a plume with initial buoyancy and momentum fluxes \( F_0 \) and \( M_0 \), and with no initial volume flux is

\[
z_{\text{max}} = 2^{-1/2} s_{1/2} \alpha^{-1/2} \pi^{-1/4} \frac{M_0^{3/4}}{F_0^{1/2}} \int_0^1 \frac{s^3}{(1-s^5)^{1/2}} \, ds.
\]

(2.80)

The integral in (2.80) can be written in terms of \( \beta \)-functions and evaluates to 0.460. The plume model provides a description of the upward part of the flow and predicts at what height the flow will reverse, but it does not predict the motion in the downwards part of the flow, nor does it take into account the interaction between the upward and downward flows.

The problem of a dense jet, initially fired vertically upwards, was studied experimentally by Turner (1966) using salt water jets. Dimensional arguments give the maximum rise height as being proportional to \( M_0^{3/4} F_0^{-1/2} \), the so-called “jet length”. From the experiments, the constant of proportionality is found to be 1.85. In comparison, assuming an entrainment coefficient of 0.1, (2.80) gives the constant of proportionality as 1.73 on theoretical grounds. In the experiments the sources have a finite (but small) volume flux which is not taken into account when comparing with the predictions of Morton (1959).

The case of a forced plume tilted at an angle was studied by Lane-Serff et al. (1993), again using an extension of the plume model. The momentum equation now needs both a vertical and a horizontal component. The appropriate spatial coordinate to use is the distance along the plume centreline, \( s \), rather than the vertical height, \( z \). This gives momentum equations

\[
\frac{d(b^2 u^2 \sin \theta)}{ds} = g' b^2
\]

and

\[
\frac{d(b^2 u^2 \cos \theta)}{ds} = 0,
\]

(2.81)

(2.82)

where \( \theta \) is the angle the tangent to the plume centreline makes with the horizontal. The equations can be integrated numerically to find the trajectory and dilution of the plume.

The predictions provided fair agreement with laboratory experiments, although some additional features, such as detrainment from the lower edge of the plume, were observed in the experiments, but not included in the model. The model also fails to take into account the curvature of the plume and any interaction between the upwards and downwards parts of the
flow. This is a particular problem for angles close to the vertical.

2.7 Summary

In this chapter some of the existing literature on gravity currents has been reviewed. It has been shown that simply integral models and shallow water models can provide an insight into the bulk features of these flows. Such models often give surprisingly good agreement with experimental measurements of the spread of the current. In the following chapters such simple model will be developed and their predictions compared with experiments and with the results of more complicated numerical simulations.

Simple models, such as these, do not provide an understanding of the detailed internal process in a gravity current. Experiments still provide the best method of obtaining this kind of information. Much of the work described in this thesis will be the results of detailed laboratory experiments into gravity currents on a slope.

Many of the experimental and theoretical results discussed in this chapter will also be useful in validating the vorticity–streamfunction model for gravity currents which is developed in chapter 6.
Chapter 3

Experimental techniques

3.1 Use of experiments to model atmospheric flows

There is a long tradition of using laboratory-scale experiments to model flows of a larger scale, such as those which occur in the atmosphere. There are several reasons for this. Firstly, laboratory experiments are carried out in a more controlled environment than larger scale field experiments. This means that more data of a higher quality can be collected. Secondly, the effect of each aspect of the flow can be carefully studied for a fuller understanding. Third, the experiments can easily be repeated, either with the same parameters or for a range of parameters. Finally, field trials are often extremely expensive or may not even be possible due to environmental considerations.

Many of the experiments into buoyancy driven flows, including those described here, are performed using salt (NaCl) dissolved in water to generate the buoyancy difference. Experiments using water are much easier to contain and control than experiments with gas. This makes them safer and easier to conduct. Salt and water are both cheap, readily available and harmless, making them ideal for laboratory use. There are a larger range of visualisation and diagnostics techniques available for saline flows than for gas flows. Some of these will be described in the following sections.

Differences can arise because of the differing scales between laboratory and field experiments. In order to maintain dynamic similarity between experiments it is desirable to maintain the relative importance of the terms in the momentum equation. Possibly the most important dimensionless group to keep constant is the Reynolds number,

\[ \text{Re} = \frac{UL}{v}, \]  

(3.1)

which measures the relative importance of advective and viscous effects. Here \( U \) and \( L \) are
CHAPTER 3. EXPERIMENTAL TECHNIQUES

Typical velocity and length scales of the flow and \( \nu \) is the kinematic viscosity. In the laboratory the length scale tends to be smaller, reducing the Reynolds number. This problem is to some extent eased by the use of water/salt experiments instead of gas experiments. The kinematic viscosity of water is \( 1.004 \times 10^{-2} \text{ cm}^2 \text{ s}^{-1} \) at 20°C, compared to 0.15 cm² s⁻¹ for air, so larger Reynolds numbers are obtained for a given size experiment.

It is also necessary to consider the differences in diffusion between saline solutions and gas. The diffusivity of salt in water is \( 1.1 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1} \), while the diffusivity of a gas in air is of the order of 0.2 cm² s⁻¹. The relevant dimensionless group in this case is the Peclet number. This is the ratio of the advective term to the diffusive term in the momentum equation so

\[
\text{Pe} = \frac{UL}{\kappa},
\]

where \( U \) and \( L \) are typical velocity and length scales for the flow and \( \kappa \) is the diffusivity of the salt or gas. Choosing the conditions of an experiment to set the initial Reynolds number of the flow fixes the Peclet number as well because the product \( UL \) is already determined. The ratio of kinematic viscosity to diffusivity is about \( 10^3 \) for salt and water compared to about 1 for air so it is not possible to keep both the Reynolds numbers and Peclet numbers the same in the laboratory experiments as compared to the atmospheric flows. In the laboratory the Peclet number will be bigger than the Reynolds number by a factor of \( 10^3 \). Provided the kinematic viscosity of the flow is much larger than the diffusivity, viscosity will be the dominant force and the mismatch between the diffusivities can be neglected.

Saline solutions are ideal for modelling Boussinesq flows where the density difference between the ambient and dense fluids is small. Salt cannot be used to create large density differences, however, as the saturation concentration at 20°C is 36.0 g per 1 kg water, giving a density of 1.2 g cm⁻³ compared to the density of 1.0 g cm⁻³ for pure water. Many problems in dense gas dispersion may involve large density ratios between the dense gas and the air, so at least initially the flow is not Boussinesq. The dilution of the current by entrainment will mean that the flow will become increasingly Boussinesq. Making the Boussinesq approximation does lead to a significant simplification in the problem and provides a useful insight into the likely solutions. The work of Gröbelbauer et al. (1993) on gravity currents in a horizontal channel with high density ratios showed that the constant Froude number front condition was still valid, although the value taken for the Froude number may depend on the density ratio between the fluids. The model of Benjamin (1968) for a cavity flow is an extreme case of a non-Boussinesq flow where the density ratio between the fluids is infinite. This model, described in §2.2.1, has been shown to provide good agreement with saline laboratory experiments, suggesting that non-Boussinesq effects may not make a large difference to a gravity
current. An understanding of Boussinesq flows is an important step towards understanding any non-Boussinesq effects.

One further difference between water and air is that water is incompressible, whereas air is not. Provided the Mach number (the speed of the air flow divided by the speed of sound) is much less than one then the compressibility of the air can be neglected. For problems in dense gas dispersion, and for many other atmospheric problems, this is the case.

Experimental work forms a large part of this thesis. The apparatus in which the experiments were carried out is described in §3.2. The use of a refractometer to measure the initial density of the release is described in §3.3. The main techniques used to gather information on the flows were based on optical and conductivity methods. These are described in §§3.4–3.7. These techniques are generally applicable to all of the experiments carried in this thesis. More specific features of individual experiments will be described in the chapters where they are used.

### 3.2 Apparatus

The experimental work in this thesis is concerned with the release of a dense fluid on a slope. The tank in which the majority of the experiments were carried out is of length 2.5 m, width 2.0 m and depth 0.85 m. The tank has opaque sides and base, and is fitted with a false bottom which can be adjusted via a winch to give a variety of slopes from 0° to about 20°. Figure 3.1 illustrates the experimental setup. The tank was filled with fresh water before each set of experiments. Both instantaneous and continuous releases of dense fluid were created. The mechanisms used to create the release are described in detail in chapters 4 and 5 respectively. The density difference was created by dissolving salt into fresh water. For most of the experiments red food colouring was added to the dense fluid to visualise the flow.

### 3.3 Measurement of salinity

In all the experiments described here buoyancy is the primary force driving the flow. To control such experiments and interpret the data it is essential to know the density of the fluids being used. The weight of salt needed to make saline solutions of a given density can be looked up in standard reference tables. Solutions can be made to the required density by carefully measuring the weight of salt and volume of water used. It is, however, desirable to be able to verify the density of a solution. This can be done using a variety of techniques.

One of the simplest techniques is to measure the refractive index of the solution, as this depends on the concentration of salt. The refractive index can be measured using a refrac-
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Figure 3.1: Illustration of the experimental setup used.

tometer. Refractometers come in both electronic and optical forms. The refractive index can be calculated by measuring the total internal reflection between the fluid and a solid prism. The optical refractometers have a scale from which the refractive index can be read. Tables of density against refractive index for saline solutions are commonly available. The electronic refractometers, being designed for medical use, give a reading in Brix which is related to the percentage of sugar in the solution. Similar tables of values can be used to read off the salinity of a solution for a given value in Brix. The refractive index also depends on the temperature of the solution so care must be taken to keep the samples at a fairly constant temperature. In practice this means allowing both the tank of fresh water and any saline solutions to adjust to room temperature before running any experiments. The water temperature was 20°C ± 0.5°C for the majority of the experiments described here. This is one of the reference temperatures at which the tables are calculated.

An optical refractometer was used to check the initial conditions for the experiments carried out in this thesis. The error in measuring the effective gravity of the flow, $g'$, (defined in chapter 2) is less than 0.5 cm s$^{-2}$. A low initial value for $g'$ in the experiments described here would be 10 cm s$^{-2}$, giving an error of less than 5% in the measurement of $g'$.

The salinity of the fluid in a flow can be measured in-situ using a conductivity probe. This is described further in §3.6.
3.4 Optical techniques

Optical techniques provide the primary source of information about the flow. The techniques are all based on the use of video equipment to record the experiment. A monochrome CCD camera (Cohu 4912) was used to capture the experiments. The output of the camera was connected to the luminescence component of a S-VHS signal fed into a Panasonic AG-7350 video recorder. The majority of the experiments were recorded onto S-VHS video tape to allow later replay and analysis. The camera and video were also connected to a PC fitted with a frame-grabber card (Data Translation DT2862) and the DigImage image processing software. (Dalziel, 1992). This allowed video images to be captured directly from the camera during an experiment, or afterwards from the video tape. Due to the low ceiling height in the laboratory (2.7 m), it was necessary to use a mirror mounted on the ceiling in order to view the whole tank. The mirror was angled and the camera was mounted on a tripod placed on the floor next to the tank. The setup is shown in figure 3.1. This arrangement allowed the camera to view all of the tank at once.

The DigImage software allows a “world” coordinate system to be set up to relate the pixel coordinates of the image to the actual physical coordinates in the tank. A grid of known size was placed on the bottom of the tank before each experiment and used to help define the world coordinates. A quadratic mapping was used to convert between image and world coordinates. The coefficients in the mapping were determined by fitting the mapping to a series of known reference points. As well as scaling distances, this mapping corrected for distortion of the image due to the mirror and the slope. For each image subsequently analysed, the position of any particular feature or edge of the flow could be measured in the world coordinates. This allowed the precise measurement of distances from the video recordings. The spatial accuracy of the digitised images is ±0.5 cm.

3.5 Dye calibration methods

Coloured dye is often used to visualise experiments. As light passes through the dye, wavelengths of the complementary colour are absorbed preferentially. The change in colour and net attenuation can be used to make quantitative as well as qualitative measurements. The intensity of the video image gives a measure of how much light has been absorbed, and hence how much dye is in various parts of the flow. Assuming that the diffusivity of the salt and dye are equal, the concentration of dye is proportional to the concentration of salt. As salt is a smaller molecule than the molecules in the food colouring, salt diffuses faster, leading to a smaller Peclet number. Provided the Peclet number based on the diffusivity of salt is much...
greater than one, the diffusion of both salt and dye are negligible over the timescales of the experiments carried out in this thesis.

The classical absorption theory of Lambert–Beer for a monochromatic light source and neglecting scatter gives

\[
\frac{d\hat{I}}{ds} = -f(c)\hat{I},
\]

(3.3)

where \(\hat{I}(s)\) is the light intensity at a point of distance \(s\) along the ray and the dye concentration is given by \(c(s)\). For low dye concentrations, \(f\) can be approximated by a linear function \(f = Ac + b\) where \(A\) represents the absorption as a result of the dye and \(b\) is the absorption due to the water in the absence of any dye. Since (3.3) is linear, we can separate the effect of the dye and the water. The measured light intensity is then

\[
\frac{\hat{I}(h)}{\hat{I}_0(h)} = e^{-Af_0(h)c(s)}ds,
\]

(3.4)

where \(\hat{I}_0\) is the intensity at the point \(h\) in the absence of dye. Experimentally \(\hat{I}_0\) is easier to measure than the intensity at \(s = 0\) with dye. We will write \(\hat{I}(h) = I\) and \(\hat{I}_0(h) = I_0\) so

\[
\frac{I}{I_0} = e^{-Af_0(h)c(s)}ds.
\]

(3.5)

The intensities given by the camera and frame-grabber are integers in the range of 0–255. These need to be calibrated to the actual light intensity. The response of the Cohu camera used is very nearly linear. Let \(I_{dig}\) be the intensity returned by the DigImage software so

\[
I = a(I_{dig} - I_{black})
\]

(3.6)

for some constants \(a\) and \(I_{black}\), where \(I_{black}\) is the intensity measured by DigImage for a completely black source. This is independent of the aperture used on the camera. One way of finding this value is to take two images with different apertures \(A_1\) and \(A_2\), and draw a scatter plot of \(I_{dig}(A_1)\) against \(I_{dig}(A_2)\), as shown in figure 3.2. The plot is linear showing that the response of the camera is indeed linear. A linear least squares fit to the scatter plot data can be calculated and the point where \(I_{dig}(A_1) = I_{dig}(A_2)\) will give \(I_{black}\). From the equation of the best fit line it can be calculated that \(I_{black} = 18\) for the camera used.

Substituting (3.6) in (3.5) gives

\[
\frac{I_{dig} - I_{black}}{I_{0 dig} - I_{black}} = e^{-Af_0(h)c(s)}ds.
\]

(3.7)

By measuring the intensity for a set of samples of known concentrations and depths, the
3.5. DYE CALIBRATION METHODS

![Intensity scatter plot](image)

Figure 3.2: Intensity scatter plot. The x-axis is the intensity, $I_{dig}(A_1)$, of points with aperture $A_1$ and the y-axis is the mean of the intensities, $I_{dig}(A_2)$, at corresponding points with aperture $A_2$. The line is a least squares best fit to the data, weighted to take into account the frequency of each intensity.

absorption constant $A$ can be calculated. The measured intensities can then be used to calculate $\int_0^h c(s) \, ds$. For vertical rays this integral is the depth integrated concentration of dye. Larger values of the integral can be due to an increase in the concentration of dye or an increase in the depth of the layer.

This technique works very well for flows where there is no variation in depth, where an approximately uniform light source can be placed behind the experiment and where there are no free surface disturbances (see e.g. Cenedese & Dalziel, 1998). For the experiments described here it was impossible to place the light source underneath the slope. Instead, the tank had to be illuminated from above, as illustrated in figure 3.3. This meant that the light passed through the fluid twice, effectively doubling the path length. The path of the light rays entering and leaving the water will not be the same because the light is reflecting off the slope and the slope is not horizontal. This leads to some smearing of the results. Provided the slope is not too large and the flow is varying slowly in space relative to the depth, the error is not too large. Light will also be scattered, as well as reflected, from the bottom of the tank again leading to smearing of the results.

A further problem was the presence of the free surface. Light rays do not enter and leave
the water perpendicular to the free surface so they will be refracted to some extent. A static refraction is corrected for by the use of the background intensity. However, there may be waves on the water surface, which lead to varying angles of incidence of the light ray on the surface, and hence variations in the amount of refraction. The primary source of waves in the instantaneous release experiments described in chapter 4 was the removal through the free surface of the cylinder containing the dense fluid. The effect of this can be assessed by conducting an experiment without any dense fluid or dye and studying the effect of the waves on the background intensity, $I_0$. Such an experiment was carried out. It was observed that as the cylinder was lifted through the surface a large number of waves radiated out from the source. These died away after a few seconds. Water continued to drip from the cylinder, producing more waves throughout the experiment, although these were smaller than those created initially. The maximum change in the light intensity level observed as a result of the surface waves was approximately 10%.

Despite these various short-comings, this technique has proved a good method for providing qualitative, and in some instances semi-quantitative, information about the whole flow. The method is particularly good where the flow is steady so the images can be time averaged to reduce the errors caused by turbulent fluctuations and surface waves.
3.6 Velocity measurement by tracking a dye front

When studying a fluid flow one of the most important variables to be measured is the fluid velocity. Methods for measuring the velocity are often based on the inclusion of a passive marker in the flow which can be followed visually. The method which was utilised here involved the use of food dye as a tracer.

For the instantaneous experiments described later in chapter 4, the dye was added at the start. As the dense fluid moved, the front of the dense region could be followed. For the experiments with dense jets in chapter 5, the velocity was measured by injecting a small amount of dye mixed with salt into the jet. This acted as a tracer which could be followed as it moved up the jet, allowing the peak velocity in the jet to be measured. This method proved simple, yet effective.

For the experiments with a continuous release of dense fluid the dye was being used not to track the density front, but to measure the speed of the flow. As the flow is steady, the pathlines marked by the dye correspond to streamlines of the flow, meaning the measured speed is the instantaneous speed of the flow. In order to minimise the effects on the flow, the densities of the dye and the jet were matched by mixing the dye with a small amount of salty water. The dye was injected into the feeder tube some distance away from the source. This allowed time for the dye to mix with the salty solution before reaching the source. It also allowed time for any disturbance caused by the injection of the additional fluid to have died down. However, if the dye was injected too far from the nozzle it would diffuse in the tube, enhanced by shear diffusion. This led to a fuzzy, rather than a sharp, interface between the dyed and undyed region, making it difficult to accurately decide where the front was. The position at which the dye was injected was chosen to provide the best balance between these two problems.

To improve the accuracy of the results and reduce the problems caused by the diffusion of the dye, the process for measurement of the front was automated using a DigImage command file. Each image was corrected for the background intensity to ensure that changes in intensity were due to changes in the dye concentration, and not to spatial changes in the illumination. A given intensity contour, chosen as the threshold for the dyed region, was traced on each image. The level was chosen so that it was near the background zero intensity, but above the level of the noise in the background level. The threshold intensity was about 1/20th of the peak intensity in the dyed region and provided a good indication of the edge of the dyed region. A minimum contour length was set for the contours to filter out small patches of high intensity caused by reflections, waves or noise. The maximum upslope position of the contour was then taken as the position of the front. This provided a much simpler, more efficient and more reproducible method of creating a distance–time graph than making measurements by
The spatial resolution of the dye front measurements was limited by the camera and framegrabber card to ±0.5 cm. Due to the diffuse nature of the dye near the front, the actual accuracy of the measurements was slightly less than that. The speed was calculated by fitting a power law curve to the front position data and differentiating the resulting expression to obtain the speed. This helped to smooth out any errors in the measurement of the front position. The time of the images was measured to within 0.04s, which is the spacing between frames of the video. The accuracy to which the speed was calculated was therefore within ±0.5 cm s\(^{-1}\). The technique works well over this range. For very large speeds, a small time interval is needed to ensure an accurate spatial resolution. The minimum time between images is fixed to 0.04s by the camera and framegrabber card. With low speeds a long time interval is needed between images to allow the dye front to move a measurable distance. For very low speeds, the rate at which the dye diffuses can become significant, leading to inaccurate measurements of the fluid speed. This is a fundamental problem with the use of a diffusive marker to measure fluid velocity. For the experiments described here the speeds were sufficiently high that diffusion was not a significant problem.

One of the most common ways of measuring velocities in a fluid is to use a technique where neutrally buoyant particles are placed in the flow and their progress followed using a variety of particle tracking and particle image velocimetry (PIV) techniques. This is difficult in this particular experiment for several reasons. Available techniques can only measure velocities in two dimensions and for best results they require the plane in which the velocities are measured to be lit by a light sheet. This is difficult to achieve on the slope. If the dense fluid is entraining lighter ambient fluid then the density of the flow will decrease with time. Particles that were initially buoyant will become dense than the fluid and begin to settle. With suitable choice of particles this difficulty can be minimised as the settling time for the particles is long compared to the duration of the experiments. As a result of these difficulties it was decided to use the dye tracking method instead.

### 3.7 Conductivity probe measurements

Another experimental technique used here is the measurement of the conductivity of the water using a conductivity probe. The conductivity of water varies, depending on the salinity and temperature of the water, allowing the probe to be used to calculate the density of the flow. The probe used here is designed to measure the conductivity at a point.
3.7. **CONDUCTIVITY PROBE MEASUREMENTS**

This method provides valuable additional measurements to the dye calibration technique described in §3.5. The conductivity probe does not require the careful background lighting needed for the dye measurements and is not affected by disturbances to the free surface. The technique can provide measurements of the density at a point instead of depth-averaged values. By traversing the probe through the flow it can also provide a measure of the depth of the dense fluid.

Figure 3.4 shows the construction of the type of aspirating probe used here. The probe consists of two concentric metal tubes separated by an insulating layer. The tip of the probe is insulated, with a small hole to the inner tube. The electrical field lines in the fluid are concentrated through the small hole so the major contribution to the conductivity comes from the fluid in the tip. The probe therefore provides a measure of the conductivity of the fluid close to the bottom of the probe. Fluid is siphoned through the probe so there is a more rapidly response to changes in the conductivity near the tip. The tube diameter is sufficiently small that the flow rate through the siphon is very low, to cause minimal disruption to the experimental flow. Flow rates of the order of $5 \text{ cm}^3 \text{ min}^{-1}$ are usual. This gives a very high fluid velocity though ($\approx 5 \text{ cm s}^{-1}$) because the tip is very small. The high velocity allows a fast response time for the probe.

In order to accurately measure the conductivity, the probe is used as one side of a standard bridge circuit. An AC voltage is applied across the bridge and the resulting AC signal is rectified and passed through a low pass filter before being measured by means of a analogue-to-digital (A/D) converter (Siliconsoft Dacqpod 12A) connected to the serial port of a PC. This allowed the taking of samples to be automated. In practice, samples rates of 10Hz were used, although the system is capable of working at a much higher rate. An AC rather than a DC voltage is used in order to prevent the build up of electrolytic deposits on the probe, which would affect the measurements and block the flow through the siphon.

The probe was calibrated using a series of solutions of known concentration to generate an expression linking the bridge circuit output (in volts) to the density of the solution (in $\text{g cm}^{-3}$). The solutions were made up to the required concentrations using a carefully weighed amount of salt dissolved in a known volume of water. The density of the solutions were checked using a refractometer, as described in §3.3. The temperature had to be controlled where possible because the conductivity of the fluid is a function of temperature as well as salinity. When the tank used in the experiments was filled with water, it was left to warm up to room temperature before any experiments were conducted. The bridge control circuitry can also drift with time as a result of thermal effects. The reading from the probe can also change as water is absorbed into the delrin tip of the probe. To account for these two effects it was necessary to check the probe against a known solution before conducting each experiment.
The calibration graph is shown in figure 3.5 for a temperature of 21°C. The A/D converter is 12 bit, i.e. it returns an integer between 0 and 4095. The graph shows the reduced gravity of the solution against the measurement from the probe and A/D converter. A linear least-squares fit to the data is also shown. This gives an equation $g' = 0.021508V - 3.639$, where $V$ is the value returned by the A/D converter. There is some deviation from the best fit line for larger densities. This may be due to the error in measuring the density with a refractometer, or due to the relation between the density and the probe reading being nonlinear. There is also some deviation for very low densities. In this case the capacitance as well as the conductivity of the probe becomes important and affects the reading. As the probe was being calibrated against the refractometer it would be expected that, like the refractometer, the errors would be larger for small reduced gravities. For effective gravities in the range of 5.0 to 30.0, the best fit provides a good match to the data. Values of $g'$ in this range are those predominantly used in the experimental work described here.

Conductivity probes provide point measurements, allowing density profiles of the flow to be built up. By looking at where the density drops to that of the fresh water in the tank, the height of the flow can be inferred. The probe can also be used to measure density fluctuations.
in the flow. The disadvantage of this technique compared to the dye calibration technique in §3.5 is that it requires a probe to be placed in the flow and this inevitably alters the flow near the probe. The use of only one probe minimises the disturbance to the flow. To gather data at several points requires a large number of probes or a large number of repeat experiments, particularly when the flow is varying with time. For quasi-steady flows, the probe can be moved around to several positions during an experiment and readings taken at each point. The use of dye calibration and conductivity probes are complementary techniques, which together provide a great deal of information about the structure of the flow.

The probe was mounted on a vertical traverse which was also controlled by the PC. Software was developed specifically for this thesis to control both the probe and the A/D converter together, allowing the movement of the probe and the sampling to be automated. For instance, in the experiments with continuous releases the probe was started on the slope and a series of samples automatically taken at height intervals of 0.5 cm through the flow. This is not only easier and quicker, but also more accurate than positioning the probe by hand. The traverse was mounted on a support across the tank. The support and the traverse could be fixed in any position above the tank to allow measurements to be made. The horizontal position of the probe was fixed for each experiment, and was measured to within 0.5 cm using the world
coordinate system described in §3.4. The traverse controller used a feedback method to ensure that the vertical position of the probe was controlled to an accuracy of within 0.2 cm.

3.8 Summary

This chapter begins with a brief discussion of the use of laboratory experiments in modelling atmospheric flows, focusing on the use of salt water to create density differences. The various advantages of laboratory experiments compared to large-scale field trials are outlined.

The experimental apparatus used for the experiments in this thesis are described and some of the experimental techniques used are discussed. The experimental data collected comes primarily from optical methods (e.g. visual measurements, dye calibration) and from conductivity probe measurements. These methods provide a large amount of information, enabling the spreading and dilution of a dense flow, such as those studied in this thesis, to be measured in detail. Further details of individual experiments are given in the following chapters along with the experimental results.
Chapter 4

Instantaneous releases

4.1 Problem

An instantaneous release of dense gas can result from the catastrophic failure of a storage tank. Such an accident might result in the spillage of several thousand kilograms of dense gas, which would correspond to several thousand cubic metres of gas at atmospheric temperature and pressure. Many gases of practical importance have densities much greater than air. In most situations the terrain around the storage tank will not be flat and the presence of any slope could have a large effect on the spreading of the dense gas. Only slopes with a fixed gradient and no obstacles or containing walls are considered in this chapter. This is the simplest case to study first, although in many applications far more complicated topographies will be involved. In most situations the gradient of the slope will relatively small, no more than maybe 10°.

It might be expected that the rapid expansion and change of temperature on release of the gas would mean that the thermodynamics of the gas would be important. This adds a great deal of complication to the problem, and is neglected here upon the assumption that it will only play a significant role near the source.

For an initially axisymmetric release with no slope, the bulk motion will remain axisymmetric even though the internal details will be three-dimensional. Experimental evidence reviewed in §2.2.2 (see Huppert & Simpson, 1980) suggests that the front of the current propagates in such a way that the Froude number, Fr = uf/√(g/hf), remains approximately constant. Here uf is the speed of the front, hf is the height of the front and g′ = g(ρ − ρA)/ρA is the reduced gravity of the flow based on the difference between the density of the current, ρ and the density of the ambient ρA. Various integral and shallow water models accurately predict the bulk motion for such cases. Once a slope is introduced into the problem, the motion is no longer axisymmetric and is therefore more complicated to model.
4.2 Previous work

In chapter 2 some of the previous work relating to gravity currents on slopes was reviewed. Much of this was focused on sloping channels. Here we are considering the problem of a release of dense fluid on an unconfined slope. The work of Webber et al. (1993) allowed for a variation in ground height, \( a \), in the shallow water equations, as described in §2.3.2. This is applicable to an initially axisymmetric releases on an unconfined slope as well as to a release in a sloping channel. A constant Froude number front is used as the front condition.

For the shallow water equations to be valid it is necessary that the horizontal lengthscale of the current is much larger than the vertical lengthscale. For a uniform slope, with an angle of inclination \( \theta \), it is also necessary that \( \tan \theta \ll 1 \). This is not a serious constraint as the slopes of interest in modelling spillages of dense gas are unlikely to have gradients larger than 1:4 i.e. \( \theta \ll 15^\circ \).

For a constant slope, this set of equations and boundary conditions has a long time similarity solution where the gravity current takes the form of a wedge shape cloud, as illustrated in figure 4.1. The width, \( b = \pi l \), and the height, \( h = l \tan \theta \), are given by the equations. Since this is a similarity solution it does not depend on the initial conditions and might be expected to be reached some time after the release.

As discussed in §2.3.2, the exact form of the wedge shape is given by the parametric equation

\[
\begin{align*}
    x &= l \cos^2 \omega \\
    y &= l (\omega + \cos \omega \sin \omega) \\
    \left\{ -\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2} \right. 
\end{align*}
\]

(4.1)

where \( x \) and \( y \) are the down-slope and across-slope coordinates and \( l \) is the length of the wedge. The overall width and height are given by \( b = \pi l \) and \( h = l \tan \theta \).

The similarity solution predicted that the wedge-shaped cloud would move downslope with a constant speed, but this is not observed in the laboratory experiments described here. Further work by Tickle (1996) took this similarity solution and, assuming that the shape in figure 4.1 was maintained, added in an entrainment term. This model predicted that the wedge would slow down. Qualitatively, this is what is seen in the experiments.

Previous experiments with an instantaneous release on a slope have been carried out by Müller & Fanneløp (1999). These experiments consisted of a release of dense gas, with small droplets added to allow visualisation. They provided some useful insights but suffered from two drawbacks. Firstly, they only used small slopes of less than 15\(^\circ\). Secondly, they did not allow the flow to develop very far, due to the limited size of the slope. They compared their experimental results with a simple integral model that assumed a circular shape for the current. For small slopes this gave a reasonable agreement, however for distance more than
Figure 4.1: A wedge shaped gravity current on a slope. The solid line is the similarity solution of Webber et al. (1993). The same shape is used for the arc shaped wedge in the integral model in §4.3. The alternative triangular shaped wedge used in the integral model is shown as a dashed line.

about 12 initial radii and for slopes steeper than 6° the model over-predicted the speed of the current. They noted that most of the fluid was in the front region rather than being spread over a circular cloud. It is hoped that the salt water experiments described here will address some of these problems and in addition provide more detailed measurements for comparison with experiment.

4.3 Theoretical model

4.3.1 The importance of a slope

It might be expected that during the initial stages the current is relatively unaffected by the slope, provided the slope is not too steep and the aspect ratio is not too low (i.e. the difference in elevation of the ground across the release area is small compared to the depth of the fluid at that point.) As the current spreads it will become wider and shallower, meaning the slope will become relatively more important.

Simple integral (or box) models are frequently used to give an estimate for the propagation of gravity currents. The simplest possible model for an axisymmetric gravity current on a flat
surface assumes that the current takes the form of a flat cylinder with radius \( r \) and height \( h \). No entrainment occurs so the volume \( V = \pi r^2 h \) is constant. The speed at which the front moves is controlled by the Froude number condition \( \dot{r} = Fr \sqrt{gh} \). Experimentally it is found that the Froude number is a constant of the flow. Such models are discussed in §2.2.2. The front position of the current is given by

\[
    r = r_0 \left( 1 + 2Fr \frac{(g'V)^{1/2}}{r_0^2 \pi^{1/2}} t \right)^{1/2} \quad (4.2)
\]

and the height is
\[
    h = \frac{h_0 r_0^2}{r^2}. \quad (4.3)
\]

Here the constants \( r_0 \) and \( h_0 \) refer to the initial radius and height of the release.

It might be expected that slope effects will become important when the aspect ratio of the current becomes comparable to the gradient of the slope, i.e. \( h/r = \tan \theta \). The angle \( \theta \) is the inclination of the slope to the horizontal. By substituting in the predictions of the integral model we can find the time and the radius at which this occurs. This gives a time,

\[
    t_\theta = \frac{r_0}{2Fr \sqrt{h_0 g}} \left[ \left( \frac{h_0/r_0}{\tan \theta} \right)^{2/3} - 1 \right], \quad (4.4)
\]

and a radius,
\[
    r_\theta = r_0 \left( \frac{h_0/r_0}{\tan \theta} \right)^{1/3}, \quad (4.5)
\]

at which the slope effects become important. The radius \( r_\theta \) also gives an estimate of the maximum upslope extent of the gravity current. It can be noted that the radius \( r_\theta \) does not depend on the initial buoyancy but is purely a geometric effect. A greater buoyancy provides a greater driving force to push the fluid upslope, but this is balanced by the fact that more energy is needed to lift the denser fluid up.

This model suggests that for any non-zero angle \( \theta \), the slope will become important after a sufficiently long time. In practice, other factors such as viscosity may well come into play first, meaning that the assumptions of the model break down. The time at which viscous effects become important can be estimated as
\[
    t \sim \left( \frac{r_\theta^2 h_0}{\nu g} \right)^{1/3}, \quad (4.6)
\]

This is obtained by balancing the viscous and inertial terms in the momentum equation. Here
4.3. THEORETICAL MODEL

ν is the kinematic viscosity of the fluid. This balance suggests that for

\[ \tan \theta < \tan \theta_c = \frac{h_0/r_0}{\left(\frac{h_0 g'}{(v^2 r_0^2)}\right)} \]  

(4.7)

viscous forces dominate before there is time for the slope to become important.

For the apparatus used in the laboratory experiments discussed in §4.4, (4.7) gives a typical critical angle of about 2.3°. If the angle of the slope is greater than \( \theta_c \) then slope effects will be significant provided \( r > r_0 \). For the experiments carried out here with a slope of 5°, \( r_0 \) is about 2.6 times the initial radius \( r_0 \). This decreases to about 1.6 times \( r_0 \) for a slope of 20°.

4.3.2 An integral model

Based on the shallow water models of [Webber et al. (1993)] and [Tickle (1996)] an integral model for the motion of a gravity current on a slope is developed here. The approach is similar to that of [Beghin et al. (1981)] and [Dade et al. (1994)] for a gravity current in a sloping channel, as described in §2.3.1. Results from the shallow water models and from experiments suggest the current may form a wedge shape. Here we shall suppose the current has the shape shown in figure 4.1. Further, we suppose the shape is self-similar, with the dimensions given by [Webber et al. (1993)] so the width, \( b = \pi l \), and the height, \( h = l \tan \theta \), where \( l \) is the length of the wedge. The top and front areas can be written as

\[ A_T = S_1 l^2 \]  

(4.8)

and

\[ A_F = S_2 l^2 \tan \theta, \]  

(4.9)

respectively, and the volume as

\[ V = S_3 l^3 \tan \theta. \]  

(4.10)

The height of the back edge of the wedge is zero. The values \( S_1, S_2 \) and \( S_3 \) are shape parameters describing the geometric form of the wedge. For the shape given by (4.1)

\[ S_1 = \frac{3\pi}{4}, S_2 = \frac{8}{3} \text{ and } S_3 = \frac{5\pi}{16}. \]  

(4.11)

The wedge integral model can be used for any self-similar shape. For a different shaped current the shape parameters will take different values to those given in (4.11). In contrast, the shape of the wedge is prescribed in the shallow water model of [Tickle (1996)], unless different boundary conditions are chosen.
An alternative shape with a triangular top was also used to compare with the experiments. The shape is illustrated with the dashed lines in figure 4.1. For this shape the shape parameters are given by

\[ S_1 = 1, \quad S_2 = \sqrt{2} \quad \text{and} \quad S_3 = \frac{1}{3}. \]  

(4.12)

In deriving the new wedge integral model the Boussinesq approximation is made, i.e. the density difference is assumed small and can be neglected, except where it multiplies the gravitational acceleration, \( g \). This is a good approximation for most laboratory experiments, where the density difference is created by dissolving salt in water. There is also an implicit assumption that the slope is gentle enough that the flow is behaving like a gravity current and being driven by horizontal pressure differences. The shallow water model from which the assumption of the wedge shape is made is also only valid for small slopes.

The volume of the current increases as it spreads, due to the entrainment of ambient fluid into the denser fluid in the current. The rate of entrainment is taken as being proportional to the area over which the current is entraining times the current speed. The constant of proportionality is the entrainment coefficient, \( \alpha \), which is a function of the layer Richardson number. The layer Richardson number

\[ \text{Ri} = \frac{g h \cos \theta}{u^2} \]  

(4.13)

measures the stability of the interface. If \( \text{Ri} \) is large, so the interface is very stable, the entrainment will be suppressed. From the experiments carried out here (see §4.4) it appears that the entrainment is mainly confined to the front region of the current. We assume that the entrainment takes place over an area, \( A_E \), near the front, of width comparable to the depth of the current, so \( A_E = S_4 l^2 \tan \theta \), for some shape parameter, \( S_4 \). Taking the width of the entraining region at a point on the front as being equal to the front height at that point, the value for \( S_4 \) can be calculated for the wedge shapes considered here. For the cylindrical wedge shape given by (4.1), \( S_4 = \pi \sqrt{2} \), and for the triangular wedge shape \( S_4 = 2 \sqrt{2} \). This assumption about the form of the entrainment, is commonly used when modelling stratified flows, such as gravity currents and plumes.

As a result of this entrainment the bulk mass conservation equation can be written as

\[ \frac{d}{dt} (V) = \alpha u A_E. \]  

(4.14)

The change in linear momentum in the down-slope direction is due to two main forces. First there is the component of the buoyancy force acting down-slope and second there is a drag force associated with the motion of the current. The drag is made up of two components, the
bottom drag and the form drag. The bottom drag term is given by a standard parameterisation of the Reynolds stress terms for a turbulent current, $C_T u |u| A_T$, where $C_T$ is the drag coefficient and $u$ is the speed of the gravity current. The form drag on the current takes the form $C_F u |u| A_F$. Since both $A_T$ and $A_F$ scale like $l^2$, both of these drag terms have the same form. Likely values for the drag coefficients are discussed later. It should be noted that adding a form drag of this type is equivalent to imposing a Froude number condition on the front, as done by Webber et al. (1993) and Tickle (1996).

To take into account the ambient fluid carried along with the dense current, the mass of the current used in the momentum equation (4.15) is increased by a constant factor, $(1 + C_A)$, to give an effective “added mass”. This is an inviscid result and so the extra drag force is only felt while the current is accelerating. The entrained ambient fluid is at rest before it is entrained, so it imparts no momentum to the gravity current. The momentum conservation equation is therefore

$$\frac{d}{dt} ((1 + C_A) Vu) = B \sin \theta - (C_T A_T + C_F A_F) u^2. \quad (4.15)$$

As expected from the Boussinesq approximation, the density only appears in (4.14) - (4.15) via the buoyancy $B = g V$, and this is conserved during the flow.

The added mass coefficient, $C_A$, cannot easily be calculated for this wedge shape. The values for a circular cylinder and a sphere translating in irrotational potential flow are given as 1 and 0.5 respectively by Batchelor (1967, p431). For a slender body of width, $b$, and length, $l$, the value is proportional to $(b/l)^2$. From this it seems likely that the value for the wedge will be of order 1 or less.

We can look for special solutions to (4.14) and (4.15) which have a constant speed, $u$, corresponding to the solution of Webber et al. (1993). For such a solution to exist $V$ must be a constant, so $l$ is also a constant and $\alpha = 0$. In this case the buoyancy force is entirely balanced by the drag on the current in (4.15).

To find a more general solution, we note that, provided the entrainment is non-zero, (4.14) gives

$$\frac{dl}{dt} \propto u = \frac{dx}{dt}. \quad (4.16)$$

where $x$ is the horizontal position of the centre of mass of the wedge shape. The origin of $x$ can therefore be chosen so that $l \propto x$. Using the initial conditions that $u = u_0$ and $x = x_0$ at $t = t_0$, and the fact that $d/dt = ud/dx$, (4.14) and (4.15) can be integrated to give

$$l = \frac{S_{4 \alpha}}{3 S_3} x \quad (4.17)$$
and

\[ u^2 = u_\infty^2 \left( \frac{x_0}{x} \right)^2 + \left( u_0^2 - u_\infty^2 \right) \left( \frac{x_0}{x} \right)^\gamma, \]  

(4.18)

where

\[ u_\infty^2 = \frac{54B S_3^2 \cos \theta}{x_0^3 \alpha^3 S_4^2 (1 + C_A) (\gamma - 2)} \]  

(4.19)

and

\[ \eta = 6 \left( 1 + \frac{C_T S_1 + C_F S_2 \tan \theta}{\alpha S_4 \tan \theta (1 + C_A)} \right). \]  

(4.20)

The parameter \( \gamma \) controls the rate at which the speed of the current decreases. Increasing the drag increases \( \gamma \), causing the current to slow down more rapidly. The speed \( u_\infty \) gives the scale for the speed for large \( x \) so \( u/u_\infty \sim x_0/x \). Both \( u_\infty \) and \( \gamma \) increase with increasing entrainment and added mass so the overall effect of different values of \( \alpha \) and \( C_A \) is not immediately obvious.

In the limit \( \alpha \to 0 \) then \( x_0 \to -\infty \) in such a way that (4.18) does not become singular. The product \( (S_4 \alpha x_0)/(3S_3) - l_0 \) is constant. Substituting \( \alpha = 0 \) into (4.14) and (4.15) and solving gives an exponential form for \( u \) as a function of \( x \).

Writing \( u = dx/dt \) and substituting in (4.18) gives a differential equation for \( x(t) \). This equation cannot be integrated exactly for arbitrary values of \( C_T \) and \( C_F \), but the solution for \( x \) can be obtained numerically.

For the no-drag case where \( C_T = 0 \) and \( C_F = 0 \), an exact solution for the centre of mass position is possible, yielding

\[ x_c = \left( \frac{(2u_\infty x_0 t)^2 - \left( \frac{u_0^2 - u_\infty^2}{u_\infty^2} \right) x_0^2}{u_\infty^2 x_0^2} \right)^{1/4}. \]  

(4.21)

This predicts that for large times \( x_c \sim t^{1/2} \). The same scaling is seen for large times in the entraining similarity solution of Tickle (1996).

A variety of similar experiments give values of order 0.1 for the entrainment coefficient (see Ellison & Turner [1959], Hallworth et al. [1996]). In many situations the Richardson number (or equivalently the inverse of the Froude number squared) is found to be constant so the entrainment is also constant. Here the entrainment coefficient is retained as a parameter to be determined experimentally to give a best fit to the data. Britter & Linden (1980) found a value of about 0.003 for the drag, \( C_T \), for a gravity current in a smooth sloping channel. It will be greater for a rougher surface, but no experimental data is available. The form drag coefficient, \( C_F \), is generally between 1.0 and 0.1 for most bodies. The wedge shaped gravity current is fairly streamlined so the value is likely to be close to 0.1. Using these values and taking slopes between 5° and 20° gives values for the exponent \( \gamma \) between 6 and about 6.3. This is a 5%
change and means that neglecting the second term in (4.20) is a reasonable approximation i.e. drag effects are small compared to the effects of entrainment and the added mass. For very small slopes, \( \gamma \) will be larger, increasing the importance of the drag term. Comparison of numerical results (not shown here) from this model, with and without the drag terms, shows there is little difference for the parameter values considered here. The difference between the front position for \( C_T = 0.01 \) and \( C_T = 0.00 \) was less than 0.05%.

As \( \theta \to 0^\circ \) then, provided the volume of the current is constant, \( A_T \to \infty \), \( A_F \to 0 \), \( u_\infty \to 0 \) and \( \gamma \to \infty \). The solution is no longer valid in this limit. This is not a contradiction as \( t_\theta \to \infty \) and \( r_\theta \to \infty \) as \( \theta \to 0^\circ \), so the wedge shaped solution is never reached and the current spreads axisymmetrically for all time.

As \( \theta \) increases then an angle will be reached, depending on the aspect ratio of the release, where the slope will always be important according to (4.4). As \( \theta \to 90^\circ \) then the current will become a dense thermal against a vertical wall. In this limit the wedge shape would become very large and flat which is not appropriate. The thermal will be a more bluff shape, as discussed by Morton et al. (1956).

The distance, \( x_c \), in (4.21) is the position of the centre of mass of the current. The front position is needed in order to compare with the experimental results in § 4.4. For a self-similar shaped wedge, the centre of mass is a fixed fraction, \( k \), of the wedge length, \( l \), from the front of the wedge so

\[
x_f = x_c + kl.
\]

Substituting (4.17) into this expression gives

\[
x_f = x_c \left(1 + k \frac{S_4 \alpha}{3S_3}\right).
\]

(4.23)

For the circular wedge shape given by (4.1), \( k = 5/12 \), while for the triangular wedge shape \( k = 1/2 \). If (4.23) is substituted into (4.14) and (4.15) to change from measuring the centre of mass position to measuring the front position then (4.14) and (4.15) remain unchanged provided that \( u_\infty \) is multiplied by the factor \( 1 + k(S_4 \alpha)/(3S_3) \) and \( \alpha \) is divided by the same factor. The effect of measuring the positions and speeds of the front rather than the centre of mass is only to alter the experimental constants and not the form of the solution.

A Fortran 90 code was written to integrate the equations of the wedge integral model, using an adaptive step size Runge–Kutta–Merson method. As this model is only valid once sufficient time has passed for the similarity solution to develop, the simple cylindrical integral model for a gravity current on a horizontal surface, described in § 4.3.2, was used for the initial stage of the release. Once the current had spread out enough for slope effects to be important, the circular shaped current was transformed into a wedge shaped current, preserving the front
position, concentration and volume. Various types of integral model were used for the initial stage, both entraining and non-entraining. The results were found not to be particularly sensitive to the exact model used, since the initial stage was quite short and the spatial position at which the changeover was assumed to occur was determined by the slope, independent of the integral model used. The numerical code allowed the effect of the parameters to be studied and predictions compared with experimental results. These are discussed further in § 4.4.

4.4 Experimental work

Initial experiments were carried out in a shallow tank of width 1.18 m, length 1.49 m and depth 0.10 m. The slope was altered by tilting the tank. The depth limited the angle of slope which could be obtained to about 2.5°. The tank was filled with water and a Perspex cylinder of radius 4.6 cm or 7.2 cm was placed in the middle. Salt was dissolved in the water inside the cylinder to create a density difference. Coloured dye was also added to the cylinder to allow the flow to be visualised. The cylinder was pulled vertically upwards to release the salt water and initiate the gravity current. Reduced gravities, \( g' = g(\rho - \rho_A)/\rho_A \), in the range 8.5 cm s\(^{-2}\) to 35 cm s\(^{-2}\) were used for these experiments. In each experiment the initial height of the release was the full depth of the water.

For these experiments on a very shallow slope the behaviour observed was not the formation of the wedge shaped current predicted by the similarity solutions of Webber et al. (1993) and Tickle (1996). The current initially spread out radially, as anticipated. Subsequently the current became slightly elongated in the downslope direction and the centre moved down the slope. The elongation was of the order of a few percent, so was difficult to measure accurately. At a later time the Reynolds number became small enough that viscous forces started to play a dominant role. From then on the flow developed as a viscous flow. Far downslope from the back of the current the viscous flow can be thought as originating from a point source. A solution for this viscous stage is described by Lister (1992).

Since the initial releases were the full depth of the ambient, there was a possibility that the difference in fractional depth of the fluid in the up and down slope directions, rather than the change in absolute height, could be playing a part in the observed asymmetry. To remove this possibility some experiments with surface currents were carried out. In this case the water in the tank was salty. Fresh water, together with dye, was put into the cylinder so that the gravity current formed was lighter than the ambient and spread along the free surface. This meant that the current did not change its absolute height (and hence its potential energy), unlike the bottom current. It was, however, still subject to the same variations in the depth of the ambient fluid as in the case of the bottom current. In the surface current experiments no significant up
4.4. EXPERIMENTAL WORK

/ down slope asymmetry was seen, suggesting that the variations in fractional depth were not responsible for the asymmetries observed in the bottom currents. Figure 4.4 shows an image from one of these experiments. The slope is small, only 1.6°. The current has spread to about 5 times its initial radius and there is no discernible asymmetry between the upslope direction (bottom of picture) and the downslope direction (top of picture).

Further experiments were carried out in a much larger purpose built tank of width 2.0 m, length 2.5 m and depth 0.85 m. The tank was fitted with a false bottom which can be adjusted to give a variety of slopes, as described in §3.2. Gravity currents were generated by a similar method to that used in the smaller tank. The cylinders used were of radius 7.15 cm, and were angled at the bottom to ensure the cylinder remained vertical when placed on the slope. Cylinders were cut to angles of 5°, 10°, 15° and 20° to match the slopes used in the experiments. For full depth releases, the salt and dye were added in the top as for the smaller tank. Reduced gravities of 10.0 cm s\(^{-2}\) to 38.4 cm s\(^{-2}\) were studied in this tank, giving initial buoyancies in the range \(1.7 \times 10^4 - 5.5 \times 10^4\) cm\(^4\) s\(^{-3}\). Details of the experiments carried out are given in table 4.1. The Reynolds numbers based on the front speed and depth of the current were in the range 3000-6000, except for the slope of 5° where they dropped to about 1500. These are discussed later on.

For some experiments a conductivity probe was placed at various points in the flow. For these experiments the downslope distance from the origin to the probe is given in table 4.1. Details of the probe used are given in §3.7. The conductivity probe allowed the concentration of salt to be measured as the gravity current passed the probe. By repeating the experiment with the probe at different heights, a picture of the vertical structure of the flow was built up. Firstly, this gave an indication of how the current entrained ambient fluid and became diluted. Secondly, it gave a measure of the depth of the current at the point.

The slopes used in the larger tank were all greater than the critical angle \(\theta_c\) given by (4.7). On the slopes of 10°-20° a wedge-like shape was seen to develop some time after the release, as expected. For the slope of 5° the wedge was not observed to fully develop before the current had reached the edges of the tank. Figure 4.3 shows an example on a slope of 15°. Rather than having a flat back, the wedge was slightly curved to form a crescent shape. Not all the fluid was contained within the wedge shape, but a thin draining layer was left behind as the wedge passed. This layer was less than a centimetre thick, and slowly moving. The presence of this draining layer makes it hard to define exactly where the back of the wedge is.

Other techniques were used to gain more information about the flow. By measuring the intensity of the light received in the camera, and correcting for the background lighting it is possible to get a measure of the amount of dye in a vertical section. This gives a measure of the height and dilution in the current. The advantage of this technique is that it gives information...
### Table 4.1: Details of initial conditions for instantaneous release gravity current experiments.

The experiments were carried out in the large tank with a sloping bottom.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>θ /°</th>
<th>$g$ / cm s$^{-2}$</th>
<th>$Q$ / cm$^3$</th>
<th>Distance to probe / cm</th>
<th>Lengthscale $Q^{1/3}$ / cm</th>
<th>Timescale $(Q^{1/3}/g)^{1/2}$ / s</th>
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</thead>
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<tr>
<td>17-24</td>
<td>10</td>
<td>22.1-25.0</td>
<td>1959</td>
<td>23.5</td>
<td>12.5</td>
<td>0.71-0.75</td>
</tr>
<tr>
<td>25-34</td>
<td>10</td>
<td>22.1-27.2</td>
<td>1959</td>
<td>49.5</td>
<td>12.5</td>
<td>0.68-0.75</td>
</tr>
<tr>
<td>35-41</td>
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<td>19.65-27.9</td>
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<td>0.67-0.80</td>
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<td>15</td>
<td>20.35-23.95</td>
<td>2168</td>
<td>65.5</td>
<td>12.9</td>
<td>0.73-0.80</td>
</tr>
<tr>
<td>48-54</td>
<td>15</td>
<td>21.4-25.35</td>
<td>2168</td>
<td>49.5</td>
<td>12.9</td>
<td>0.71-0.78</td>
</tr>
<tr>
<td>55-60</td>
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<td>20.0-23.5</td>
<td>1767</td>
<td>51.0</td>
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<td>0.74-0.76</td>
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<td>11.9</td>
<td>0.64-0.65</td>
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<td>1.05</td>
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</tbody>
</table>
4.4. EXPERIMENTAL WORK

Figure 4.2: Formation of a wedge shape during experiment 96. The initial conditions for the release are given in table 4.1. The top of the slope is to the right of the picture and the bottom of the slope to the left. The release point is marked by a black x. The black outline curve is the integral model prediction for this experiment. The slope is 15°. The images are at non dimensional times a) 0.7, b) 2.1, c) 3.5 and d) 4.9 after release.

about the whole of the gravity current. Unlike the conductivity probe, it does not give a direct measure of the height or concentration though. Further details of this technique are given in §3.5.
CHAPTER 4. INSTANTANEOUS RELEASES

Figure 4.3: Development of a wedge shape during experiment 96. These images are a continuation of the release shown in figure 4.2. The initial conditions for the release are given in table 4.1. The top of the slope is to the right of the picture and the bottom of the slope to the left. The release point shows up as the dark circle on the right of the picture. The black outline curves are the integral model predictions for this experiment with the two different assumed shapes for the wedge. The slope is 15°. The images are at non-dimensional times a) 2.1, b) 9.0, c) 16.0 and d) 22.9 after release.

4.5 Comparison of experiment and theory

Figures 4.2 and 4.3 show plan views of a gravity current on a slope of 15° at different times. The images are taken from the video recordings of experiment 96 and are in false colour to improve the contrast. The black corresponds to where the current is deep or dense and the white corresponds to where there is no dense fluid. The integral model predictions are superimposed in black for comparison.

The pictures in figure 4.2 were taken at non-dimensional times 0.7, 2.1, 3.5 and 4.9 after the release and show the initial development of the current. The predictions of the axisym-
4.5. COMPARISON OF EXPERIMENT AND THEORY

Figure 4.4: View of a surface gravity current with a sloping bottom. The slope is 1.6° and slopes down from the bottom of the picture to the top of the picture. The initial radius of the release was 7.25 cm and at the time shown the radius is about 35 cm. The reduced gravity of the release was $-14.4 \text{ cm s}^{-2}$ and the volume of the release was 1180 cm$^3$, giving an initial buoyancy of $1.7 \times 10^4 \text{ cm}^4 \text{s}^{-3}$.

The development of the current in figure 4.3 is shown for later times by figure 4.4. These pictures are at non dimensional times of 2.1, 9.0, 16.0 and 22.9 after release. They show the wedge shape moving down the slope. The predictions of the wedge integral model are superimposed on the figures for both the cylindrical and the triangular shaped wedges. It can be seen that the shape of the wedge formed is more triangular than the circular shape predicted.
Neither shape fully captures the rear of the wedge.

Figures 4.5-4.14 show some of the results from the experiments. The results are all non-dimensionalised to allow comparison of different experiments. Dimensionally, the results are only expected to depend on the initial volume, $V_0$, and effective gravity, $g'_0$, of the release. Using these uniquely determines a lengthscale $V_0^{1/3}$, a timescale $(V_0 / g'_0^3)^{1/6}$ and an effective gravity scale $g'_0$ for the problem. The results may also depend on the slope and the dimensionless aspect ratio $h_0/r_0$ of the release. For all the experiments carried out here the aspect ratio was about 1.5. Each figure shows the results for slopes of $5^\circ$, $10^\circ$, $15^\circ$ and $20^\circ$. The $\times$ show the results from the conductivity probe readings. These readings gives the front height, concentration and arrival time at set distances from the source. By fitting the integral model to the front position data, a best fit value for the entrainment coefficient was calculated. The lines show the predictions of the wedge integral model using the best fit entrainment value. The blue lines are for the arc shaped wedge given by (4.1) and the red lines are for the triangular wedge. The shape parameters are given by (4.11) and (4.12), respectively. For the comparisons the drag coefficients $C_F$ and $C_T$ were set to zero and the added mass coefficient was set to 1.

Figure 4.5 shows the position of the front as a function of time. The results from several experiments with four different slopes ($5^\circ$ - $20^\circ$) and different values of $g'_0$ are shown along with the predictions of the integral model and the similarity solution of Webber et al. (1993). It can be seen that the integral model developed here agrees well with the experiments for a suitable value of the entrainment coefficient. The non-entraining similarity solution over-predicts the rate at which the current moves downslope. The entraining similarity solution of Tickle (1996) gives similar predictions to the integral model presented here.

The entrainment coefficient, $\alpha$, was chosen for each slope to provide the best fit to the front position data. This is why the entraining similarity solution of Tickle (1996) and the integral model for both wedge shapes used here give very similar predictions for the front position in figure 4.5. The numerical values obtained for the entrainment coefficients were about 0.1. The exact value differed slightly between the similarity solution and the integral model, although this is to be expected as any slight difference in the shape parameters used in the integral model can be compensated for by changing the entrainment. The effects of the slope, and comparisons with previous work, are discussed later on in this section.

The conductivity probe measurements allowed the depth of the current to be measured at various points down slope. Figure 4.6 shows the height of the front as a function of time. From the experimental results it can be seen that the height of the cloud varies only slowly with time. For the $5^\circ$ and $10^\circ$ slopes the height is nearly constant, with values of about 0.3 and 0.4 respectively in non-dimensional units. For the slopes of $15^\circ$ and $20^\circ$ the height increases
4.5. COMPARISON OF EXPERIMENT AND THEORY

Figure 4.5: Front position of the gravity current against time for slopes of 5°, 10°, 15° and 20°. The × are results from the conductivity probe. The +, ■, △ and diamond are measurements from the video. The lines are the integral model prediction with the best fit entrainment coefficients. The blue line is for the circular wedge and the red line is for the triangular wedge. Since these lines are a best fit to the front position they give predictions which are indistinguishable on the graph. The green line is the non-entraining similarity solution of Tickle (1996). Distances are non-dimensionalised with respect to \( V^{1/3} \) and times with respect to \( (V/g)^{1/6} \). Results are shown for experiments 95-99, 132-135, 160, 162 and 164.

with distance from about 0.4 to 0.7 over the length of the tank. The integral model results over-predict the height of the cloud, giving a steady increase in height with time as more ambient fluid is entrained.

Figure 4.7 shows the length of the gravity current against the downslope distance for the same slopes as in figure 4.5. As mentioned before, the measurement of length is somewhat subjective since the back is not straight and the distinction between the wedge and the draining layer behind is not always clear cut. The back position was taken as the imaginary line joining the two back corners of the wedge. Since the back was curved the wedge length obtained this way is larger than the length measured by only looking along the wedge centreline. The experimental results are plotted on the graph, along with the theoretical predictions of the integral model developed here. For slopes of 10°-20° the integral model appears to slightly
over-predict the cloud length. With a smaller slope of $5^\circ$, the integral model predictions are much too low. In the experiments the gravity current remains nearly axisymmetric so the measured length remains much larger than for the equivalent wedge.

Figure 4.8 shows the width of the gravity current against time. The width of the current increases with time, as expected, although the experimental results are smaller than the values predicted by the integral model, particularly for the $5^\circ$ slope.

The integral model appears to over-predict the size of the gravity current. One possible explanation for this is that the model does not take into account the fluid left behind the current in the draining layer. This draining layer is observed in the experiments and can cover a large area behind the wedge. Even if the layer is thin, the volume in the layer may be significant compared to the volume of the current once the current has travelled some distance down slope.

Figure 4.9 shows the maximum effective gravity of the front as a function of time. This quantity is subject to a fairly large error. The billowing nature of the flow and the discrete
Figure 4.7: Length of the gravity current against time for slopes of 5°, 10°, 15° and 20°. The + and □ are measurements from the video. The lines are the integral model prediction with the best fit entrainment coefficients. The blue line is for the circular wedge and the red line is for the triangular wedge. Distances are non-dimensionalised with respect to $V^{1/3}$ and times with respect to $(V/g^{1/3})^{1/6}$. Results from experiments 95-99, 135,159 and 160 are shown.

sampling in both time and space mean that the maximum value recorded can vary between experiments, even for nominally identical starting conditions. The peak value does however give some indication of the dilution taking place in the gravity current. The trend in the peak concentrations is to decrease with time, which agrees with the integral model. In general the integral model appears to over-predict the dilution of the current. Since the total buoyancy in the current is conserved, this is consistent with the model over-predicting the size of the wedge. It is not an entirely fair to compare the peak value with the model predictions. The data from the experiments shows that the lower part of the gravity current is relatively dense, while the thicker top layer is much more dilute. The integral model assumes a uniformly mixed current. An depth integrated effective gravity would be more representative and would give lower experimental values, more in line with the integral model. Note that the effective gravity predicted by the integral model does not depend on the wedge shape chosen, but only on the entrainment.

The conductivity profiles in figure 4.10 show that the current is divided into a relatively
narrow and deep (≈ 4–6 cm) head region, with a much wider and shallower (≈ 1 cm) region behind where the fluid is slowly draining away. Viscous forces are likely to play a significant role in the shallower region. This is verified by the visual images of the flow in figures 4.2 and 4.3. The thicker crescent shaped band of fluid which makes up the head is visible in the picture as a darker region, with a lighter thinner draining region behind it. The wedge integral model predicts a linear decrease in height from the maximum at the front to zero height at the back of the current. This does not agree precisely with the experiments.

The speed of the front as a function of time is shown in figure 4.11. The speed is calculated by fitting a polynomial through the front position data and differentiating to get the speed. This has the advantage over the more obvious finite differencing method of smoothing out any errors in the experimental measurements. As would be expected, the integral model and the experimentally derived data agree well.

Figure 4.12 shows the intensity profiles along the current centreline at intervals during an experiment. The intensity profiles give an indication of how the depth of the current changes...
4.5. COMPARISON OF EXPERIMENT AND THEORY

Figure 4.9: Effective gravity of the front of the gravity current against time for slopes of 5°, 10°, 15° and 20°. The × are results from the conductivity probe. The lines are the integral model prediction with the best fit entrainment coefficients. The blue line is for the circular wedge and the red line is for the triangular wedge. Effective gravities are non-dimensionalised with respect to $g'$ and times with respect to $(V/g'/3)^{1/6}$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Probe Height / cm</th>
<th>Non-dimensional peak</th>
<th>Peak $g'/g'_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.5</td>
<td>0.041</td>
<td>0.293</td>
</tr>
<tr>
<td>81</td>
<td>1.0</td>
<td>0.083</td>
<td>0.284</td>
</tr>
<tr>
<td>82</td>
<td>2.0</td>
<td>0.165</td>
<td>0.298</td>
</tr>
<tr>
<td>83</td>
<td>3.0</td>
<td>0.248</td>
<td>0.268</td>
</tr>
<tr>
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<td>4.0</td>
<td>0.331</td>
<td>0.119</td>
</tr>
<tr>
<td>85</td>
<td>5.0</td>
<td>0.413</td>
<td>0.217</td>
</tr>
<tr>
<td>86</td>
<td>6.0</td>
<td>0.496</td>
<td>0.127</td>
</tr>
<tr>
<td>87</td>
<td>7.0</td>
<td>0.579</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 4.2: Height of probe and peak effective gravity for experiments 80-87
Figure 4.10: Effective gravity against time at a series of different heights above the bottom (see Table 4.2). The probe is a non-dimensional distance of 1.2 from the origin. The slope is $15^\circ$. Effective gravities are non-dimensionalised with respect to $g$, and times with respect to $\sqrt{g_\text{R}}$. 

Reduced gravity

Time

Height of probe

0.5 cm
1.0 cm
2.0 cm
3.0 cm
4.0 cm
5.0 cm
6.0 cm
7.0 cm

0.000 0.050 0.100 0.150 0.200 0.250 0.300 0.350
Figure 4.11: Speed of the front of the gravity current against time for slopes of 5°, 10°, 15° and 20°. The + and □ are results from video measurements. The lines are the integral model prediction with the best fit entrainment coefficients. The blue line is for the circular wedge and the red line is for the triangular wedge. Speeds are non-dimensionalised with respect to \((g'V^{1/3})^{1/2}\) and times with respect to \((V/g'g^{3/6})\).

as the current develops. The light intensity, \(I\), is proportional to \(\exp(-\int g' dz)\) as described in §3.5. The vertically integrated \(g'\) is the pressure difference or head which is driving the gravity current forward. The values plotted are in arbitrary units so that an increase in the plotted value corresponds to an increase in the head. The profiles show a clear head region to the current with a much thinner tail behind. It can be seen that as the current moves down the slope the head is reduced slightly, which explains why the current slows down. The width of the head region is also seen to increase as the current develops. Behind the head the intensity varies over a range of 0.1 to 0.2. These variations correspond the the billows observed behind the head during the experiments.

Figure 4.13 shows the width / length ratio of the gravity current against time. In the integral model the current is assumed to be self similar so should be a constant. For the circular wedge shape this ratio is \(\pi\), while for the triangular shape it is 2.0. The experimental data is consistent with the ratio being constant, although there is a fair degree of scatter in the data, mainly due to the difficulty of precisely defining the back and sides of the current. There appears to be
Figure 4.12: Centre-line profiles of the light intensity at 3 s intervals (a non-dimensional time of 4.2). The profiles show the progress of the current downslope and the development of the head. The intensity scale gives a measure of the vertical integral of $g'$. The experiment was on a slope of 15°. For the initial conditions see experiment 96 in table 4.1.

a slight trend for the ratio to decrease with time, but this too may be due to difficulties in defining the current length. The experimental values for the slope of 5° is much lower than for the larger slopes, with a value of about 1.0. This is because the wedge solution did not have time to develop and the current still appeared to be circular, although the downslope front was deeper than the upslope front. The value of 1.0 would be expected for a circular current.

Figure 4.14 shows the height / length ratio divided by $\tan \theta$ plotted against time. The integral model assumes a flat top for the current, so this ratio is 1.0 for both choices of wedge shape. The experimental results are again consistent with this assumption, but with a large scatter. The uncertainty in measuring the length is compounded by the uncertainty in the height measurements made using the conductivity probe.

The extent to which the current spreads upslope is likely to be governed by viscous forces, since the upslope layer is thin and slow moving. Figure 4.15 shows the maximum upslope extent as a function of the slope. The height to which the current rises is non-dimensionalised with the initial height of the release. It can be seen that the current rises to about half the initial height.

The values for the entrainment coefficient, $\alpha$, were obtained by doing a least squares best
4.5. COMPARISON OF EXPERIMENT AND THEORY

Figure 4.13: Width / length of the front of the gravity current against time for slopes of 5°, 10°, 15° and 20°. The × are results from the conductivity probe. The lines are the assumed values of π for the circular wedge and 2.0 for the triangular wedge. Times are non-dimensionalised with respect to \((V/g^3)\)^{1/6}.

The results from Beghin et al. (1981) for saline gravity currents in a sloping channel and from Tickle (1996) for a three-dimensional release of dense gas on a slope are also shown. The results from Tickle (1996) agree well with the results here. The values from Beghin et al. (1981) also agree well, although they are on the whole slightly lower than the values for the three dimensional gravity currents. In Beghin et al. (1981) the entrainment is assumed to occur over the whole head so the slight difference in entrainment coefficient may be due to a difference in the area over which the entrainment is occurring. The entrainment coefficient is seen to be an approximately constant function of the slope. The work of Hallworth et al. (1996) for both two-dimensional and three-dimensional currents on a smooth horizontal surface, entraining over the whole head, gave values of between 0.08 and 0.09 for the entrainment coefficient, which agree with the results here for a gravity current on a slope. The results from Tickle (1996) are recalculated assuming the entrainment is limited to the head region. This assumption, which agrees with the experimental observations, removes most of the slope dependence seen by Tickle (1996) and Ellison & Turner (1959) in previous experiments. This suggests that for moderate slopes the fundamen-
Figure 4.14: Height / (length $\times \tan \theta$) for the front of a gravity current against time for slopes of 5°, 10°, 15° and 20°. The $\times$ are results from the conductivity probe. The line is the assumed values of 1.0 for the wedge integral model. Times are non-dimensionalised with respect to $(V/g^3)^{1/6}$.

tal entrainment processes may not be significantly different from those in a gravity current on a horizontal surface. The constant entrainment coefficient also suggests that the Richardson number stays fairly constant in the flow as well. The variation in the Froude number (which is related to the Richardson number by $Ri = 1/\text{Fr}^2$) is discussed below.

There was some variability in the experimental results, which was mainly due to experimental difficulties. Any small disturbances to the initial current, caused by for example the removal of the cylinder, grew as the current flowed down the slope. Thus two nominally identical experiments could give quantitatively different results, as can be seen from the spread of experimental results in figure 4.5. In several of the experiments the current developed a noticeable asymmetry between the two sides of the downslope flow. This meant that the front of the cloud did not always lie directly downslope from the release point. If it was to one side then the front position was taken as the downslope component of the distance. In the experiments described here the front-most point was up to 10cm from the centreline by the time the current reached the end of the tank. Another cause of variability is the billowing nature of the gravity current. This can be seen from the picture in figure 4.3 and is also visible in the conductivity probe measurements. Figure 4.17 shows the results from a probe for 5 nominally
identical experiments. The probe was located 1.0 cm above the bottom and 51 cm from the origin on a slope of $15^\circ$. While the graph shows that the arrival time, general shape of the front and following flow are similar in all the experiments, there are differences caused by passing billows.

Unlike the shallow water model, the integral model does not include any explicit Froude number condition. Instead the drag of the slope and the ambient fluid is parameterised using the drag coefficient and added mass terms in the momentum equation. The predicted Froude number can be calculated and compared with other models and with experiments. For large distances the model predicts that

$$Fr \to \left(\frac{6S_3 \cos \theta}{\alpha S_3 (1 + C_A)(\gamma - 2)}\right)^{1/2}.$$  \hspace{1cm} (4.24)

This value is a function of the slope as well as the parameters $C_A$, $C_T$, $C_F$ and $\alpha$. For the values used here, the asymptotic form in (4.24) gives a Froude number in the range $0.75 - 1.07$ which is slightly lower than the generally accepted value of about 1.2 for gravity currents on a flat surface, but of the same order of magnitude. The dependence on the slope is very weak for moderate angles, $\theta$, since $\cos \theta \approx 1$ for small $\theta$ and $\gamma - 2 \approx 4$ provided $\theta \geq 5^\circ$. 

Figure 4.15: Maximum upslope extent of the gravity current as a function of slope. The height is non-dimensionalised with respect to the initial height of the release.
CHAPTER 4. INSTANTANEOUS RELEASES

Figure 4.16: Comparison of entrainment coefficients as a function of slope from different experiments. The + are best-fit values to the integral model using the experiments described here, the □ are values from Beghin et al. (1981) and the ○ values from Tickle (1996). The values of Beghin et al. (1981) and Tickle (1996) are corrected to assume the entrainment is located near the front of the gravity current.

Figure 4.18 shows that, for a given experiment, the Froude number remained constant to within about 20%, once the current was established. This is reasonable given that both the experimental front height and speed values are subject to an error of about 10%. The values calculated from the experiments were in the range 0.75 – 1.2, depending on how developed the flow was. This is in fair agreement with the predictions of the integral model. These values also compare well with the values cited in Tickle (1996). The experimental evidence seems to support the suggestion that the constant Froude number condition widely used for a gravity current on a horizontal surface is still applicable for a gravity current on a sloping surface. The constant Froude number condition also implies that the Richardson number of the flow is constant, which in turns suggests that the entrainment coefficient will not vary much during the experiments. A fixed entrainment coefficient in the integral model has been shown to provide good agreement with the experimental results.

Figure 4.19 show the Reynolds number of the gravity current front against distance from source for a slope of 10°. The experimentally calculated values are generally slightly lower than the integral model predictions, but of the same order of magnitude. The Reynolds number
Figure 4.17: Effective gravity against time for a series of identical experiments at a height of 1.0 cm above the bottom and 51 cm from the source. The slope is 15°. Effective gravities are non-dimensionalised with respect to $g'$ and times with respect to $(Y/g'^{3})^{1/6}$.
Figure 4.18: Comparison of experimentally calculated Froude numbers with theoretical predictions as a function of downslope distance for slopes of 5°, 10°, 15° and 20°. The × are experimental values. The lines are the predictions of the integral model developed here with the using best-fit entrainment coefficients obtained from the front position data. All distances are non-dimensionalised with respect to $V^{1/3}$.

is of the order of several thousand in the region of interest, so the modelling assumption that we can neglect viscosity is reasonable.

### 4.6 A wedge integral model with a draining region

As discussed in §4.5, a significant proportion of the fluid from the gravity current may eventually be left in the draining layer behind the wedge. The experimental measurements using the conductivity probe, described in §4.4, suggest that the draining layer may be of the order of 0.5 cm thick. The integral model can be adapted to take this draining layer into account.

Suppose that the layer left behind the wedge has a constant thickness, $h_d$. The conservation of volume equation, (4.14), becomes

$$\frac{dV}{dt} = \alpha u A - bh_d u.$$  \hspace{1cm} (4.25)
4.6. A WEDGE INTEGRAL MODEL WITH A DRAINING REGION

Figure 4.19: Experimentally calculated Reynolds numbers as a function of downslope distance for slopes of 5°, 10°, 15° and 20°. The × are experimental values. The lines are the predictions of the integral model developed here. All distances are non-dimensionalised with respect to $V^{1/3}$. 
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The conservation of momentum equation, (4.15), is similarly altered to give

\[ \frac{d}{dt} \left( (1 + C_A) Vu \right) = B \sin \theta - (C_T A_T + C_F A_F) u^2 - bh_d u^2 \]  

(4.26)

as a result of the fluid left behind. Buoyancy is no longer conserved in the wedge and the rate of change of buoyancy is given by

\[ \frac{dB}{dt} = -g' bh_d u. \]  

(4.27)

The code used to obtain numerical solutions to the wedge integral model in §4.3.2 was adapted to solve these modified model equations.

To study the effect of the draining layer on the integral model predictions the initial conditions corresponding to experiment 96 in table 4.1 were used. Values of 0.0 cm, 0.2 cm and 0.5 cm were taken for the draining height, \( h_d \). As before, the entrainment was determined by performing a best fit of the front position to the experimental data. This ensured that the predictions for the front position were very similar for the three cases. The entrainment was found to decrease as \( h_d \) increased. This led to a large difference in the width and length of the wedge, but a negligible difference in the density, as shown in figures 4.20 and 4.21. At non-dimensional time \( t = 25 \) the width of the wedge for a current with no draining layer was 3.5, as opposed to 2.3 for a current with a draining layer of height \( h_d = 0.5 \) cm.

The potential slight improvement in the accuracy of the model predictions is balanced by the need to determine another experimental parameter, \( h_d \), in the problem. For many purposes the added degree of freedom cannot be justified. The draining layer observed in the laboratory experiments is primarily a result of viscous forces. In large scale atmospheric flows there will not be the same kind of draining layer so the extensions to the model will not apply.

4.7 Summary

The experimental work reported here demonstrates that, even for relatively gentle slopes, the presence of a sloping boundary can have a significant effect on a gravity current. Further, the role of entrainment is significant in slowing down the current as it flows down the slope.

Using the scaling in §4.3.1, the time \( t_\theta \) for the slope to become important is given by (4.4). Experimental results have shown that for very small slopes the similarity solution is not reached in the available sized tank. For larger slopes a behaviour similar to the similarity solution was observed. Once the slope has become important, the gravity current develops into a wedge shape which moves down the slope. The similarity solution from the shallow
Figure 4.20: Effective gravity of the front of the gravity current against time for a slope of 15°. The lines are the integral model prediction for various draining layer depths, $h_d$, using the best fit entrainment coefficients. Effective gravities are non-dimensionalised with respect to $g'$ and times with respect to $(V/g')^{1/6}$.
Figure 4.21: Width of the gravity current against time for a slope of 15°. The lines are the integral model prediction for various draining layer depths, \( h_d \), using the best fit entrainment coefficients. Lengths are non-dimensionalised with respect to \( V^{1/3} \) and times with respect to \((V/g^{1/3})^{1/6}\).
4.7. SUMMARY

The water model of Webber et al. (1993) does not accurately predict the motion of the gravity current as it fails to take into account the entrainment of the ambient fluid into the current. The model of Tickle (1996) includes the entrainment in a simple manner, which leads to better prediction of the front position, but does not accurately capture the shape of the gravity current. In particular, the width is over-predicted and the wedge is seen to have more of a triangular shape than the circular shape predicted by the shallow water models.

The wedge integral model developed here is more successful than the non-entraining shallow water model in predicting the front position once slope effects become important. Neither model is valid for the initial stages of the flow where the slope can be neglected. For a suitable choice of the shape parameters in the integral model, a good fit to the current shape, width and length can be obtained. The model also provides a reasonable prediction of the reduced gravity of the current. The entrainment coefficient is found to be approximately constant, provided the entrainment is taken to occur in a region near the front of the gravity current of width comparable to the wedge height. Experimentally, this is the region where the most vigorous mixing is observed and where the bulk of the fluid is located. The values for the entrainment coefficient agree well with previous work in similar situations.

Both the experiments and the integral model suggest that, once the wedge shaped gravity current has developed, the front Froude number is a constant of the flow, independent of time and slope. For an axisymmetric gravity current on a horizontal surface this result is well known, but has not been reported previously for a sloping bottom.

The entraining wedge integral model does not predict all of the remaining details of the flow accurately. The actual shape of the gravity current observed in the laboratory is more complicated than the simple wedge shapes assumed by the models. In fact it is more of a crescent shape and behind the crescent is a thin layer of draining fluid. Within this thin layer viscous forces are likely to be important. This region can be included in the wedge integral model in an ad hoc manner. This modification resulted in a smaller wedge, but almost no change in the front speed or the effective gravity of the current. It appears that the bulk motion of the current is controlled by the front of the current, as for an axisymmetric current on a horizontal surface. Any integral model which predicts this part of the flow reasonably accurately with give a fair prediction for the bulk motion of the current. This helps to explain why the integral model does so well, despite not including many of the more complicated aspects of the flow.

For dense gas dispersion in the atmosphere the draining region may be different as the dominant effect is likely to be turbulent diffusion rather than viscous diffusion. In addition any wind will be important, but that is neglected here. We have shown that for smooth surfaces the bottom drag is not significant in determining the bulk motion of the current, however
for a rough surface, such as tall vegetation or buildings, the results may be affected by the bottom drag term. Nonetheless, the experimental data and wedge integral model presented here provide a useful check for more complicated existing and future dense gas dispersion models. The experiments demonstrate that the presence of a relatively gentle slope may have a large influence on the area affected by a spillage of dense gas and also on the dose received at a given point.
Chapter 5

Continuous releases

5.1 Problem

A continuous release of dense gas can result from the rupture of a storage tank leaving a hole through which gas can escape. Such an accident might result in the spillage of several thousand kilograms of dense gas, as for an instantaneous release, but over a period of minutes or hours rather than instantaneously. At atmospheric temperature and pressure this would correspond to a total volume of several thousand cubic metres of gas and a flow rate of a few cubic metres a second. Such releases can be complicated. The gas may be liquefied in the container and flash into gas on release. The thermodynamics of the release add extra complications and are not discussed further here. Instead we concentrate on the role of the density difference. Away from the source, once the initial expansion and cooling has taken place, it might be expected that the thermodynamics will only have a small effect on the flow. As in chapter 4, only slopes with a fixed gradient and no obstacles or containing walls are considered here. This is the simplest case to study first, although in many applications more complicated topographies will be involved. In most real-world situations the gradient of the slope will be relatively small, no more than maybe 15°.

The aim of this chapter is to consider a negatively buoyant forced plume, initially directed either upslope or at an angle to the slope. A negatively buoyant jet or forced plume has a source with a finite momentum flux and buoyancy flux. This means that the initial momentum is driving the flow up the slope, while the buoyancy acts downslope to slow the plume down and eventually reverse the flow. Of particular interest is the transition from plume to gravity current and how far up the slope the flow reaches. From experiments it can be seen that near the source the flow looks similar to a wall jet or plume. The flow is predominantly in one direction, with the lateral speeds being small. As the plume moves upslope it entrains ambient
fluid and slows down. This leads to an increase in the cross sectional area. As a result of the
density difference between the plume and the ambient fluid the plume spreads sideways. Since
work needs to be done to raise the dense fluid, vertical spreading is suppressed on energetic
grounds. Once the current has reached its maximum upslope position it spreads sideways. By
this stage the cross slope speed is comparable or greater than the up slope speed and the flow
can no longer be considered as a plume. The dense fluid then flows back down the slope under
the action of gravity and the flow forms a large and shallow draining layer. As described, the
flow consists of several stages. We shall develop a model to describe these different stages.
First we shall consider the initial upward phase of the flow during which the flow behaves like
a plume.

5.2 Plume model

Suppose that the initial upward part of the flow is considered as an inclined negatively buoyant
jet or forced plume, as shown in figure 5.1. The density difference will tend to suppress
spreading in the vertical direction as it requires work to be done against buoyancy, but will
enhance the lateral spreading.

The analysis in this section is based on the work of Morton et al. (1956). The standard
plume equations of Morton et al. (1956) provide an effective way of modelling dense vertical
plumes and jets, as discussed in §2.6. They have been very successful in describing such
phenomena and predictions agree well with experimental evidence. These ideas have been
extended to look at vertical forced plumes or negatively buoyant jets by Morton (1959) and
Turner (1966). Lane-Serff et al. (1993) used a modified form of the plume equations to look
at tilted rather than vertical forced plumes. These equations are taken as a basis for the model

![Figure 5.1: Definition diagram of a forced plume on a slope.](image-url)
developed here for a forced plume on a slope, taking into account the buoyancy-induced reduction in the entrainment, and the drag on the slope.

For a vertical plume, the cross section can be assumed to be circular so the radius completely specifies the size. However, for a plume directed up a slope, the width and the depth of the plume will no longer necessarily be equal and an extra equation is needed to specify the relation between the them. Here the plume is assumed to have a top hat profile i.e. the density and speed at a given height are uniform inside the plume and zero outside. The upslope distance from the source, \( x \), is taken as the independent variable. In order to calculate fluxes in the plume it is necessary to know the cross sectional area of the plume. In general, the area can be written as \( S_1 x bh \), where \( S_1 \) depends on the shape of the plume cross section, which may vary with distance from the source. Here it is assumed that the plume is self similar i.e. that the cross sectional shape of the plume does not change with \( x \), although the size may. For a self-similar shape \( S_1 \) is a constant. In practice the plume may not be self-similar, but, provided that \( S_1 \) changes sufficiently slowly, the local dynamics will not notice the \( dS_1 / dx \) and changes in \( S_1 \) can be neglected. For a rectangular cross section \( S_1 = 2 \) and for a semi-elliptical cross section \( S_1 = \frac{1}{4} \pi \).

The mean speed of the flow in the plume, \( u \), is defined so that the volume flux in the plume is \( Q = S_1 h bu \). The speed of the flow outside the plume is assumed to be zero. The speed, \( u \), is a function of upslope distance, \( x \). This gives the volume flux in the plume as \( S_1 h bu \). The volume flux increases through entrainment of ambient fluid into the plume. The standard assumption is made that the entrainment is proportional to the plume perimeter times the speed of the plume. The constant of proportionality is the entrainment coefficient, \( \alpha \). The plume perimeter can be written as \( S_3 (x) (h + b) \) for a suitable shape parameter \( S_3 \). Again \( S_3 \) will be a constant if a self-similar shape is assumed. For a rectangular cross section, \( S_3 = 2 \). For a semi-elliptical cross section there is no simple formula but a series solution gives \( S_3 = \sum_{n=0}^{\infty} \left( \frac{1}{n} \right) \left( \frac{1-h/b}{1+h/b} \right)^{2n} \). For a given cross section shape the ratio \( h/b \) is a constant so \( S_3 \) is also constant. Note that for the case of a circle where \( h = b \) this formula gives \( S_3 = \pi/2 \), as expected. Similarly, in the limit \( h \to 0 \) we recover \( S_3 = 2 \). The equation for the volume flux in the plume can therefore be written as

\[
\frac{d}{dx} (S_1 h bu) = S_3 \alpha u (h + b). \tag{5.1}
\]

The entrainment coefficient, \( \alpha \), may be a function of the slope and the other plume variables. Here it is assumed that \( \alpha = \alpha(\theta) \) only.

The momentum flux, \( S_1 h bu^2 \), changes for several reasons. Firstly, there is a contribution as a result of the work done against gravity. Secondly, there is a drag term to take into account the presence of the wall. Finally, there is a term as a result of the horizontal pressure gradient
due to the change in depth of the plume. This gives rise to a momentum equation of the form

$$\frac{d}{dx} \left( S_1 h b u^2 \right) = -S_1 h b g' \sin \theta - 2C_D b u^2 \frac{1}{2} \frac{d}{dx} \left( S_2 g' h^2 b \cos \theta \right). \quad (5.2)$$

The first term on the right hand side of (5.2) is to take into account the change in momentum as the dense plume rises. The second term is a drag term to take into account the effect of the slope surface. The drag coefficient, $C_D$, needs to be theoretically or experimentally determined. The value of $C_D$ is discussed below. The final term, to account for the horizontal pressure gradient, is the $x$ derivative of $\int g' z \cos \theta \, dA$. This integral can be written in the form $\frac{1}{2} S_2 g' h^2 b \cos \theta$, so for a rectangular cross section $S_2 = 2$ and for a semi-elliptical cross section $S_2 = 4/3$.

In an unstratified ambient the buoyancy flux, $S_1 h b g' u$, in the plume is conserved so

$$\frac{d}{dx} \left( S_1 h b g' u \right) = 0. \quad (5.3)$$

Integrating gives $S_1 h b g' u = F_0$, where $F_0$ is the initial buoyancy flux.

Equations (5.1)–(5.3) are not sufficient to close the problem. A further equation is needed to relate the width and depth of the plume. In stratified flows the Richardson number,

$$Ri = \frac{g' h \cos \theta}{u^2}, \quad (5.4)$$

is an important dimensionless number which provides a measure of the stability of the flow. The entrainment into the flow is a function of the Richardson number. In many situations it is observed that the flow quickly adjusts to a state where the Richardson number is constant, i.e.

$$Ri = Ri_c. \quad (5.5)$$

Examples include the work of Ellison & Turner [1959] on the flow of dense layers in a sloping channel. For moderate slopes of $9^\circ$–$23^\circ$ the flow was found to adjust to a state with a Richardson number of between 0.1 and 0.3 depending on the slope and the Reynolds number of the flow. Once the transition to this constant Richardson number has occurred, setting the Richardson number to a constant value gives an extra equation for the flow and closes the problem.

Initially the plume is driven predominantly by its momentum rather than by the density difference. In such a situation the plume might be expected to behave in a manner similar to a vertical plume. It is reasonable to assume that in the initial stages the plume has a semi-circular
shape with

\[ b = h. \]  \hfill (5.6)

Morton et al. (1956) and many later workers formulate their models using the volume, momentum and buoyancy fluxes \((Q, M \text{ and } F)\) respectively as the variables. This works well for problems with vertical plumes. However, in the case of a tilted plume the presence of the derivative on the right of (5.2) makes it difficult to write the equations in this form. Instead the equations will be rewritten in terms of the plume width, depth and the local Richardson number, \(Ri\). This has the advantage that the extra conditions (5.5) and (5.6) can easily be used to rearrange (5.1) and (5.2) in terms of the two independent variables \(h\) and \(Ri\) or \(h\) and \(b\). This approach was used by Ellison & Turner (1959) in studying the flow of a dense layer of fluid in a sloping channel and is generalised here to the three-dimensional case.

After rearranging, (5.1) and (5.2) become

\[
\frac{dh}{dx} = \frac{S_3 \alpha}{S_1} \left( 1 + \frac{h}{b} \right) - \frac{2h db}{3b dx} + \frac{h}{3 Ri} \frac{dRi}{dx}.
\]  \hfill (5.7)

and

\[
\frac{h}{3 Ri} \frac{dRi}{dx} \left( 1 - \frac{S_2}{S_1} \frac{Ri}{Ri} \right) = \frac{Ri \tan \theta + 2CP}{S_1} + \frac{S_3 \alpha}{S_1} \left( 1 + \frac{h}{b} \right) \left( 1 + \frac{S_2}{2Si} \right) - \frac{h db}{3b dx} \left( 1 + \frac{S_2}{2Si} \right).
\]  \hfill (5.8)

The only dimensional quantities remaining in equations (5.7) and (5.8) are the lengths \(h\), \(b\) and \(x\). These all appear as the ratio of two lengths, so we can non-dimensionalise the problem with any arbitrary lengthscale and leave (5.7) and (5.8) unchanged. The choice of the non-dimensional Richardson number as a variable is effectively non-dimensionalising the problem.

The further condition that \(h = b\) or \(Ri = Ri_c\) can be applied to these equations to give a pair of ordinary differential equations in two unknowns.

Suppose that initially \(h = b\) then (5.7) and (5.8) become

\[
\frac{dh}{dx} \left( 1 - \frac{S_2}{S_1} \frac{Ri}{Ri} \right) = \frac{1}{2} \frac{Ri \tan \theta + CP}{S_1} + \frac{2S_3 \alpha}{S_1} \left( 1 - \frac{2S_2}{5S_1} \frac{Ri}{Ri} \right)
\]  \hfill (5.9)

and

\[
\frac{2h}{5Ri} \frac{dRi}{dx} \left( 1 - \frac{S_2}{S_1} \frac{Ri}{Ri} \right) = \frac{Ri \tan \theta + CP}{S_1} + \frac{8S_3 \alpha}{3S_1} \left( 1 + \frac{S_2}{2S_1} \frac{Ri}{Ri} \right).
\]  \hfill (5.10)

Once the Richardson number has reached the critical value of \(Ri_c\) it is assumed to remain...
constant. In order to ensure the problem is not over specified the width and depth are allowed to vary independently. This means that $S_3$ is no longer a constant. Substituting $R_i = R_i_c$ into (5.7) and (5.8) gives

$$\frac{h}{b} \frac{dh}{dx} \left(1 + \frac{S_2}{2S_1}R_i_c\right) = \frac{S_3\alpha}{S_1} \left(1 + \frac{h}{b}\right) \left(1 + \frac{S_2}{2S_1}R_i_c\right) + R_i_c \tan \theta + \frac{2C_D}{S_1}$$

(5.11)

and

$$\frac{dh}{dx} \left(1 + \frac{S_2}{2S_1}R_i_c\right) = -\frac{S_3\alpha}{S_1} \left(1 + \frac{h}{b}\right) \left(1 + \frac{S_2}{2S_1}R_i_c\right) - 2R_i_c \tan \theta - \frac{4C_D}{S_1}.$$  

(5.12)

Initially the volume flux and the reduced gravity are known at the source. The initial values for $u$, $h$ and $b$ and hence $R_i$ can be calculated. This provides suitable initial conditions for the equations.

No simple analytically solutions have been found for this model. Solutions were obtained by integrating the equations numerically. A Fortran 90 program using a fourth order Runge–Kutta method was written for this purpose. The plume evolution can be calculated using this program for various initial values of $h$ and $R_i$. The model will break down when the velocity tends to zero and the plume area becomes infinite. The point at which the velocity becomes zero may be identified with the top of the plume, although it is likely that in the laboratory either viscous forces or the interaction of the upward and downward flows will become important before that point is reached.

The initial fluxes of volume, momentum and buoyancy can be written in terms of the speed, $U_0$, and the reduced gravity, $g'_0$, of the plume at the source and the radius of the source, $r_0$. The volume flux is given by $Q_0 = \pi r_0^2 U_0$, the momentum flux is $M_0 = \pi r_0^2 U_0^2$ and the buoyancy flux is $F_0 = \pi r_0^2 g'_0 U_0$. Given these three fluxes then two independent lengthscales can be formed. The “jet length”, $l_{MF} = (M_0^3/F_0^2)^{1/4}$ is based on the momentum and buoyancy fluxes. The other lengthscale, $l_{QM} = Q_0/M_0^{1/2}$, is based on the volume and momentum fluxes. The initial Richardson number can be rewritten in terms of these lengthscales to give

$$Ri_0 = \frac{r_0 g'_0}{U_0^2} \cos \theta = \left(\frac{l_{QM}}{l_{MF}}\right)^2 \pi^{-1/2} \cos \theta.$$  

(5.13)

Since $Ri_0 << 1$ for the range of experimental parameters considered here then initially the buoyancy-momentum lengthscale is much larger than the volume-momentum lengthscale, suggesting that the buoyancy and momentum fluxes are initially more important than the volume flux. It is therefore anticipated that the maximum upslope distance will scale on the so-called “jet length”, $(M_0^3/F_0^2)^{1/4}$, based on the source fluxes of momentum and buoyancy.
5.2. PLUME MODEL

Figure 5.2: Graphs of the plume model predictions for the maximum upslope extent of the plume against the jet length for slopes of a) 10° and b) 15°. x mark the values from the plume model and the solid line gives the best fit line to the data.
Figure 5.3: Graphs of the plume model predictions for the maximum upslope extent of the plume against the jet length predictions for slopes of 5°-20°. x mark the values from the plume model and the solid line gives the best fit line to the data.

For a vertical plume this is found to be the case both experimentally and in the forced plume model of Morton (1959). For a forced plume on a slope the model presented here is more complicated and it is not immediately obvious that the same scaling will apply. Figure 5.2 shows the maximum upslope extent predicted by the plume model plotted against the jet length. Slopes of 10° and 15° are used. The graphs show that the plume model predictions do have a linear relation with the jet length, although there is an offset from the origin. This offset could suggest that a virtual origin correction is needed to take into account the finite volume flux at the source and make the relationship linear with no offset. This kind of correction is discussed by Morton (1959). The constant of proportionality relating the model predictions and the jet length is seen to be a function of angle.

In the case of a sloping plume, the appropriate buoyancy flux to use for the jet length may be \( F_0 = S_l h_{\text{bug}} \sin \theta \) since only the \( \sin \theta \) component of gravity is working against the plume. This gives a jet length

\[
I_{MF} = \frac{Q_0}{\frac{\pi^{3/4}}{4} \frac{r_0^{3/2}}{8} \frac{1}{\theta^{1/2}} \sin^{1/2} \theta}.
\]

This suggests that the data from figure 5.2 should collapse when rescaled by \( \sin^{1/2} \theta \). However,
we find that a better collapse is obtained by using a factor of $\sin^{-1/4} \theta$ rather than $\sin^{-1/2} \theta$ in the jet length. Figure 5.3 shows the plume model predictions plotted against this slope-corrected jet length for slopes of $5^\circ$-$20^\circ$. The collapse is seen to be very good. In their work on the related problem of forced angled plumes in the absence of a slope, Lane-Serff et al. (1993) also found that the simple geometric correction to the jet length of $\sin^{1/2} \theta$ did not produce the best collapse because it did not take into account the conservation of horizontal momentum flux.

The dependency of the model predictions on the entrainment coefficient, $\alpha$, and the critical value of the Richardson number, $Ri_c$, is examined in figure 5.4. It can be seen that increasing the critical Richardson number from 0.25 to 1.0 results in a small increase in the maximum distance upslope that the plume travels since the plume takes longer to adjust to the constant Richardson state. Decreasing the entrainment from 0.1 (the value typical of a plume) to 0.05 (the value typical for a jet) has a much larger effect on the plume. With the lower entrainment rate the plume travels about 50% further upslope.
5.3 Experimental work

Experiments were carried out in the large tank described in §3.2. A small nozzle of diameter 5.75mm was placed near the bottom of the slope, pointing up the slope and parallel to it. The nozzle was of the form of a circular metal tube protruding from a support. The bottom of the source was 10mm above the slope. Salty water, with dye added for visualisation, was fed into the nozzle at a constant flow rate from a constant head tank. The flow rate was measured using a “gap meter” flow rate meter. The meter scale was calibrated for each solution used as the calibration is sensitive to the viscosity of the fluid.

Reduced gravities in the range 11.05cm s$^{-2}$ to 32.95cm s$^{-2}$ were used in conjunction with slopes of 10° and 15°. The flow rate was controlled to give a range of values from 8.0cm$^3$s$^{-1}$ to 24.4cm$^3$s$^{-1}$. This corresponded to buoyancy fluxes in the range 92cm$^4$s$^{-3}$ to 575cm$^4$s$^{-3}$. For all the experiments carried out the initial Richardson number, $Ri$, is in the range $1.14 \times 10^{-3} - 9.01 \times 10^{-3}$. The details of the experiments carried out are given in table 5.3.

The experiments were recorded on video for later analysis using DigImage, as described in §3.4. This allowed the shape and extent of the plume to be measured. Information about the depth and dilution of the plume could be inferred from the light intensity, using the techniques in §3.5.

For some experiments the dye was injected into the flow after the plume had become established. This was used to measure the speed of the flow in the plume, as discussed in §3.6. In other experiments a conductivity probe was used to infer the mean effective gravity in the plume at various positions and heights. This technique also measured the depth of the plume. The Richardson number at various parts of the flow could be calculated using the speed measurements from tracking the dye, and the depth and effective gravity measurements from the conductivity probe.

Some qualitative observations from the flow will be discussed before going on to make more detailed comparison with the plume model in the next section. Figure 5.5 shows a typical plume from the experiments. Previous experimental and theoretical work on a vertical plume in the absence of a wall showed that the width increases linearly with distance from the source. Near the source this appears to be the case. However, at larger distances from the source the plume appears to increase in width with distance at a rate greater than linear. Once the plume reaches its maximum upslope distance, there is a large cross slope spreading of the fluid before it drains away down the slope. The top region of the flow no longer looks much like a plume. Fluid can be seen to drain away down the sides of the plume.

The experiments in which dye was injected to track the velocity of the front allowed the
### 5.3. EXPERIMENTAL WORK

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Table 5.1: Details of initial conditions for experiments with plumes on a slope.

Flow inside the plume to be visualised. Figure 5.6 shows a series of three images from one of these experiments. It can be seen that near the source (figure 5.6(a)) the fluid looks like a plume, with the flow being predominantly upslope, with some across slope spreading. The plume becomes diluted as ambient fluid is entrained in. Further up, the across slope spreading becomes greater as gravity slows the flow down and a large, relatively slow moving region forms at the top of the plume. This is shown in figure 5.6(b). The flow eventually begins to move back down the slope. From figure 5.6(c) it can been seen that this flow is down the sides of the upward plume. The diagram in figure 5.7 sketches the flow in the plume. Almost none of the dense fluid flows back over the top of the upwards plume. This means that, from above, the flow appears two-dimensional. This has important consequences for
Figure 5.5: A typical plume from an experiment showing the spreading of the plume, the maximum height reached and the draining away of the fluid. The image is corrected for the background lighting and shown in false colour. Different colours correspond to different light intensities. Regions where the fluid is dense or deep are shown in pink or blue and regions where there is no dense fluid are white. The black lines shows the predicted edge of the plume using the plume model with $\alpha = 0.05$ and no drag and with $\alpha = 0.03$ and no drag. The plume reaches further upslope with reduced entrainment.

Since the plume is relatively shallow compared to it’s width, the top area is much greater than the side area. It would therefore be expected that the bulk of the fluid entrained into the plume would be through the top rather than through the sides. The upward plume is therefore mainly entraining ambient fluid from above, rather than denser fluid for the downwards part of the plume.

5.4 Comparison of theory and experiments

The maximum upslope distance reached can be compared to the jet lengthscale and the plume model predictions. These comparisons are shown in figures 5.8 and 5.9.

In figure 5.8 the experimentally measured maximum upslope extent is plotted against the jet length divided by $\sin^{1/4} \theta$. This rescaled jet length was found to give a good collapse of the plume model predictions for various slopes. It can be seen that the experimentally measured
5.4. COMPARISON OF THEORY AND EXPERIMENTS

Figure 5.6: A series of images of a plume. Dye was injected over a short period of time so the motion of the dye can be used to track the motion of the fluid in the plume. The images are from experiment 11.

upslope distance scales on the jet length for a given slope. If the jet length is divided by the factor of $\sin^{1/4} \theta$, then the experimental data collapses quite well for slopes of 10° and 15°. The best fit line gives a relationship between the experimental maximum upslope distance, $x_{\text{max}}$, and the jet length, $l_{\text{MF}}$ of

$$x_{\text{max}} = 2.46 + 4.09 \frac{l_{\text{MF}}}{\sin^{1/4} \theta},$$  

(5.15)
Figure 5.7: Sketch showing the upward and downward parts of the flow in a forced plume directed up a slope.

Figure 5.8: Graph of the experimental results for the maximum upslope extent of the plume against jet length / $\sin^{1/4} \theta$. Results are plotted for slopes of 10° ($\times$) and 15° (+). The solid line gives the best fit line to the data.
Figure 5.9: Graph of the experimental results for the maximum upslope extent of the plume against the plume model predictions. Results are plotted for slopes of $10^\circ$ (×) and $15^\circ$ (+). The solid line gives the best fit line to the data.

where

$$l_{MF} = \left( \frac{M_0^3}{F_0} \right)^{1/4} = \left( \frac{\pi r_0^2 u_0^4}{g^2} \right)^{1/4}. \quad (5.16)$$

Figure 5.9 shows that the plume model predictions for the maximum upslope distance with entrainment coefficients $\alpha = 0.05$ are slightly smaller than the experimental values, but they appear to be linearly related. The difference is most likely to be due to the interaction of the upward and downward flows or to a failure to take into account the different behaviour near the top where the flow is no longer behaving like a plume and viscous forces may be important. Since only the bottom half of the current can be described as a distinct plume, it might be expected that the plume model would not accurately describe all of the flow. Predictions of the plume model show that the Reynolds number of the flow remains high (at least a few hundred) until right at the top of the plume. This suggests that the discrepancies between the model and the experimental results may not be due entirely to viscous effects.

Rather than comparing the maximum upslope distance reached by the plume we can consider the maximum vertical rise height. This allows easier comparison with the previous work of Morton (1959) and Turner (1966) on vertical plumes. The maximum upslope distance
can be converted to the maximum rise height by multiplying both sides of (5.15) by \(\sin \theta\). Figure 5.10 shows the maximum vertical rise height plotted against the jet length multiplied by \(\sin^{3/4} \theta\) \((= \sin^{-1/4} \theta \sin \theta)\). The experimental data is seen to collapse well. The best fit line gives an expression

\[
h_{\text{max}} = 0.208 + 4.18 l_{MF} \sin^{3/4} \theta
\]

for the maximum vertical rise height \(h_{\text{max}}\). In the experiments the plume has a source of finite area and volume flux, whereas the scaling is for an idealised plume with no zero volume flux. This may lead to the zero offset in (5.17). Figure 5.11 shows the vertical rise height divided by jet length as a function of slope. The experimental data from slopes of 10° and 15° is shown. Also shown are the theoretical value of 1.73 for a vertical forced plume from Morton (1959), the experimental value of 1.85 for a vertical forced plume from Turner (1966) and the limit of zero rise height as the slope tends to zero. It can be seen that the maximum rise height in jet lengths increases steeply from 0° to 15°. The increase is much greater than for the unconstrained tilted plume studied by Lane-Serff et al. (1993), where the rise height at 15° is between 0.25 and 0.3 jet lengths. In the present experiments the plume is constrained to remain on the slope, whereas for a tilted free plume the plume can bend, reducing its rise height. The scatter in the data is partly due to experimental errors and partly to the zero offset in (5.15).

While there are no data for slopes between 15° and 90° we might expect that the ratio of vertical rise height to jet length will not vary much in this region. The buoyancy may act to stabilise a tilted plume and reduce the entrainment so increasing the rise height. To counteract this the slope will exert some drag on the plume, reducing the plume height. The plume on a slope is unlikely to reach much higher than the value for a free vertical plume. Similarly, it would be expected that the drag of the slope would be less for larger slopes so the ratio of rise height to jet length is unlikely to become smaller than the value at 15°. This suggests that over a fairly large range of moderate to steep slopes the vertical rise height will not vary much with slope, for a given jet length. It also suggests that further experiments in this intermediate regime are needed. For small slopes, up to about 15°, the ratio will increase rapidly with slope. The scaling expression in (5.17) seems to over-predict the rise height in the limit \(\theta \rightarrow 90°\). The dependence on \(\theta\) cannot be accurately obtained from experiments on only two different slopes.

More detailed comparisons between the experimental plumes and the model predictions were made. Figure 5.12 shows cross sections of the plume at distances of 1/6 and 1/3 the maximum upslope extent of the plume. The cross sections were obtained from the video recordings of experiments 7-9. Using the DigImage system, a time-averaged image of each experiment was made. The images were corrected so that the intensity is proportional to the depth inte-
5.4. COMPARISON OF THEORY AND EXPERIMENTS

Figure 5.10: Graph of the maximum vertical rise height of the plume against the jet length. Experimental results are plotted for slopes of 10° and 15°.

Figure 5.11: Graph of the maximum vertical rise height of the plume divided by the jet length. Experimental results are plotted for slopes of 10° and 15°. Also plotted are the value for a horizontal plume and the theoretical value of Morton (1959) and the experimental value of Turner (1966) for a vertical plume.
grated effective gravity in the plume. The intensities along the line of the cross section were recorded and saved to a file using DigImage. The data from these cross sections are plotted in non-dimensional form in figure 5.12. The cross slope distance is non-dimensionalised with respect to the maximum extent of the plume (which has been demonstrated to scale with the jet length). Experiments 7-9 all have the same slope of 15°, but different initial conditions. The data are seen to collapse well, confirming that the jet length is the relevant lengthscale for the plume and that the plumes are self-similar. The step-like nature of the cross sections is a result of the discretisation of the intensities by the framegrabber card and not an indication of any step-like behaviour in the experiments.

The effective gravity of the plume is plotted against the distance up the slope in figure 5.13. The values are from the experiments using the conductivity probe. The effective gravity is non-dimensionalised with the initial effective gravity and the distance upslope is non-dimensionalised with the maximum upslope extent of the plume. The solid blue line is the plume model prediction with $\alpha = 0.05$ and the solid red line is the plume model prediction with $\alpha = 0.03$. It can be seen that for the slopes of 10° and 15° either of these provides a fair estimate of the effective gravity of the plume, although the stretched line appears to give a better fit further up the slope.

The depths determined using the conductivity probe are plotted against the upslope distance in figure 5.14. Again the blue line is the model prediction with $\alpha = 0.05$ and the red line is the prediction with $\alpha = 0.03$. Both lengths are non-dimensionalised on the maximum upslope distance. It can be seen from the experimental results that there is some uncertainty in the measurements but that the depth appears to be relatively constant over a large proportion of the plume. The plume model gives a sharper increase then decrease in the plume depth than is observed in the experiments. The accuracy of the experimental results is not sufficient to be more definite about the actual trend. Most of the data points appear to collapse, apart from one point on the graph for 15° which is significantly higher than the rest.

Effective gravity (or concentration) profiles obtained from the conductivity probe measurements in experiment 10 are shown in figure 5.15. The profiles are at different distances up the slope ranging from 0.11 to 0.43 of the maximum upslope extent of the plume. It can be seen that the depth of the plume (as measured by the height at which the effective gravity drops to zero) does not vary greatly over the length of the plume. This is in agreement with the results in figure 5.14. The effective gravity (or equivalently the density) within the plume is seen to decrease from a maximum near the slope to zero at the top of the plume. The maximum value of the effective gravity decreases with distance upslope since the plume is entraining ambient fluid and becoming more dilute. For the profile nearest the source it can be seen that the maximum value actually occurs just above the slope. This is because the source
Figure 5.12: Intensity cross sections of plumes at distances of 1/6 and 1/3 the maximum height. The across slope distance is non-dimensionalised with respect to the maximum upslope extent. The intensity is proportional to the depth integrated effective gravity. The results are from experiments 7-9 in table 5.3.
Figure 5.13: Graphs of the non-dimensional effective gravity of the plume against the distance upslope for slopes of $10^\circ$ and $15^\circ$. The x are the experimental data and the solid lines are the plume model predictions for $\alpha = 0.05$ (blue) and $\alpha = 0.03$ (red). For both lines $Ri = 0.25$ and there is no drag. The distance is non-dimensionalised with respect to the maximum upslope extent of the plume and the density with respect to the initial density of the plume.
5.4. COMPARISON OF THEORY AND EXPERIMENTS

Figure 5.14: Graphs of the depth of the plume against the distance upslope for slopes of 10° and 15°. The x are the experimental data and the solid lines are the plume model predictions for $\alpha = 0.05$ (blue) and $\alpha = 0.03$ (red). For both lines $Ri = 0.25$ and there is no drag. The distances are non-dimensionalised with respect to the maximum upslope extent of the plume.
Figure 5.15: Effective gravity profiles at different distances upslope during the same experiment. The maximum extent of the plume in this experiment was 94 cm. The effective gravity is non dimensionalised with respect its value at the source. The results are for experiment 10.

nozzle is not quite on the slope so until the plume develops and grounds the maximum density will be just above the slope too.

Figure 5.16 shows the Richardson number plotted against the distance upslope for slopes of 10° and 15°. It can be seen from figure 5.15 that the density profile in the plume is not constant, but is closer to linear. The reduced gravity used in calculating the Richardson number was taken as half the peak value. This gives a better approximation to the depth averaged reduced gravity in the plume. The Richardson number is a derived quantity and depends on the depth, reduced gravity and speed of the plume so there is some uncertainty in the experimental results. The results are not inconsistent with an adjustment to a constant Richardson number of 0.25. This value also provides fair agreement between other measurements of the experiments and the model predictions.

In the region where a distinct plume is visible, the plume model solution with $\alpha = 0.03$ given in figure 5.5 shows a good agreement with the measured plume width as a function of distance up slope. Above this region the current can no longer be considered as a distinct plume since dense fluid flowing back down the slope is interacting with the fluid flowing up. The value of 0.03 for the entrainment is low compared to the values measured for vertical jets
5.4. COMPARISON OF THEORY AND EXPERIMENTS

Figure 5.16: Graphs of the Richardson number of the plume against the distance upslope for slopes of 10° and 15°. The x are the experimental data and the solid lines are the plume model predictions for $\alpha = 0.05$ (blue) and $\alpha = 0.03$ (red). For both lines $Ri = 0.25$ and there is no drag. The distance is non-dimensionalised with respect to the maximum upslope extent of the plume.
and plumes. The stabilising effect of the horizontal density interface may act to reduce the entrainment compared to the vertical case.

5.5 The viscous regime

If the Reynolds number of the flow becomes small at any point in the flow then viscous forces will begin to play an important role in the bulk motion of the fluid. In practice, this means if the flow becomes either very thin or very slow moving. In the laboratory experiments it was observed that the fluid in the downward flowing region formed a thin layer and was only slowly moving. It seems likely that viscous forces are playing a role in this region.

For a gravity current on a slope where viscous forces are much larger than inertial forces, the current can be modelled using the standard equations of lubrication theory. The draining region can be considered as a viscous flow from a distributed source (effectively a line source of finite length). It is of interest to see how the flow from such a source compares with the flow from a point source, as described by Smith (1973) and Lister (1992) and discussed in §2.5. The presence of the upward plume in the experiments inhibits the spread of fluid back into the central region around the source, providing a difference from the case of both the point and line source.

The position of the outer edge of the draining region was measured from the video recordings of the experiments. Using DigImage, a threshold intensity contour was plotted on the time averaged image of the plume. The intensity was selected to correspond to the edge dividing the downward plume and the ambient fluid. Figure 5.17 shows the plume. The x-y position of the contour was saved and plotted on a log–log graph as shown in figure 5.18. The upper solid line is a power-law best fit to the data. The lower solid line is the predicted shape for a point source as described in §2.5. This is calculated by using the plume model in §5.2 to give a flow rate and effective gravity at the top of the plume and then using these as the initial conditions for the viscous draining solution. The power law best fit from figure 5.18 was also plotted on figure 5.17 using DigImage to allow comparison with the experimental results.

It can be seen by comparing the curves in figure 5.18 that the experimental draining region is significantly bigger than that predicted for a point source. The distributed nature of the effective source of the draining region might be expected to lead to a wider draining region. It may also be the case that the fluxes at the top of the slope given by the plume model are not correct, giving the wrong initial conditions for the draining region.

The best-fit line to the experimental data has a slope of 0.43, within 1% of the 3/7 predicted by the similarity solution, suggesting that a similar process may be happening here as for the point source, but with a suitable correction to the initial conditions. If the draining
5.5. THE VISCOUS REGIME

Figure 5.17: Image of experiment 9 with the curve $y = 9.4x^{0.43}$ plotted on top. This provided the best power law fit to outer edge of the plume.

Figure 5.18: Graph showing the edge of the draining region on a log–log plot. The results are from experiment 9.
region was turbulent, with a high enough Reynolds number that viscous forces could be neglected, then the current would be expected to behave similarly to the model of Bonnecaze & Lister (1999) described in §2.3.3 where \( y \propto x^{3/8} \) if entrainment is neglected, and \( y \propto x \) including the effects of entrainment. The experimental data clearly does not agree with either of these power laws suggesting that, at least in the laboratory experiments, viscous forces are important in the downward region and can be modelled using lubrication theory. For large-scale atmospheric flows, such as the spread of a dense gas, atmospheric turbulence will ensure that the flow remains at a high enough Reynolds number that viscous forces will not play a dominant role.

5.6 Tilted plumes

5.6.1 A model for tilted plumes

In practice, a continuous release of dense gas may be initially directed at any angle relative to the upslope direction. The model in §5.2 for plumes directed upslope can be extended to plumes initially directed at an angle to the upslope direction, as shown in figure 5.19. Let \( x \) and \( y \) be the upslope and across coordinates, with velocities \( u \) and \( v \) in those directions. The speed \( w \) is such that \( w^2 = u^2 + v^2 \). The distance along the plume centreline is measured by \( s \). The angle the plume makes with the \( x \) axis is \( \phi \) so \( u = w \cos \phi \) and \( v = w \sin \phi \). The depth of the plume, \( h \), and the width from the centreline to the edge of the plume, \( b \), are defined as for the previous model. The assumption that the plume is self-similar is kept here. In addition, there is an assumption that the radius of curvature of the plume is large compared to the plume width, so that the curvature does not significantly affect the perimeter of the plume or give rise to the plume interacting with itself.

The development of the plume can be described by a system of equations similar to (5.1)–
The distance along the plume centreline, $s$, takes the place of the distance upslope, $x$. The conservation of volume equation is effectively the same, but expressed in terms of the centreline distance, $s$, rather than the upslope distance, $x$, so

$$\frac{d(S_1 h bw)}{ds} = S_3 \alpha w (h + b). \tag{5.18}$$

The volume flux of the plume changes only as a result of fluid being entrained in at the perimeter of the plume. There are now two momentum equations, one in the upslope direction and one in the across slope direction, rather than the one equation in §5.2. In the across slope direction

$$\frac{d(S_1 h bv w)}{ds} = -2C_D b v w - \frac{1}{2} \frac{d}{ds} \left( S_2 g' h^2 b \cos \theta \sin \phi \right), \tag{5.19}$$

so the total $y$ momentum is changed only as a result of the drag on the plume and the horizontal across slope pressure gradient. In the upslope direction there is an extra term for the buoyancy force acting downslope due to gravity so

$$\frac{d(S_1 h bu w)}{ds} = -S_1 h bg' \sin \theta - 2C_D b u w - \frac{1}{2} \frac{d}{ds} \left( S_2 g' h^2 b \cos \theta \cos \phi \right). \tag{5.20}$$

For flow where the ambient fluid is unstratified, buoyancy is conserved in the plume giving

$$\frac{d(S_1 h bg' w)}{ds} = 0. \tag{5.21}$$

For a tilted plume the Richardson number, $Ri$, is

$$Ri = \frac{g' h \cos \theta}{w^2}. \tag{5.22}$$

The system of equations (5.18)-(5.21) can again be expressed in terms of the variables $h$, $b$ and $Ri$ with the addition of an equation for $\phi$. Changing to these variables gives the equations

$$\frac{dh}{ds} = \frac{S_3 \alpha}{S_1} \left( 1 + \frac{h}{b} \right) - 2h \frac{db}{3b \frac{ds}{ds}} + \frac{h}{3 Ri} \frac{dRi}{ds}, \tag{5.23}$$

$$\frac{h}{3 Ri} \frac{dRi}{ds} \left( 1 - \frac{S_2}{S_1} Ri \right) = \frac{Ri \tan \theta \cos \phi + 2C_D}{S_1} + \frac{S_3 \alpha}{S_1} \left( 1 + \frac{h}{b} \right) \left( 1 + \frac{S_2}{2S_1} Ri \right) - \frac{h \frac{db}{3b \frac{ds}{ds}}}{1 + \frac{S_2}{2S_1} Ri} \tag{5.24}$$
and
\[ h \frac{d\phi}{ds} \left( 1 + \frac{S_2}{2S_1} \right) = \frac{Ri \tan \theta \sin \phi}{ds} \]  \(5.25\)

Although there is now an additional equation compared to the model in §5.2, there is also a new variable, \(\phi\), so there is still the need for another equation to close the problem. As in §5.2, the condition that \(h = b\) is used until the Richardson number has reached some value \(Ri_c\). After that the condition that the Richardson number is kept constant is used so \(Ri = Ri_c\).

Initially, where \(h = b\), (5.23) and (5.24) become
\[ \frac{2h}{5Ri} \frac{dRi}{ds} \left( 1 - \frac{S_2}{S_1} \right) = \frac{Ri \tan \theta \cos \phi}{2Ri} + \frac{2C_D}{S_1} \left( 1 + \frac{S_2}{2S_1} \right) \]  \(5.26\)
and
\[ \frac{dh}{ds} \left( 1 - \frac{S_2}{S_1} \right) = \frac{1}{2Ri} \tan \theta \cos \phi + \frac{C_D}{S_1} + \frac{2S_3 \alpha}{S_1} \left( 1 - \frac{2S_2}{5S_1} \right) \]  \(5.27\)

In contrast, once the Richardson number has reached a constant value \(Ri_c\) (5.23) and (5.24) become
\[ \frac{h}{3b} \frac{db}{ds} \left( 1 + \frac{S_2}{2S_1} \right) = \frac{S_3 \alpha}{S_1} \left( 1 + \frac{h}{b} \right) \left( 1 + \frac{S_2}{2S_1} \right) + \frac{Ri_c \tan \theta \cos \phi}{2Ri} + \frac{2C_D}{S_1} \]  \(5.28\)
and
\[ \frac{dh}{ds} \left( 1 + \frac{S_2}{2S_1} \right) = -\frac{S_3 \alpha}{S_1} \left( 1 + \frac{h}{b} \right) \left( 1 + \frac{S_2}{2S_1} \right) - \frac{2Ri_c \tan \theta \cos \phi}{4C_D} - \frac{4C_D}{S_1} \]  \(5.29\)

Given initial conditions for \(h\), \(Ri\) and \(\phi\), the system of equations (5.25)-(5.29) can be numerically integrated. The solution gives the plume variables as a function of the distance along the plume centreline, \(s\). In order to calculate the position of the plume it is also necessary to integrate the equations
\[ \frac{dx}{ds} = \cos \phi \]  \(5.30\)
and
\[ \frac{dy}{ds} = \sin \phi \]  \(5.31\)
to give the plume trajectory.

Two sample solutions are shown in figure 5.20 with \(\phi = 10^\circ\) and \(\phi = 45^\circ\). The slope is 15\(^\circ\), the initial reduced gravity is 11.05 cm s\(^{-1}\) and the initial flow rate is 19.01 cm\(^3\) s\(^{-1}\). These conditions correspond to experiment 7 for a plume directed upslope (see table 5.3) and are typical of the values used in the laboratory experiments described in the next section. It can be seen that for values of \(\phi\) close to 0\(^\circ\) the prediction is that the edges of the plume cross over
5.6. TILTED PLUMES

Figure 5.20: Trajectories for two plumes initial directed at angles of a) $\phi = 10^\circ$ and b) $\phi = 45^\circ$ to the across slope direction. The slope was 15°, the initial reduced gravity is 11.05 cm s$^{-2}$ and the initial flow rate was 19.01 cm$^3$ s$^{-1}$. Each figure shows the plume centreline position and the plume edges.
each other, and thus the solution cannot be a physically viable one. This interaction between
the different parts of the plume needs to be taken into account. For larger values of $\phi$ the edges
of the plume follow a smooth trajectory. In some sense this makes it an easier problem than
that in §5.2 for a plume directed straight up the slope, as there are no interactions between
different parts of the plume to consider.

From figure 5.20 it can be seen that far from the source the across slope component of the
flow dies away and the plume moves downslope under the action of gravity. The distance over
which this occurs depends on the value of the drag parameter, $C_D$, which controls how quickly
the horizontal momentum of the plume is dissipated. However, even with $C_D = 0$ the flow
rapidly adjusts to become predominantly downslope. The effect of the initial conditions and
direction become less important far from the source, as would be expected. If only predictions
in the far field are required then it may be reasonable to ignore these source effects altogether
or to just calculate a virtual origin correction to take them into account, without considering
all the details.

The model is more complicated than the simple vertical plume models of, for example,
Ellison & Turner (1959), however there are still simple scalings underlying them. On dimen-
sional grounds it would still be expected that the plume would scale on the jet length

$$l_{MF} = (M_0^4/F_0^2)^{1/4} = \frac{Q_0}{\pi^{3/4} r_0^{3/2} \delta_0^{1/2}},$$

as for a plume directed upslope. Unfortunately, the dependence on the angle of the slope, $\theta$,
and the angle of the plume to the slope, $\phi$, cannot be found from dimensional arguments.

For the maximum upslope extent of the plume it can be supposed that the angular de-
pendence is of the form $\sin^\beta \theta \cos^\gamma \phi$, since this is the component of the fluxes in the ups-
lopes direction. Solutions of the model were found for slopes in the range $\theta = 5^\circ - 40^\circ$ and
$\phi = 30^\circ - 60^\circ$. The jet lengths used were in the range 4.5 cm – 14.1 cm, which corresponds to
initial conditions similar to those in the experiments described below.

A good collapse of the plume model predictions for the maximum centreline extent is ob-
tained with $\beta = -1/4$ (as for the plume directed up the slope) and $\gamma = 5/4$. Figure 5.21 shows
the maximum upslope extent of the plume centreline against the scaling $l_{MF} \sin^{-1/4} \theta \cos^{5/4} \phi$.
The data can be seen to collapse well for this range of parameters. The best fit line gives the
maximum upslope extent, $x_{max}$, as

$$x_{max} = -0.76 + 1.60 l_{MF} \frac{\cos^{1/4} \phi}{\sin^{1/4} \theta} \cos \phi,$$

The distance across slope that the plume travels is expected to scale on the distance the
plume travels upslope, with a factor of $\tan\phi$ to take into account the angle the plume makes with the y-axis. Figure 5.22 shows the maximum across slope extent of the plume centreline against the scaling $l_{MF} \sin^{-1/4} \theta \cos^{5/4} \phi$ in (5.34). Again the collapse is good over the range of parameters studied. The best fit line gives

$$y_{\text{max}} = -0.56 + 2.81 \frac{Q_0}{\pi^{3/4} \rho^{3/2} \sigma_0^{1/2} \sin^{1/4} \theta} \cos^{1/4} \phi \sin \phi$$

(5.34)

for the maximum cross slope extent, $y_{\text{max}}$.

The results are plotted with an entrainment coefficient $\alpha = 0.1$, which is the typical value found in a plume. Using a smaller value of $\alpha = 0.05$, which is more typical of a jet, reduces the dilution of the plume and leads to it travelling further in both the upslope and across slope directions by a factor of about 1.6. This is consistent with the previous findings for a plume directed upslope.

The value taken for the constant Richardson number, $\text{Ri}_c$, to which the flow adjusts is not derived theoretically. The value of 0.25 is consistent with the findings described in this chapter for experiments on plumes directed upslope. Figure 5.23 shows the model predictions for three different values of the Richardson number. Increasing the Richardson number increases the
across slope distance from the centreline to the source. This is because the plume initially takes longer to adjust to the larger value and so travels further in the cross slope direction. The value chosen for the Richardson number has a much bigger effect on the plume width. A value of 0.25 results in the plume spreading out greatly as it moves downslope, while a value of 1.0 gives the plume pinching in again downslope of the source. The intermediate value of 0.5 gives the most realistic looking shape, with the width increasing as the plume moves downslope, but at a slower rate than for $R_i = 0.25$. This strong dependence on the constant value chosen for the Richardson number is in contrast to the relative insensitivity to $R_i$ of the model for a plume directed straight up the slope, as illustrated by figure 5.4. The reason for this lies in the fact that for a plume directed upslope the plume model breaks down when the velocity becomes zero and the width becomes infinite. This means there is only a relatively small region where the Richardson number is kept constant, so the actual value chosen makes only a small difference. In contrast, for a tilted plume most of the trajectory is in the constant Richardson number regime, so the value of the constant has a much bigger effect.

Figure 5.22: Graph of jet length against maximum across-slope centreline position from the plume model.
5.6. TILTED PLUMES

Figure 5.23: Plume model predictions for experiment 8 with $\text{Ri}_c = 0.25$ (blue), $\text{Ri}_c = 0.5$ (red) and $\text{Ri}_c = 1.0$ (green). The centreline and the edges of the plume are shown for each value. The centreline line is thicker than the line for the edges. The slope is upwards from the left to the right of the page.

5.6.2 Experiments with tilted plumes

Laboratory experiments were carried out for plumes where $\phi$ is non-zero i.e. the plume is initially directed at an angle to the upslope. Angles of 45° and 60° were investigated. Details of the experiments carried out are given in table 5.2.

As for the case with a plume fired directly upslope (see §5.4), and for the tilted plume model in §5.6.1, the lengthscale of the problem is the “jet length”. Figure 5.26 shows the maximum upslope extent of the current against the jet length. The dependence on the angles $\theta$ and $\phi$ is the same as that found for the tilted plume model. The results are shown for two different slopes, $\theta = 10^\circ$ and $15^\circ$, and for three different angles of tilt, $\phi = 0^\circ$, $30^\circ$ and $45^\circ$, as well as for a variety of initial plume conditions. The data collapses well over the range of experiments. The best fit to the experimental data gives an expression

$$x_{\text{max}} = 7.36 + 3.84 l_M \cos^{5/4} \phi / \sin^{1/4} \theta$$  \hspace{1cm} (5.35)

for the maximum upslope distance.
<table>
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<th>Experiment</th>
<th>$\theta/^{\circ}$</th>
<th>$\phi/^{\circ}$</th>
<th>$g / \text{cm s}^{-2}$</th>
<th>$Q / \text{cm}^3 \text{s}^{-1}$</th>
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Table 5.2: Details of experiments with tilted plumes.

Figure 5.24: Experimental view of the plume in experiment 9, scaled on the jet length, with predictions of the plume model superimposed. $R_i = 0.25$, $\alpha = 0.05$ and $C_D = 0.0$ for the model results.
5.6. TILTED PLUMES

As for the plume directed up the slope, we can also compare the maximum vertical rise height. Figure 5.28 shows the maximum rise height plotted against the jet length times $\cos^{5/4} \phi \sin^{3/4} \theta$. This rescaled jet length is obtained by multiplying (5.35) by $\sin \theta$. The data collapses reasonably for a range of initial conditions. The best fit line to the data gives a maximum rise height

$$h_{max} = 1.22 + 3.94 l_{MF} \cos^{5/4} \phi \sin^{3/4} \theta.$$ (5.36)

Figure 5.29 shows the maximum rise height divided by the jet length as a function of the slope. In addition to the experimental results for 10° and 15°, the theoretical value of Morton (1959) and the experimental value of Turner (1966) for vertical plumes are shown. The scatter in the data is large. This is partly due to the error in the experimental data, but is also due to the zero offset in the linear relation between the height and the jet length.

While the plume model qualitatively predicts the features observed in the flow it does not provide precise quantitative predictions, as can be seen in figures 5.24 and 5.25. The centreline position is well predicted for the initial plume part of the flow, where the slope is relatively unimportant, however the model predicts that the plume will be bent towards the downslope direction more rapidly than appears to happen in the experiments. The upslope
edge is predicted fairly well, although the agreement is not exact. In general, the model appears to over-predict the horizontal spreading of the plume during the constant Richardson number phase of the flow. The model predictions for the downslope edge of the plume are not accurate since that part of the flow is not behaving as a plume. Fluid is being detrained as a result of gravity as well as being entrained by the turbulence. The plume model includes no mechanism for detraining. It may also be that viscous forces are important near the edges of the plume and again this is not included in the model.

The assumption of constant Richardson number, used to close the problem, has not been rigourously verified. It may be that some other condition is more appropriate to provide a link between the plume depth and width. The model assumes that the curvature in the plume has no effect. It can be seen in the experiments that the curvature is large enough to make a significant difference to the inside and outside perimeter of the plume in plan form, which will have an effect of the plume depth and entrainment. On a related note, the model assumes that there is no interaction between the upslope and downslope parts of the plume. For values of $\phi$ close to $0^\circ$, the model breaks down when the plume crosses itself. Even if the plume does not cross, there may be some re-entrainment of dense fluid. One further effect which is present

Figure 5.26: Graph of maximum upslope extent from experiments against jet length $\times \cos^{3/4} \phi / \sin^{1/4} \theta$. Data points are for slopes of $\theta = 10^\circ$ (blue) and $15^\circ$ (red), angles of tilt $\phi = 45^\circ(+), 30^\circ(+) \text{ and } 0^\circ(\times)$, and various initial conditions for the plume.
5.6. TILTED PLUMES

Figure 5.27: Graphs of the plume model predictions for the maximum upslope extent of the plume against the experimental results for slopes of 10° and 15°. The + and upper line are the results and best fit line for the predicted maximum upslope extent of the plume from the plume model. The × and lower line are the maximum upslope extent of the centreline.
Figure 5.28: Graph of the maximum vertical rise height of the plume against the jet length. Experimental results are plotted for slopes of $10^\circ$ and $15^\circ$.

Figure 5.29: Graph of the maximum vertical rise height of the plume divided by the jet length. Experimental results are plotted for slopes of $10^\circ$ and $15^\circ$. Also plotted are the limit of no slope, the theoretical value of Morton (1959) and the experimental value of Turner (1966).
in the tilted plume experiments, but not in the upslope plumes is the detrainment of dense fluid from the downslope edge of the plume. This can be best observed near the source. In figure 5.24 dense fluid can be seen draining down the slope on the opposite side of the source from the direction the plume is pointing in.

5.7 Summary

In this chapter the problem of a dense forced plume fired upslope, or at an angle to the slope, has been considered.

For a plume directed upslope a simple plume model has been presented and laboratory experiments have been carried out. It has been found that both the model predictions and the experimental results scale on the jet length

\[ l_{MF} = (M_0^3/F_0^2)^{1/4}, \]  

although the constant of proportionality depends on the slope. Multiplying the jet length by \( \sin^{-1/4} \theta \) gave a good collapse to the experimental and plume model data over a range of slopes. The plume model qualitatively predicts the features observed in the experiments. The quantitative agreement between experiments and the plume model is fair, although the model tends to under predict the extent to which the plume will spread up the slope. In the experiments both viscous effects and the interaction of the upward and downward flows can play a role and neither is included in the model. The flow near the top can no longer be realistically considered as a plume and the model breaks down in this region.

Further experiments were carried out with plume tilted at an angle to the upslope direction. The plume model was generalised to this situation. Again the experimental and model results were found to scale on the jet length. Multiplying the jet length by \( \cos^{5/4} \phi \sin^{-1/4} \theta \) gave a good collapse of the experimental and the plume model data for a range of slopes and angles of tilt. Quantitative agreement between the experiments and the plume model was not particularly good. The complications due to curvature and interaction between different parts of the flow are not included in this model.

The experimental results in this chapter show that this is a rich and complicated problem. The underlying scalings are simple, but to successfully model such flows it is necessary to include the interaction between the upward and downward parts of the flow. How this should be achieved is still an open question, even for a vertical forced plume (see Baines et al., 1990; Bloomfield & Kerr, 1998, 1999). For tilted plumes the curvature of the plume centreline also needs to be taken into account. This is a problem worthy of further study.
Chapter 6

A numerical model for gravity currents with slope and wind

6.1 Problem

Industrial situations where an accidental release of dense gas can occur have been discussed in chapter 4. As noted, such releases may well occur on a slope and this can greatly alter the evolution of the gas cloud. Other factors can also affect the spreading and dilution of the gas. One particularly important factor in the environment is the ambient wind. The wind can act to slow down or speed up the spread of the current, leading to significant differences in the dose and exposure time at a given location and altering the area affected by the spillage.

As the interaction of slope and wind is a complicated problem, only the two-dimensional case is considered here. The flow can either be in a channel or axisymmetric. The relevance of a wind in the axisymmetric case will be discussed later. The problem is sketched in figure 6.1. A two-dimensional vorticity–streamfunction model for the flow is developed in §6.2. The details of the numerical scheme used to solve it are given in §6.3. Various test cases were used to validate the code. The test cases were chosen because they have been the subject of previous theoretical or experimental work. This work can be compared with the numerical model predictions. In addition, some new experimental results are used for comparison. Details of the validation are given in §6.4 and §6.5. This validation is important because, although such vorticity–streamfunction models have been used before, there has been little published work to compare the predictions with experimental results and other theoretical models. If such models are to be utilised it is necessary to understand their strengths and limitations.
6.2 Inviscid vorticity–streamfunction model

In this section a model for gravity currents on a slope and with a background wind will be developed, based on the vorticity–streamfunction formulation of the Euler equations. The equations, boundary conditions and solution methods will be described in the following sections.

6.2.1 Governing equations

Assuming that the flow is inviscid and incompressible, the Euler equations can be written in the form

\[ \rho \frac{Du}{Dt} = \rho g + \nabla p, \]  
\[ \frac{D\rho}{Dt} = 0, \]

where \( u \) is the velocity of the flow, \( \rho \) is the density of the fluid and \( g \) is the acceleration due to gravity. Since the flow is assumed inviscid there is no explicit term to include the effects of viscosity.

The Boussinesq approximation is made so that density variations are neglected, except where they appear combined with gravity. This simplifies the equations and is a reasonable approximation provided \( \Delta \rho / \rho \ll 1 \). For saline laboratory experiments this condition generally holds. It may not be true for the initial stages of a dense gas release, but once the cloud has diluted through entrainment it will become a better approximation. Under the Boussinesq approximation (6.2) becomes

\[ \frac{Dg'}{Dt} = 0, \]  

where

\[ g' = g \frac{(\rho - \rho_a)}{\rho_a} \]
is the effective gravity of the flow and \( \rho_a \) is the density of the ambient fluid.

Taking the curl of (6.1) and making the Boussinesq approximation gives

\[
\frac{D\omega}{Dt} = \nabla \times g',
\]

(6.5)

where

\[
\omega = \nabla \times u
\]

(6.6)

is the vorticity of the flow. This is useful as it eliminates the pressure term in (6.1) so reducing the number of variables. Equations (6.3) and (6.5) describe the time evolution of the flow.

If the flow is entirely two-dimensional then in Cartesian coordinates

\[
\mathbf{u} = (u, 0, v) \quad \text{and} \quad \mathbf{\omega} = (0, \omega, 0).
\]

(6.7)

Similarly for an axisymmetric flow,

\[
\mathbf{u} = (u, 0, v) \quad \text{and} \quad \mathbf{\omega} = (0, \omega, 0)
\]

(6.8)

in cylindrical polar coordinates, \((r, \phi, z)\).

It can be shown that for a two-dimensional flow there always exists a streamfunction, \(\psi(x, z)\), with

\[
\mathbf{u} = \left( -\frac{\partial \psi}{\partial z}, 0, \frac{\partial \psi}{\partial x} \right).
\]

(6.9)

Similarly for an axisymmetric flow a streamfunction, \(\Psi(r, z)\), exists with

\[
\mathbf{u} = \left( -\frac{1}{r} \frac{\partial \Psi}{\partial z}, 0, \frac{1}{r} \frac{\partial \Psi}{\partial r} \right).
\]

(6.10)

Writing the equations (6.1) and (6.2) using the streamfunction ensures the incompressibility condition, \(\nabla \cdot \mathbf{u} = 0\), is automatically satisfied, making the equations easier to solve. The velocity field can always be recovered by calculating the partial derivatives of \(\psi\).

Substituting the streamfunction in (6.6) gives

\[
\nabla^2 \psi = -\omega
\]

(6.11)

for the two-dimensional case. This is used to relate the vorticity \(\omega\) and the streamfunction, \(\psi\), in a two-dimensional channel flow. Given the vorticity, (6.11) is inverted to obtain the streamfunction, from which the velocities can be easily calculated. For an axisymmetric flow,
the relation between $\Psi$ and $\omega$ is

$$\nabla^2 \Psi - \frac{2}{r} \frac{\partial \Psi}{\partial r} = -\omega. \quad (6.12)$$

This formulation of the Euler equations is frequently used because it separates the problem into two parts. Firstly, the vorticity–streamfunction relationship is inverted to obtain the streamfunction and hence the velocity field. Secondly, the velocity is used to advect the vorticity and effective gravity. This has one fewer variables than the initial problem, since there is no need to explicitly solve for the pressure field. Solving the pressure field is often the most difficult part of computational fluid dynamics problems.

Such vorticity–streamfunction models have been used to study a variety of buoyancy-driven fluid flows such as particle-laden gravity currents (Huppert, 1998), Rayleigh–Taylor instability (Dalziel, 1998) and convection (Leppinen, 1997).

### 6.2.2 Boundary conditions

In order to simulate flow in a channel or an axisymmetric flow we assume that the top and bottom edges of the domain are solid. The ends of the domain can be either solid or open. The model equations (6.1)-(6.2) are inviscid, so the boundary condition for a solid wall is that there is no flow through the wall. The absence of any viscous terms means that the momentum equation is first order and the condition of no slip on the wall cannot be imposed as well as no flow through the wall. The boundary conditions on the vorticity and the effective gravity are that there is no flux through the solid boundaries.

For an open boundary, the streamfunction is specified on the edges of the domain in order to simulate the wind. Specifying the streamfunction gives the wind speed and profile into and out of the domain. The variables $\omega$ and $g'$ are assumed to be advected out of the domain at the downwind end. Once $\omega$ and $g'$ have left the domain they have no further influence on the flow. No $\omega$ or $g'$ is advected in at the upwind end of the domain. The cases studied have either a linear streamfunction with height (i.e. uniform wind speed) or a streamfunction quadratic with height (i.e. uniform shear flow and linear wind profile). In principle any horizontal wind profile can be used.

The assumption that $\omega$ and $g'$ are lost out of the downstream end of the domain forever is not physically realistic as it allows no information in the form of disturbances or waves back into the domain. Any such disturbances could alter the flow. In practice the assumption is a good one for simulating a homogeneous ambient fluid of infinite extent where there are no internal waves and no end walls or obstacles to generate reflections. One way to minimise the problems with the open boundary conditions is to ensure the gravity current remains well
6.3. METHOD OF SOLUTION

away from the ends. This requires a large domain, which is computationally expensive. The
imposed uniform wind speed at the boundaries of the domain is also not completely realistic.
The gravity current will not affect the background flow far upwind provided that the wind
speed is high enough. At the downwind boundary the flow may well be altered by the pres-
ence of the gravity current. It would be better to let the velocity adjust itself to whatever value
it needed to be. However, the multigrid method described in §6.3.1 needs the streamfunction
to be prescribed on the boundary in order to solve for the streamfunction in the domain. The
effect of this approximation can again be minimised by ensuring that the gravity current re-
mains well away from the ends of the domain. The effect of the domain length is examined in
§6.5.

In the atmosphere the wind profile may be more complicated than the linear shear profile
considered here. The wind speed will also be affected by the topography. The wind speed
at the summit of a hill will be higher than the wind speed before the obstacle. The changes
in wind speed as a result of the terrain are not taken into account in this work, nor are more
complicated wind profiles.

6.3 Method of solution

Equations (6.3) and (6.5), together with (6.11) or (6.12) are numerically solved in a rectangular
domain with suitably applied initial and boundary conditions. The solution consists of two
stages. First, a full multigrid method is used to solve (6.11) or (6.12) to find \( \psi \) given \( \omega \). From
\( \psi \) the velocities \( u \) and \( v \) can be calculated. In the second stage, \( \omega \) and \( g' \) are advected using
these velocities. These two stages will now be described in more detail.

6.3.1 Multigrid solution of Laplace’s equation

Given the vorticity field, it is necessary to solve (6.11) to obtain the streamfunction for the
flow. For a rectangular grid with uniform spacing the multigrid method provides an efficient
way of achieving this. A full multigrid method, similar to that described in Press et al. (1992)
is used.

A naïve approach to solving (6.11) is to discretise it on a grid as a system of linear equa-
tions of the form

\[
\frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{\Delta x^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta z^2} = -\omega_{i,j},
\]

(6.13)

where \( \Delta x \) and \( \Delta z \) are the grid spacing in the \( x \) and \( z \) directions. The subscripts give the \( x \) and
\( z \) coordinates of the variable. This system of linear equations can be solved directly, but for a
large grid this is a very expensive operation. The Fourier transform method for example (see Press et al., 1992) can solve this is in $O(M^2 \ln M)$ operations for an $M \times M$ grid. There are alternative approaches however.

The system of linear equations (6.13) can be rearranged into an iterative formula for $\psi_{i,j}$ of the form

$$
\psi_{i,j}^{n+1} = \left( \frac{\psi_{i+1,j}^n + \psi_{i-1,j}^{n+1}}{2(\Delta x^2 + \Delta z^2)} \right) + \frac{\Delta x^2 \Delta z^2}{2(\Delta x^2 + \Delta z^2)} \omega_{i,j},
$$

(6.14)

where the superscript indicates the timestep at which the variable is given. The Gauss-Seidel method sweeps across the grid, applying this formula at each point and using the updated values as they are calculated. For a grid size of $M \times M$ this approach is $O(M^2)$ for each pass, and requires $O(M)$ iterations to achieve convergence.

To improve on this first consider a Fourier decomposition of the solution. Analysis of the error in the Gauss-Seidel method shows that the high frequency, non-smooth components of the answer converge quite rapidly, but the smooth, low frequency components which cover many grid points, take much longer. At each pass information is propagated from one grid point to the next so many iterations are needed to smooth out the components which cover many grid points. The fundamental idea of the multigrid method is to solve the problem
6.3. METHOD OF SOLUTION

on several different sized grids. On a coarse grid, the set of linear equations can be either solved exactly or solved rapidly with an iterative method. This coarse grid only gives a crude approximation to the solution, but does include the low frequency components. The coarse grid solution can then be interpolated to give an approximate solution on a finer grid. Since the low frequency components are already well approximated, far fewer iterations are needed to converge to an answer on this finer grid. The process can be repeated with a hierarchy of different sized grids. The convergence on each grid is measured by calculating the residual,

\[ r^n_{i,j} = \frac{\psi^n_{i+1,j} - 2\psi^n_{i,j} + \psi^n_{i-1,j}}{\Delta x^2} + \frac{\psi^n_{i,j+1} - 2\psi^n_{i,j} + \psi^n_{i,j-1}}{\Delta z^2} + \omega_{i,j}. \] (6.15)

For an exact solution of the discretised equations the residuals are zero. The residual can be thought of as a measure of how much the approximate solution differs from being an exact solution to the set of linear equations.

For an \( M \times M \) grid, the multigrid method is \( O(M^2) \), which is significantly better than the simple Gauss-Seidel method. It also appears to be better than the direct Fourier transform method mentioned above which is \( O(M^2 \ln M) \). In practice, the coefficient in front of the \( M^2 \) for the multigrid method is large so for moderate grid sizes the efficiency of the two algorithms is similar. The multigrid method has the added advantage of being applicable to a more general class of problems.

For the axisymmetric case a similar iterative formula can be formed from (6.12) to give

\[ \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{\Delta r^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta z^2} - \frac{1}{i \Delta r} \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2 \Delta r} = -\omega_{i,j}. \] (6.16)

The multigrid method works best for homogeneous boundary conditions. Since the Poisson problem is linear, this can be achieved by subtracting off the vorticity and streamfunction resulting from the background flow and solving only for the perturbation. Since the boundary values of the streamfunction are given by the background flow this leads to a problem with homogeneous boundary conditions.

The rate of convergence was improved by using the previous solution for \( \omega \) to extrapolate an estimate of the new solution which can then be used as the initial guess for the multigrid method. This extrapolation is carried out on the finest grid. The approximate solution is then transferred onto successively coarser grids. The problem is solved on the coarsest grid and the solution interpolated back onto the finer grids, carrying out a few passes of the iterative formula at each stage to smooth the solution. The residual was monitored to ensure convergence. This scheme was found to provide an efficient means of solving the Poisson problem for the streamfunction.
6.3.2 The advection scheme

At each timestep the vorticity, $\omega$, and the effective gravity, $g'$, are advected by the velocity field calculated using the multigrid method. A staggered finite-volume scheme is used for the timestep. The vorticity, $\omega$, and the effective gravity, $g'$, are stored at the centre of each grid square, and the streamfunction is stored at the corners of the square. Using central differencing $u$ is calculated on the sides of each square and $v$ on the top and bottom of each square, as required to calculate the fluxes.

The time stepping of the equations of motion is done using either a linear central differencing scheme, which is second order in space, or a third order in space QUICK scheme (see Leonard, 1979) or SHARP scheme (see Leonard, 1988). All three schemes are first order in time.

It can be shown that no linear schemes higher than first order in space are guaranteed to be monotonic. Oscillations usually occur where there are sharp gradients in the solution. First order schemes are excessively diffusive however and smear out the solution. The central differencing scheme, being second order in space, is subject to spurious oscillations. A more sophisticated scheme is generally necessary.

The QUICK method uses quadratic upwind interpolation to calculate the effective gravity and the vorticity on the boundaries of the cells. Where the velocity is zero the method falls back to using a linear central differencing scheme. The velocity is already known on the boundaries so the fluxes of effective gravity and vorticity can be calculated. These fluxes are then used to update the values of $\omega$ and $g'$ in each cell volume. The QUICK method, first used by Leonard (1979), proves a significant improvement over the central differencing scheme. It still has difficulties coping with sharp gradients such as those that occur at the front of a gravity current.

The SHARP scheme (see Leonard, 1988) uses quadratic upwind differencing with a flux limiter term to prevent the growth of spurious oscillations. The flux limiter term works so that the scheme is third order where the solution is smooth, but near extremum the scheme is reduced to first order to help maintain monotonicity and prevent spurious oscillations. A check is made afterwards to ensure that the effective gravity does not become negative or greater than the initial effective gravity, which are both physically unrealistic.

In general, suppose that $\phi$ is the variable to be advected. Let $\phi_f$ be the unknown value on the face of a cell, $\phi_u$ the value in the upwind cell, $\phi_d$ the value in the downwind cell and $\phi_{uu}$ the value in the next but one upwind cell. The general expression for $\phi$ in a higher-order scheme is

$$
\phi_f = \phi_u + 0.5B(r)(\phi_u - \phi_{uu}),
$$

(6.17)
where $B(r)$ is a function of
\[
B(r) = \frac{\phi_d - \phi_u}{\phi_u - \phi_{uu}}
\]
which characterises the scheme. The variable $r$ is a measure of the local curvature. If $r < 0$ then there is an extremum.

The simplest scheme is the first order upwind scheme for which $B(r) = 0$. For the central difference scheme $B(r) = r$, while for the QUICK scheme $B(r) = 0.75r + 0.25$. The SHARP scheme is more complicated and the function $B(r)$ is made of several parts, depending on the value of $r$, so
\[
B(r) = \begin{cases} 
-\frac{5}{4} & r \leq -2 \\
\frac{3}{4}r + \frac{1}{4} & -2 \leq r \leq -\frac{1}{3} \\
0 & -\frac{1}{3} \leq r \leq 0 \\
\frac{5}{4}r - \frac{1}{4} + 0.2609\frac{(1+3r)[1-r]}{1+r} - 0.13613\frac{(1-r)^3}{(1+r)^2} & 0 \leq r \leq \frac{3}{7} \\
\frac{5}{4}r - 1 + \frac{1}{2}\left(\frac{r^2-1}{1+r}\right)^2 & \frac{3}{7} \leq r \leq \frac{7}{3} \\
r & \frac{7}{3} \leq r.
\end{cases}
\]

In practice it was found necessary to use the SHARP scheme in order to prevent spurious oscillations in the solution near the interface where there are steep density gradients.

The baroclinic term on the right hand side of (6.5) can be incorporated either as an extra term in the flux or as an additional forcing term after the fluxes in and out of each cell have been summed. Physically it can be considered as a forcing term rather than a flux term so it makes more sense to include it this way. Both methods to discretise the baroclinic term were tried and it was found that the method used can make a difference of 5-10% in the predicted front position of the gravity current. A separate forcing term was chosen, using second order upwind differencing for the derivative, as it gave results which were consistent with experiments and did not create oscillations. At the boundaries the baroclinic torque was calculated using half the difference between $g$ in the two cells nearest the wall.

To ensure stability of the numerical scheme it is necessary that the Courant number,
\[
c = \frac{u\Delta t}{\Delta x}
\]
is less than 1, where $u$ is the fluid speed, $\Delta t$ is the timestep and $\Delta x$ is the grid spacing. In practice the timestep was chosen by setting $c = 0.25$ so
\[
\Delta t = 0.25 \min_{i,j} \left( \frac{\Delta x}{u_{i,j}}, \frac{\Delta y}{v_{i,j}} \right).
\]
6.3.3 Numerical viscosity

While the model equations used in this chapter are inviscid, the discretisation process introduces some artificial or numerical viscosity to the problem. This is important in stabilising the numerical method and is also responsible for some of the flow details observed. It is important to be able to quantify this numerical viscosity. By considering the leading order neglected terms in a Taylor series expansion of the equations, the numerical viscosity for the first order upwind scheme can be found (see for example [Toro, 1997]), giving

\[ \nu_{\text{num}} = \frac{1}{2} \Delta x u (1 - |c|), \]  
(6.22)

where \( \Delta x \) is the grid spacing, \( u \) is the speed and \( c \) is the Courant number given by (6.20). For a typical simulation carried out in this chapter, (6.22) gives a maximum value of about 0.04 cm\(^2\) s\(^{-1}\). This is higher than the value of about 0.01 cm\(^2\) s\(^{-1}\) for the real kinematic viscosity of water. The value is lower where the speed of the fluid is less.

For the higher order QUICK scheme the numerical viscosity is lower over all of the flow and takes the form of a fourth order superviscosity acting on the fourth derivative rather than the second derivative. For the SHARP scheme the numerical viscosity is lower over most of the flow, except near extremum where the scheme reverts to first order upwind to preserve monotonicity. The value of the numerical viscosity for the first order scheme therefore puts an upper bound on the total numerical viscosity.

6.4 Validation of the code

The numerical code was validated by running simulations of gravity currents with no slope or wind, and currents with either a slope or a wind. The predictions of these simulations can be compared with previous work in order to validate the code. Each of these test cases will now be considered in more detail.

6.4.1 A gravity current in a flat channel

The simplest test case for a gravity current model is an instantaneous release in a flat channel. This case has been extensively studied (see chapter 2). The numerical predictions from the model are compared with a laboratory experiment and with theoretical predictions from several different models.

[Huppert & Simpson (1980)] proposed an integral model for a gravity current with a slump- ing phase during which the flow developed to a self-similar phase. The model is described in
6.4. VALIDATION OF THE CODE

Figure 6.3: Density of a gravity current in a flat channel - a) numerics and b) experiments

2.2.2. The front position in the slumping phase is given as a function of time by

\[ l_f = l_0 \left(1 + \frac{7}{12} \left( \frac{g^3VH^2}{l_0^{3/2}} \right)^{1/6} \right)^{6/7} t \]  \hspace{1cm} (6.23)

for \( l_f < l_s \), where \( V = l_f h_f \) is the volume per unit width of the gravity current and

\[ l_s = \frac{V}{0.075H} \] \hspace{1cm} (6.24)

is the distance at which the fractional depth \( \phi = 0.075 \).

For \( l_f > l_s \), the Froude number is assumed to remain constant and the front position is given by

\[ l_f = l_s \left(1 + \frac{3}{2} \times 1.19 \frac{\sqrt{gV}}{l_s^{3/2}} (t - t_s) \right)^{2/3}, \] \hspace{1cm} (6.25)

where

\[ t_s = \frac{12}{7} \left( \frac{l_0^7}{g^3VH^2} \right)^{1/6} \left[ \left( \frac{h_0}{0.075H} \right)^{7/6} - 1 \right] \] \hspace{1cm} (6.26)

is the time at which \( l_f = l_s \).

An experiment was also conducted in a rectangular tank of length 200cm, width 20cm and height 25cm. The tank was filled with water to a depth of 15cm and a barrier was placed 20cm from one end to form a lock. Salt was dissolved in the lock to create a density difference and dye was added to aid visualisation. The lock gate was removed to release a gravity current and the motion of the resulting front was measured.

Figure 6.3 shows the density field from the numerics and an experimental image at the
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Figure 6.4: Front position against time of a gravity current in a flat channel. The graph shows the front position measured from the numerics and from experiments. The slumping solution of Huppert & Simpson (1980) and the shallow water similarity solution are also shown. Lengths are non-dimensionalised with respect to $V^{1/2}$ and times with $V^{1/4}/g^{1/2}$.

same instant in time. It can be observed that there is a clear similarity between the two. The front position agrees well and the presence of a head region with a thinner tail can be seen in both pictures, although the tail appears to be thicker in the experiments. This is partly a result of the experimental profiles being averaged over the width of the tank rather than the cross section seen in the numerics.

Another observed feature of the numerical predictions is the continued existence of two-dimensional structures. The Kelvin–Helmholtz type billows on the interface between the current and the ambient are much more pronounced in the numerical results. In the experiments the two-dimensional billows are themselves unstable to three-dimensional instabilities, giving rise to the characteristic “lobe and cleft” structure of the gravity current head. The three-dimensional instability helps to limit the growth of the two-dimensional billows.

Figure 6.4 shows the front position of the gravity current against time. The experimental and numerical results are plotted, along with the slumping solution given by (5.23) and (5.25) and the shallow water similarity solution. The agreement between the various models and experiments can be seen to be good, although the similarity solution slightly under predicts the front position and the numerical model slightly over predicts it.
6.4. VALIDATION OF THE CODE

Figure 6.5: A snapshot of the height of the gravity current plotted against distance from the origin. The graph shows the averaged height \( \int g' h dz / g'_0 \) measured from the numerics and the shallow water similarity solution. Lengths are non-dimensionalised with respect to \( V^{1/2} \).

Figure 6.5 shows a snapshot of the height of the current against distance from the origin. The profiles are at the same time as the density images in figure 6.3. The experimental height was found by using the Dig/mage software to measure an intensity contour corresponding to the edge of the current. The steps in the experimentally measured height are a result of the resolution of the frame grabber card used to digitise the images. It was difficult to measure the exact height of the current from the results of the numerical simulations due to the billows. Instead the averaged height given by \( \frac{1}{g'_0} \int g' dz \) is plotted. This can be thought of as the equivalent height in the absence of any mixing. Since this is a snapshot in time the presence of the Kelvin–Helmholtz type billows can clearly be seen by the oscillations in the averaged height. The shallow water similarity solution does not include any mixing so an exact comparison between the numerics and the shallow water model can be made. The agreement is good. The experimentally measured height is larger than that predicted by either the numerics or the shallow water solution. Since entrainment will be occurring in the experiments, the height would be expected to increase with a decreasing effective gravity. There is a good qualitative agreement between the predicted height from the numerics and the measured height in the experiments. The numerics successfully capture the head region and also some of the re-
Figure 6.6: Front position against time of a gravity current in a flat channel. The graph shows the front position measured from the numerics. The × are for a channel the same height as the release and the + are for a channel twice as deep. All other initial conditions are the same for each simulation. Lengths are non-dimensionalised with respect to $V^{1/2}$ and times with $V^{1/4}/g^{1/2}$.

remaining disturbances near the lock. With more carefully controlled experiments the DigImage software could be used to measure the averaged height from the experiments to allow better comparison with the numerical results. Experimentally, the head height is close to half the channel height, as predicted by the inviscid model of Benjamin (1968).

The integral slumping model of Huppert & Simpson (1980) predicts that the gravity current will initially travel faster in a deep ambient fluid. However, once the ambient fluid becomes sufficiently deep that the fractional depth of the current is less than 0.075, any further increase in depth does not have a significant effect. This is also observed in the numerical results and is likely to be a blocking effect as the ambient fluid attempts to flow back above the gravity current to replace the dense fluid in the lock. Figure 6.6 shows the numerical predictions for the front position for a full depth and a half-depth release. All other initial conditions are identical. It can be seen that for a half-depth release the current travels more rapidly in the initial stages of the flow since there is less blocking.

While the numerics and experiments are for the same initial problem there are several differences between the two situations. The numerics does not include any explicit viscosity
and so has a free slip boundary condition on the bottom of the tank, in contrast to the no-slip condition in the experiments. The work of Simpson & Britter (1980) has shown that a gravity current with a free slip boundary condition will travel slightly faster than a current with a no-slip boundary condition. Surface gravity currents with a stress free boundary current are also observed to travel slightly faster than a bottom gravity current (see for instance H¨artel et al., 2000b). At a Reynolds number of $10^4$, comparable with those in the laboratory experiments, the difference in speed was approximately 5-10% which is close to the 5% difference observed here between the numerical simulations and the experiments. There will be less dissipation in the numerical simulations since there is no explicit viscosity, which could also lead to the current travelling slightly faster. These factors may explain the slight discrepancy between the numerical and experimental front positions.

### 6.4.2 An axisymmetric gravity current

A similar analysis to that in §6.4.1 can be carried out for an axisymmetric gravity current. In the slumping phase, for $r_f < r_s$,

$$r_f = r_0 \left( 1 + \frac{2}{3} \left( \frac{g^3 V H^2}{\pi r_0^5} \right)^{1/6} t \right)^{3/4}, \quad (6.27)$$

where $V = \pi r_f^2 h_f$ is the volume of the gravity current and $r_s$ is the distance at which $\phi = 0.075$. After the slumping phase, the constant Froude number condition gives

$$r_s = \left( \frac{V}{0.075 \pi H} \right)^{1/2}. \quad (6.28)$$

For $r_f > r_s$ then

$$r_f = r_s \left( 1 + 2 \times 1.19 \frac{\sqrt{g^3 V \pi}}{r_s^2} (t - t_s) \right)^{1/2}, \quad (6.29)$$

where

$$t_s = \frac{3}{2} \left( \frac{\pi}{g^3 V H^2} \right)^{1/6} \left( r_s^{4/3} - r_0^{4/3} \right) \quad (6.30)$$

is the time at which $r_f = r_s$.

Figure 6.7 shows the front position of the gravity current against time. The numerical results are plotted, along with the slumping solution given by (6.27) and (6.29), and the shallow water similarity solution. The slumping solution of Huppert & Simpson (1980), the numerics and the experiments are all in fair agreement for the front position of the gravity current. The shallow water similarity solution over predicts the front position because it does not take into
Figure 6.7: Front position against time for an axisymmetric gravity current on a flat surface. The graph shows the measurements made from the numerics, the slumping solution of Huppert & Simpson (1980) and the shallow water similarity solution. Lengths are non-dimensionalised with respect to \( V^{1/3} \) and times with \( V^{1/6} / g^{1/2} \).

Figure 6.8: Density of an axisymmetric gravity current on a horizontal surface from numerics.

account the initial development of the flow. The numerics do appear to tend towards the similarity solution some time after the release. The experimental results shown on this figure are subject to a reasonably large error as they are from a rough experiment conducted in the small tank described in §4.4.

A snapshot of the density field from the vorticity–streamfunction model is shown in figure 6.8. Compared to the density field for a current in a horizontal channel it can be seen that the axisymmetric current is much shallower as a result of the radial expansion. This effect is purely geometric and is also observed in the experiments. In order to get accurate results from the numerical model a fine enough grid spacing has to be chosen to ensure the detail in the thin layer is captured. This can lead to increased computation time for the axisymmetric case.
6.4.3 A gravity current in a sloping channel

Beghin et al. (1981) carried out experiments on an instantaneous release gravity current in a sloping channel. They looked at a range of different slopes and measured the shape, growth and movement of the gravity current head or “thermal” that developed. The head was found to be slightly higher for a current flowing down slope than for a similar current on a horizontal surface. They developed an integral model for the motion of the gravity current head and compared it with experiments as described in §2.3. For large $x_f$, the non-dimensional front velocity, $u_f x_f^{1/2} / (g-V_0)^{1/2} = K_f \sin^{1/2} \theta$, was found to be a weakly decreasing function of slope, from a value of about 2.6 at 10° to about 1.5 at 90°. The integral model predicts the front position as a function of time to be

$$x_f = x_0 \left(1 + \frac{9K_f^2}{4} \frac{g-V_0 \sin \theta}{x_0^3} t^2 \right)^{1/3}.$$  \hspace{1cm} (6.31)

To compare with the numerical predictions an experiment was conducted in the rectangular tank described in §6.4.1. To create a slope the tank was tilted at an angle of 5°. The tank was filled with water so the depth of water varied between 6 cm and 27 cm along the length of the tank. The barrier was placed at the top of the slope, a distance 20 cm from the end wall, to form a lock. The barrier was perpendicular to the slope. Salt was dissolved in the lock to create a density difference and dye was added to aid visualisation. The barrier was removed to release a gravity current and the motion of the resulting front was measured. Figure 6.9 shows the front position of the gravity current against time. The experimental and numerical results are plotted, along with the semi-empirical solution given by (6.31). The numerics differ from the experiments in that the numerics have a channel of constant depth, while in the experiments the depth of the water in the channel varied. Numerical results are given for a channel with the same depth as the current and a channel twice as deep as the current. This can be seen to give rise to a large difference in the predicted front position, as was observed for a horizontal channel. The experiments agree much better with the predictions for a deep channel. The deepening of the channel with distance in the experiments appears to significantly reduce the blocking as a result of the full depth release. The experimental front position agrees well with the numerical predictions for a deep channel over the length of tank used in the experiments. The numerics also agree well with the similarity solution of Webber et al. (1993) and the integral model of Beghin et al. (1981) in this case.

Figure 6.10 shows snapshots of the density field from the numerics and the camera images from the experiments. The snapshots are both at the same time. It can be seen that there is a similarity between the numerical predictions and the experimental results. It is observed that
Figure 6.9: Front position against time of a gravity current flowing down a sloping channel. The graph shows the measurements made from the numerics and experiments, along with the solution for the integral model of Beghin et al. (1981) and the similarity solution of Webber et al. (1993). Two sets of results are shown for the numerics, one with the channel the same depth as the release and one with the channel twice the depth of the release. Lengths are non-dimensionalised with respect to $V^{1/2}$ and times with $V^{1/4}g^{1/2}$.

Figure 6.10: Density of a gravity current flowing down a sloping channel - a) numerics and b) experiments
6.4. VALIDATION OF THE CODE

Figure 6.11: Front position against time of a gravity current flowing down a sloping channel. The geometry of the release is the same as in the experiments of Beghin et al. (1981). The measurements from the numerics are shown along with the predictions of the integral model of Beghin et al. (1981) and the similarity solution of Webber et al. (1993). Lengths are non-dimensionalised with respect to $V^{1/2}$ and times with $V^{1/4} g^{1/2}$.

The gravity current head is slightly flatter and longer in the experiments. This could be a result of the variation in depth along the tank caused by the slope. It is noticeable that even for a slope of 5° the current head is increased in size for a downslope current.

The experiments of Beghin et al. (1981) were carried out in a longer and much deeper tank that the one used here. Their integral model was a one layer model so the results might not be exactly applicable to the case described here with a full depth release. The numerical model was also run for a release similar to that described in Beghin et al. (1981) and the results are shown in figure 6.11 along with the similarity solution and the integral model predictions. The values for the constants $K_f$ and $x_0$ in the integral model are taken from Beghin et al. (1981). Figure 6.11 shows that the numerics and the similarity solution predict a greater speed than the integral model, and hence a larger value for the front position. This behaviour was also observed by Beghin et al. (1981) who noted that for slopes of less than 45° the speed predicted by the integral model was 10-20% less than the speed measured in the experiments.

An experiment was also carried out with the lock at the bottom of the slope to generate a gravity current flowing upslope. The comparison between the numerics and the experiment...
Figure 6.12: Front position against time of a gravity current flowing up a sloping channel. The graph shows the measurements made from the numerics and experiments. Lengths are non-dimensionalised with respect to $V^{1/2}$ and times with $V^{1/4}/g^{1/2}$.

is shown in figure 6.12. The initial agreement between the experiments and the numerics is good but the gravity current is slowed down more rapidly in the numerics so does not reach as far up the slope. One possible explanation for the difference between the numerics and the experiments lies in the two-dimensional nature of the numerics. The persistent presence of the billows means that the momentum within the current is spread over a larger volume. In the experiments a smaller volume of fluid at the head carries a higher density of momentum and is able to travel further up the slope. Figure 6.13 shows that the shape of a gravity current travelling upslope is quite different to that travelling downslope. The head is thinner and there is a much deeper and more pronounced tail.

6.5 Gravity currents with a background flow

The presence of a background flow can be expected to influence the spread and development of a gravity current in a channel. The previous work on this problem is reviewed in §2.4. By using a flume tank with a moving floor, Simpson & Britter (1980) carried out a series of experiments looking at steady state gravity currents with a head wind or a tail wind. The speed
of the moving floor was adjusted until it equalled the speed of the current, bringing the front to rest. The current was fed with saline solution from behind to maintain the head and any excess fluid drained out over a weir. The speed of the flow through the tank could also be controlled. A flow speed greater than the speed of the floor simulated a head wind and a flow speed less than the speed of the floor simulated a tail wind. Simpson & Britter (1980) discovered that the speed of propagation of the current was altered by approximately $3/5$ of the applied wind speed when compared to the same gravity current in the absence of any ambient flow.

In this section the numerics developed here are compared with the work of Xu (1992), described in §2.4, and with the experimental findings of Simpson & Britter (1980).

### 6.5.1 An inviscid model for an gravity current with shear

The classic analysis of Benjamin (1968) for an air cavity intruding into a channel can be extended for the case of an inviscid gravity current moving in a channel with a mean flow. Since the analysis is inviscid, a uniform flow merely has the effect of translating the solution with a speed equal to the wind speed. The case of a uniform shear flow has been studied by Xu (1992) and leads to more interesting behaviour, as described in §2.4. The work of Xue et al. (1997) extended this to the case of a constant shear layer with a layer of constant wind speed above it. If this shear layer is very thin then it may be expected to model some of the effects of a thin boundary layer near the surface with a uniform wind above it. Xue et al. (1997) looked at the effect of different heights for the shear layer but they did not consider the limit as the shear layer thickness goes to zero.
When taking the model of Xue et al. (1997) and letting the shear layer thickness go to zero, it is necessary keeping the speed, $U_1$ at the top of the shear layer constant, i.e. keeping the uniform wind speed constant. This leads to a simplification of the model equations in the case where the boundary layer is much smaller than the height of the gravity current. The presence of the thin shear layer means that the problem does not simply reduce to the model of Benjamin (1968) with a uniform flow. As will be seen, this will capture some of the effects of the viscous boundary layer which are lost by using an inviscid model. Figure 6.14 illustrates the problem.

The equations of Xue et al. (1997) are similar to those for a uniform shear flow presented in §2.4. In the limit of an infinitely thin shear layer we obtain

$$C_2^2 = 2(1 - H)$$

from Bernoulli’s equation,

$$C_1 - U_1 = (C_2 - U_2)H$$

and

$$C_1 U_1 - \frac{1}{2} U_1^2 = C_2 U_2 - \frac{1}{2} U_2^2$$

by conservation of mass and

$$U_1^2 + \frac{1}{2} C_1^2 - 2C_1 U_1 = C_2^2 H - 2C_2 U_2 H + U_2^2 H - \frac{1}{2}(1 - H^2)$$

from the flow force balance. The wind speed, $U_1$, is prescribed. The other variables are then given by

$$H = \frac{1}{2}(1 - U_1),$$

$$H = \frac{1}{2}(1 - U_1),$$
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\[ C_1 = \frac{1}{2} (1 + U_1) \quad (6.37) \]

and

\[ C_2 = (1 + U_1)^{1/2}. \quad (6.38) \]

The change in the speed, \( C_1 \), of the current is found to be \( 1/2 \) of the applied wind speed, as opposed to the \( 3/5 \) found by Simpson & Britter (1980) in their experiments.

There is no simple analytic solution available for the constant shear model of Xu (1992) and his published results were obtained from numerical solutions of the model. For small \( \Omega \) we will derive a new asymptotic expansion for comparison with the experimental fit of Simpson & Britter (1980) and other theoretical models. We assume an expansion for \( H \) and \( C_1 \) in powers of \( \Omega \). Substituting these expansions into the equations of Xu (1992) given in §2.4 leads to the expressions

\[ H = \frac{1}{2} - \frac{1}{4} \Omega + O(\Omega^2) \quad (6.39) \]

and

\[ C_1 = \frac{1}{2} + \frac{1}{4} \Omega + O(\Omega^2) \quad (6.40) \]

for the fractional height and speed of the gravity current. The average wind speed is \( U_1 = \Omega/2 \) so, to first order in the wind speed, the model with constant shear and the model with an infinitely thin shear layer give the same predicted front speed. The change in speed of the current is half of the applied wind speed. This suggests that, at least for small winds, the current speed may not be too susceptible to the exact profile used for the wind speed. Note that in both cases the wind speed at the ground is zero.

The inviscid models of Xu (1992) and Xue et al. (1997) can be used to give an estimate of the effect of the background flow, however, for the Boussinesq saline experiments described here the assumptions of this analysis are no longer strictly true. Since the two fluids are not inviscid or immiscible, the effects of mixing between the two fluids may be important, but this is not taken into account by the analysis in this section. The same limitations are present in the work Benjamin (1968), yet his results have been shown to provide good agreement with experiment, so it is hoped that these models incorporating shear will be as successful.

6.5.2 Comparison with numerical simulations - a uniform flow

The numerical code developed in this chapter can be compared against the model of §6.5.1 and the experimental results of Simpson & Britter (1980) for the test case of a current in a channel with a background flow. The simulations in this section were all carried out with a full depth release of dense fluid in the middle of the domain. The aspect ratio of the release
was 1.1 (based on the height divided by half the length of the release.) The mean wind speed was taken as the average wind speed over the height of the initial release.

First, the code was run twice with identical initial conditions, apart from the length of the domain. The release was for the centre of the domain. Comparing the two results allowed the effect of the end boundary conditions to be assessed. The front positions are plotted against time in figure 6.15. This illustrates that the effect of imposing a constant wind profile at the end is not significant for the propagation of the current. The perturbation to the mean flow streamfunction for both runs, at a time just before the current reaches the downwind end of the shorter domain is shown in figure 6.16. It can be seen that there is almost no difference between the flow in the two cases (a) and (b). Image (c) shows the difference between the images multiplied by a factor of 10. This shows that the deviation in the domain is tiny, with the deviation at the end only slightly more. The maximum difference between the two images (a) and (b) was about 5%. This suggests that the effects of imposing the flow on the downstream end of the domain are not significant, except right at the end. The perturbations to the mean streamfunction are predominately confined to the region near the head. The finding that the current needs to be very close to the end of the domain before the finite extent of the domain is important agrees with the earlier findings of Hārtel et al. (2000b) for their simulations of lock release gravity currents.

The numerical results for various uniform wind speeds are shown in figure 6.17. The wind speed is given as a fraction of the speed \( \left( \frac{gh}{2} \right)^{1/2} \), which gives a velocity scale for the gravity current. Positive values correspond to a tail wind and negative values to a head wind. It can be seen that in this computational domain the gravity current head is observed to become arrested for a head wind of about 0.4 or higher. It would be expected that for any head wind the current will eventually become arrested, although for a lower wind speed the current will travel further before being brought to rest. It can be seen that the presence of the wind makes a large difference to the movement of the gravity current.

The model equations (6.1) and (6.2) are invariant under a change of reference frame. This means that mathematically, the effect of an imposed uniform wind speed is just a translation of the gravity current. Figure 6.18 shows the same numerical results as figure 6.17 but in a frame of reference moving with the wind. In this case the data collapses well, confirming that the predominant effect of the wind is to advect the gravity current. The collapse is particularly good for those runs with a tail wind. With a head wind the collapse is less good. For an exact solution to the model equations, the symmetry between the upwind and downwind parts of the gravity current should be maintained. In the numerical simulations this is not quite achieved due to the upwind bias in the QUICK and SHARP advection schemes. The imposed wind speed affects the definition of “upwind” leading to an asymmetry. The stability of the scheme
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Figure 6.15: Front position against time of a gravity current in a channel with an imposed wind. The measurements from the numerics for two different domain lengths are shown to illustrate the effects of the end boundary conditions. Lengths are non-dimensionalised with respect to $V^{1/2}$ and times with $V^{1/4}/g^{1/2}$.

relies on this upwind differencing. If upwind is calculated using the speed relative to the mean flow then the scheme was found to become unstable. Differences also arise because the numerical grid is fixed in a stationary frame of reference rather than moving with the current so the up and down wind heads are not identically modelled. The down wind head crosses more cells than the upwind head, which affects the numerical viscosity (see equation 6.22), and hence the shape of the head. The asymmetry due to the upwind bias could be overcome by using an implicit central difference scheme. Central differencing ensures symmetry but the scheme would need to be implicit in time for stability. As already discussed, the linear central scheme does not cope with the sharp gradients near the head of the current. An implicit scheme would be computationally much more expensive than the relatively fast schemes described here so it was decided not to implement one.

6.5.3 Comparison with numerical simulations - a uniform shear

For a linear wind profile, the background flow imposes a shear across the gravity current. The problem can no longer be reduced to the case with no wind by choosing a suitable frame of
Figure 6.16: The perturbation from the mean-flow streamfunction for two identical simulations except (a) is in a short domain and (b) is in a domain twice the length. The release was in the middle of the domain. The figure for the streamfunction in the long tank is truncated to the size of the small domain. The figures show the streamfunction just before the gravity current reaches the end downwind of the shorter domain. Figure (c) shows the difference between the two images. The difference is increased by a factor of 10 to make it visible.

reference. It would be expected that this would lead to an asymmetry between the upwind and downwind heads of the current. The linear profile has the wind speed falling to zero at the ground, so there is very little opposing the gravity current, provided it remained thin. Figure 6.19 shows the front position against time for a variety of different mean wind speeds. The wind has the expected effect of speeding up the current for a tail wind and slowing down the current for a head wind. Figure 6.20 shows the front position in a frame of reference moving with 3/5 the wind speed at half the initial release height. This is a wind speed typical of that which the current will experience. It can be seen that this leads to a good collapse in the data, provided the wind speed is not too large. For larger wind speeds and at later times the head of the current runs out and eventually becomes arrested by the wind. The resulting deviation from the data collapse can be seen in figure 6.20. The collapse of the data based on 3/5 of the mean wind speed is in agreement with the experimental work of Simpson & Britter (1980) and suggests that using a linear wind profile may give better agreement than
using a constant wind speed. Figure 6.21 shows that with a head wind the front of the gravity current becomes flattened and more pointed. With a tail wind the current is higher and has a more blunt front. This agrees with the experimental observations of Simpson & Britter (1980).

Figure 6.22 shows the front position against time for a full depth release in a channel. The numerical predictions and the model predictions of §6.5.1 are shown. It can be seen that the inviscid Benjamin-type analysis slightly over-predicts the speed of the current. Even with no background flow the predictions are too large. The model does appear to show the same trends as the numerics.

Comparison by Klemp et al. (1994) of two-dimensional non-hydrostatic simulations with the analysis of Benjamin (1968) for gravity currents in a flat channel showed that the inviscid Benjamin solution over-predicted the front speed and height. The current height in the simulations of Klemp et al. (1994) was observed to be less than the value of half the channel depth predicted by Benjamin (1968). Comparison between the front speed expected for the lower height and the numerical simulations showed good agreement. This suggests that the analysis was correct, but the effects of mixing, which reduce the current height, need to be accounted for.

Comparison between the simulations carried out here and the predictions of the analysis in...
Figure 6.18: Front position against time of a gravity current in a channel in a frame moving with the wind. The numerical results for a range of applied wind speeds are shown. The wind speeds are non-dimensionalised with \((g_0^2h_0/2)^{1/2}\) which gives a typical speed scale for the gravity current. Lengths are non-dimensionalised with respect to \(V^{1/2}\) and times with \(V^{1/4}/g^{1/2}\).

§6.5.1 and Xu (1992) is difficult because of the finite nature of the releases in the simulations. It is expected that the head will appear to be steady in the initial slumping region before information about the finite extent of the release has time to reach the head of the current. For a current in a channel with no background flow this slumping phase occurs over the first 10 lock lengths. In the simulations it is difficult to decide where behind the head the height should be measured. The problem is compounded by the fact that the Kelvin–Helmholtz billows in the numerics are much larger than those observed in the experiments, increasing the apparent height. The billows are larger because the code is two-dimensional. In the experiments the billows break down sooner through a three-dimensional instability, however this possibility is not available in the numerics. Figure 5.23 shows the density field for two numerical runs with \(\Omega = 0\) and \(\Omega = 0.1\). This shows the change in the height as a result of a head and tail wind. With a head wind the following flow is only changed a little. With the tail wind the flow behind is lower as it is swept into the head. In all three cases the height of the flow behind the head is significantly less than half the total depth of the channel.
Figure 6.19: Front position against time of a gravity current in a channel with a constant shear head or tail wind. The numerical results for a range of applied wind speeds are shown. The wind speeds are taken as the mean wind speed in the channel and are non-dimensionalised with \((g_0 h_0/2)^{1/2}\) which gives a typical speed scale for the gravity current. Lengths are non-dimensionalised with respect to \(V^{1/2}\) and times with \(V^{1/4}/g^{1/2}\).
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Figure 6.20: Front position against time of a gravity current in a channel in a frame moving with $3/5$ the wind speed in a constant shear wind. The wind speeds are taken as the mean wind speed in the channel and are non-dimensionalised with $(g' h_0/2)^{1/2}$ which gives a typical speed scale for the gravity current. Lengths are non-dimensionalised with respect to $V^{1/2}$ and times with $V^{1/4}/g^{1/2}$.

Figure 6.21: A gravity current in a flat channel with a linear wind profile, blowing from left to right. The mean wind speed was $0.4 \ (g' h / 2)^{1/2}$. 
Figure 6.22: Front position against time of a gravity current in a channel - predictions from numerical code and from the model of §6.5.1. The fronts are for values of $\Omega = \pm 0.1$. 

Downwind prediction
Upwind prediction
Downwind theory
Upwind theory
Figure 6.23: A gravity current in a flat channel with a linear wind profile, blowing from left to right. The release was a full depth lock release in the middle of the channel. The wind speed was chosen so (a) $\Omega = 0.0$ and (b) $\Omega = 0.1$. Only the part of the solution near the head is shown.
6.6 Summary

In this chapter a numerical model for two-dimensional and axisymmetric gravity currents has been described. The model uses the vorticity–streamfunction formulation of the inviscid Navier–Stokes equations. The method of solution is briefly discussed in §6.3. In §6.4 the model is validated by comparison with previous theoretical models and with some new experiments. The model is seen to provide good agreement with the existing work on gravity currents in both flat and sloping channels, and axisymmetric gravity currents on a horizontal surface. In §6.5 the effect of different wind profiles on the numerical predictions is considered. The results are compared with the previous work of Simpson & Britter (1980), Xu (1992) and Xue et al. (1997). Some new asymptotics to the inviscid models of Xu (1992) and Xue et al. (1997) are also presented. The numerical model is again seen to agree well with previous work.

The close agreement between the numerical predictions and previous experimental and theoretical work, for a range of different conditions, provides confidence in the use of the numerical model to study some new gravity current problems. Two such problems will be considered in the next chapter.
Chapter 7

Two new problems involving slope and wind

7.1 Introduction

A numerical model for gravity currents, based on the vorticity–streamfunction formulation of the Euler equations, was developed in chapter 6. The model can be used for two-dimensional and axisymmetric gravity currents and includes both slope and wind effects. In this chapter the model will be used to look at two new problems. The first is an instantaneous release of dense fluid on a cone. The second is a gravity current in a sloping channel with a background wind.

In many industrial situations a release of dense gas may occur on the top of a hill or in the bottom of a valley. In this case it may be more appropriate to consider the geometry as axisymmetric rather than as channel flow or a uniform slope. The instantaneous release of a gravity current on a cone provides an idealisation of this problem. The flow need not be over the whole of the cone. For instance, a valley on a mountain side may well become wider with distance down the mountain. A sector of a cone could provide a good model for the flow in such a valley.

The numerical model in chapter 6 can be used to study this situation. A simple integral model and a shallow water model are also developed here. In §7.2 the predictions of the numerical code are compared with the integral model, with the shallow water model and with some new laboratory experiments.

The second problem, discussed in §7.3, is the flow of a gravity current in a sloping channel with a background flow. In many industrial, environmental and geophysical applications both the topography and the ambient flow will play an important role. The previous work on
the individual effects of slope and ambient wind on a gravity current has been reviewed in §2.3 and §2.4 respectively. Some of this work was used in the previous chapter to validate the numerical model. Very little work has considered a gravity current where the combined effects of wind and slope are important. In such circumstances it is necessary to understand how the slope and wind may interact. The simplest situation to consider is an instantaneous release gravity current in a sloping channel with a background flow. It is hoped that this will provide important insights into the more general case. The combined effects of the wind and slope are studied using the vorticity–streamfunction numerical model. By varying the initial conditions the effects of the wind and slope can be understood. No experimental results are available for this case.

7.2 Axisymmetric gravity currents on a cone

The problem of an instantaneous release gravity current on a cone will be considered from several points of view. In §7.2.1 a simple integral model of the flow is developed. A shallow water model, similar to that of Webber et al. (1993) is also presented in §7.2.2. The predictions of these two models are compared with the vorticity–streamfunction model predictions in §7.2.3 and with the results of some new experiments described in §7.2.4.

7.2.1 An annular integral model

The release of a instantaneous volume of dense fluid on a cone can be modelled using an integral model approach similar to that in §4.3.2. Suppose the gravity current forms an annular shaped cloud which travels down the cone. Let \( r \) be the distance of the annulus from the tip of the cone. Let \( l \) be the width of the annulus and \( h \) the height of the annulus. We assume the annulus is self-similar so that the height \( h = S_1 l \), the cross sectional area is \( S_2 h l \) and the perimeter of the annulus is \( S_3 l \) for some constants \( S_1 \), \( S_2 \) and \( S_3 \). The volume of the gravity current is thus \( V = 2\pi r S_1 S_2 l^2 \cos \theta \).

The equation for conservation of mass is

\[
\frac{dV}{dt} = 2\pi r \cos \theta S_3 l \alpha u, \tag{7.1}
\]

where

\[
u = \frac{dr}{dt} \tag{7.2}
\]

is the speed of the annulus and \( \alpha \) is the entrainment coefficient.
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The conservation of momentum equation becomes

\[
\frac{d}{dt}(Vu) = g'V \sin \theta - C_D 2\pi r \cos \theta l u^2,
\]  

(7.3)

where \(C_D\) is a drag coefficient.

By writing \(d/dt = u d/dr\), (7.1) can be integrated to give

\[
l = \frac{\alpha S_3}{3S_1 S_2} \left( r - \frac{r_0^{3/2}}{r^{1/2}} \right) + \frac{l_0 r_0^{1/2}}{r^{1/2}},
\]

(7.4)

where \(l = l_0\) at \(r = r_0\). In the far field \(l \propto r\), as was found for the wedge on a uniform slope in §4.3.2. If \(l \propto r\) then from (7.3) we have \(u \propto 1/r\) for large \(r\) and \(r \propto t^{1/2}\). More precisely (7.3) can be integrated to give

\[
u^2 = u_0^2 \left( \frac{r_0^{3/2} - r_*^{3/2}}{r^{3/2} - r_*^{3/2}} \right) \gamma + \frac{9 S_1 S_2 g'V \tan \theta}{\pi \alpha^2 S_3^2 \left( r^{3/2} + r_*^{3/2} \right)^\gamma} \int_{r_0}^{r} \left( s^{3/2} + r_*^{3/2} \right)^{\gamma-2} ds,
\]

(7.5)

where

\[
r_*^{3/2} = \frac{3 S_1 S_2 l_0^{1/2}}{\alpha S_3} - r_0^{3/2}
\]

(7.6)

and

\[
\gamma = 4 \left( 1 + \frac{C_D \cos \theta}{\alpha S_3} \right).
\]

(7.7)

Equation (7.5) is complicated and in general the integral on the right hand side cannot be evaluated analytically. If \(C_D = 0\), so \(\gamma = 4\), then the integrand is just a series of terms in \(s^3\), \(s^{3/2}\) and \(s^0\) which can be integrated. However, the resulting equation is not enlightening and has to be integrated again in order to get the front position against time. The solution of the second integration cannot be written in a closed form. For non-zero values of \(C_D\) it is necessary to numerically integrate the three equations (7.1) – (7.3). This was done here by writing a code using the fourth order Runge–Kutta method. These numerically integrated solutions for the integral model will be compared with the numerical results from the vorticity–streamfunction model in §7.2.3 and the experimental results in §7.2.4.

There is some difficulty in choosing the initial conditions for the integral model since the model is effectively looking for a long time similarity solution of the equations. The initial conditions were chosen to ensure that the front of the annulus coincided with the front of the lock and that the volume of the annulus was the same as the lock. Problems can occur because the solution becomes singular at the origin, \(r = 0\). If the volume of the release is too large then it is not possible to maintain the assumed self-similar shape and to keep the length of the
annulus less than the distance to the origin. Starting the integral model solution at some later
time when the gravity current has moved away from the origin will solve this problem, but
then some alternative theory is needed to model the initial stage of the flow.

The predictions of the annulus integral model will be discussed further in the following
sections, where comparisons are made with the other numerical and experimental results.

7.2.2 Shallow water model

Provided the slope of the cone is not too large, the shallow water equations can be used to
describe the flow of a gravity current down a cone in a manner analogous to the solutions
of Webber et al. (1993) in a channel and on a uniform slope described in §2.3.2. In polar
coordinates the shallow water equations for mass and momentum are

\[ \frac{\partial h}{\partial t} + \frac{1}{r} \frac{\partial (ruh)}{\partial r} = 0 \]  
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + g \frac{\partial h}{\partial r} - g' \tan \theta = 0, \]  

where \( h \) is the height above the cone, \( u \) is the speed of the flow and \( r \) is the distance from the
origin of the cone. The front boundary condition that

\[ Fr = \frac{u_f}{(g' h_f)^{1/2}} \]  

is a constant is again applied. The subscript \( f \) denotes the value of the variable at the front of
the gravity current. The similarity solutions of Webber et al. (1993) for a gravity current in
a sloping channel and for a current on an unconstrained uniform slope are both special in the
sense that the speed is constant. A constant speed means that the momentum equation is purely
a balance between the change in height and the slope effect terms. This leads to a solution with
a flat top and a constant height so the constant Froude number condition can be satisfied. For
an axisymmetric current the radius increases as the current moves down the slope. To maintain
a constant volume the cross sectional area of the current must decrease. If a self-similar
solution existed then the height of the current would have to decrease. In order to maintain
a constant Froude number the speed of the current would also have to decrease. Numerical
solutions of the shallow water equations on a cone show a hydraulic jump forming at the
back of the current and propagating towards the head. The simple numerical code written
to integrate the shallow water equations in this case cannot cope with such a discontinuity
so it cannot be run for long enough to see if any long time similarity solution develops. The
current appears to be getting thinner and deeper, leading to an increase in speed. The hydraulic
7.2. AXISYMMETRIC GRAVITY CURRENTS ON A CONE

Figure 7.1: Profiles of a gravity current on a cone from the shallow water model. Profiles are at non-dimensional time intervals of 5.

jump becomes too steep after about 10 initial radii, or at a non-dimensional time of about 24. This is shown by the cross-sections in figure 7.1. The front height against time is shown in figure 7.2. It can be seen that initially the height decreases as the cone spreads out. However, as the hydraulic jump at the back catches up with the front the current gets deeper again. The constant Froude number condition means that the speed is proportional to the square root of the front height, so the speed shows the same trends as the height. The speed only changes by a relatively small amount in the calculations performed here. The shallow water equations may not be an appropriate way of modelling this type of flow as the deep and thin current violates the shallow water approximation that the horizontal lengthscale of the flow is much greater than the vertical lengthscale.

7.2.3 Numerical results

Figure 7.3(a) shows the predictions of the numerical code, the annulus integral model and the shallow water model for a gravity current released on a cone. The integral model results were calculated assuming a semi-circular shaped head and a typical value for the entrainment coefficient of 0.1 (see chapter 4 and the earlier work of Ellison & Turner 1959 and Hallworth et al. 1996) and a drag coefficient, $C_D = 0$. The precise predictions of the annulus integral
model depend on the exact values taken. The integral model and the numerical code both give
similar predictions for the front position of the gravity current, although the values from the
numerics are slightly higher. Given the uncertainty in the choice of the shape parameters in
the integral model, this difference is to be expected. Figure 7.3(a) shows that the discrepancy
is due to the starting conditions. After a distance of approximately half the initial radius the
predicted speed from the numerics and from the integral model are very similar. Initially the
release in the numerics does not have the self-similar shape assumed by the integral model
and some adjustment period is needed to account for this.

The shallow water model predicts a nearly constant speed for the initial conditions used
here. The speed is higher than both the numerical predictions and the integral model predic-
tions. Since the shallow water model contains no entrainment this is to be expected. The
shallow water predictions for a gravity current on a uniform slope discussed in chapter 4 were
also found to over-predict the front position when compared with experiments.

7.2.4 Experimental results and comparison with theory

Experiments with this axisymmetric geometry were carried out using a sector tank tilted at
an angle, rather than using a cone. This has the advantage of taking up much less space so a
longer slope can be used. It also enables the flow to be viewed from the side, allowing easier

Figure 7.2: Front height of a gravity current on a cone against time.
Figure 7.3: Front position against time from numerics and experiments for a gravity current flowing (a) down and (b) up a cone. For the flow down a cone the integral model and shallow water predictions are also shown. Lengths are non-dimensionalised with respect to $V^{1/3}$ and times with $V^{1/6}/g^{1/2}$.  

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and more accurate comparison with the numerical code. Provided the sector is reasonably wide, it can be expected that the presence of the side walls will not play a significant role in the flow. Previous experiments on horizontal axisymmetric gravity currents have been carried out in a sector tank (see e.g. Huppert & Simpson [1980]) and have been found to agree with experiments using a cylindrical release on a flat surface.

When the sector is tilted, the tank does not exactly reproduce the geometry of a cone as the bottom of the tank is flat not curved in the azimuthal direction. The width of the tank is also not quite correct. However, the error is small for gentle angles of slope and a small sector angle. The sector tank used for the experiments described here was 235 cm long, 41 cm deep and had a sector angle of 9.5°. The tank was tilted to give a slope angle of up to 6°. Both the sector angle and the angle of the slope are sufficiently small to make the experiments a good approximation to a conical geometry.

In addition to comparing the predictions of the numerical model, the annular integral model and the shallow water model, figure 7.3(a) shows results from a typical experiment. In this experiment the sector tank was tilted down at an angle of 5.6° to model the flow of a release on top of a cone. The lock was 60 cm long starting at the point of the sector tank, and the depth at the apex was 13.5 cm.

The numerical simulation assumes a rigid lid so the depth of the channel is constant; in contrast to the experiments where the free surface is horizontal and the depth of the channel varies. To take this into account, an average lock depth was used as the channel depth for the numerics to ensure that there was the same volume of fluid in the lock in each case. The experimental and numerical results agree very well despite the differences in the geometry. The fractional depth is less important in an axisymmetric geometry than in a channel as the current becomes thinner much more rapidly. The integral model appears to under predict the front position of the gravity current. This could be due to the uncertainty in the initial conditions. Choice of different shape parameters and values for the drag and entrainment coefficients can also alter the predictions. The shape parameters were chosen to provide a simple shape which approximated the experimental observations. Although the sector tank is only about 4 times the length of the lock it appears that the flow is approaching a speed similar to that predicted by the integral model. This distance agrees well with the observed distance of about 3 lock lengths for the development of an axisymmetric gravity current on a horizontal surface. After an initial adjustment period the trend in both the experiments and the integral model is the same.

The experiment was repeated with the sector tank tilted in the other direction to simulate a gravity current flowing up the sides of an upturned cone. The slope in this case was 5.5°, the lock was 60 cm in length and 36.8 cm deep at the apex. The numerical results are for a lock of
the same length and initial volume as the experiment, with the same slope. The comparison between the experimental results and the numerical results is shown in figure 7.3(b). Again, the experimental and numerical results agree well. In the limit \( t \to \infty \), the fluid will end up at the bottom of the cone under the force of gravity. This means that there cannot be a long time similarity solution for the flow up a cone. The integral model cannot be valid for flow up a cone as it assumes that such a similarity solution exists.

Figure 7.4 shows images from the experiments and the density fields from the numerics. Each image was taken 12s after the release. It can be seen that in addition to accurately predicting the front position of the gravity current, the numerics provide a good description of the shape and size of the current. The head of a current flowing down a cone is seen to be well defined and contain most of the dense fluid. Behind is a thin layer of slightly less dense fluid. This layer does not stretch all the way back to the origin. In contrast, for a current flowing up an inverted cone, the head of the current is much less clearly defined. The gravity current takes the form of a dense layer of fluid which is slightly higher and denser at the front than at the point of the cone. These features are qualitatively similar to those seen in figures 6.10 and 6.13 for a gravity current in a sloping channel.

One interesting prediction of the annulus integral model is that the cross sectional area of the gravity current head in a vertical plane only changes relatively slowly for moderate values of \( r \). The increase in size as a result of entrainment is nearly balanced by the decrease as a result of the increasing perimeter of the annulus. This corresponds to the \( r \) and \( r^{-1/2} \) terms in (7.4). The nearly constant head height is also observed in the shallow water model (see figure 7.2) and in the experiments.

Flows directed towards the origin were not considered in this work, although they can be modelled using the vorticity–streamfunction code, provided that care is taken at the origin. Such a flow could result from a release of dense gas near the head of a valley.

### 7.3 Results for wind / slope interactions in a channel

One of the aims of this thesis is to study the combined effects of wind and slope on a gravity current. In this section the motion of a gravity current in a sloping channel with an imposed wind is investigated using the vorticity–streamfunction model.

The work of Simpson & Britter (1980) has shown that the effect of a wind with mean speed \( U \) on the speed of a gravity current is approximately \( 0.6U \). Using the integral model of Beghin et al. (1981) for a thermal on a slope the difference in speed of a gravity current on a
Figure 7.4: Density of a gravity current flowing down and up a sloping cone - (a), (c) numerics and (b), (d) experiments
slope compared with the same current on a horizontal surface is about

\[ (K_f \sin \theta - 1.2) g^{1/2} V^{1/4} x_f^{-1/2}, \]  

(7.11)

where \( x_f \) is the non-dimensional front position and \( K_f \sin \theta \) is a weak function of slope, varying from about 2.6 at 15° to about 1.5 at 90°. The value of 1.2 comes from the constant Froude number condition for a gravity current in a flat channel.

Using these two estimates for the effects of the wind and slope leads to a dimensionless group

\[ S = \frac{0.6U}{(K_f \sin \theta - 1.2) g^{1/2} V^{1/4}} \]  

(7.12)

which compares the relative strength of the two effects. The ratio of the change in speed as a result of the wind to the change in speed as a result of the slope is \( S x_f^{1/2} \), so eventually the wind will be expected to dominate the effects of the slope. If \( S << 1 \) then initially the slope will play the major role in determining the speed of the gravity current, while for \( S >> 1 \) the wind will be dominant almost immediately. The distance at which the transition between the slope dominated regime and the wind dominated regime occurs is given by \( 1/S^2 \). This analysis is not valid for currents travelling upslope as the integral model of Beghin et al. (1981) only considers gravity currents travelling downslope.

Experiments on gravity currents in the presence of both wind and a slope were not possible in the flume available in the laboratory. The results in this section are all from a series of numerical simulations consisting of an instantaneous release of a dense fluid. All the measurements and results given in this section are non-dimensionalised. The aspect ratio of the release (defined as half the release length divided by the release height) was 0.875. The channel was twice as deep as the release. Slopes of 0°, 10° and 15° were used. A linear wind profile was used for some of the simulations, with the wind speed at the release height being 0.18, 0.36 or 0.72. For slopes of 10°-15°, \( K_f \sin \theta \approx 2.6 \), so \( S \) is about 0.43 of the dimensionless wind speed, i.e. \( S = 0.08, 0.15 \) or \( 0.30 \). With no wind \( S = 0 \), and with no slope \( S = \infty \). This suggests that for the simulations carried out here the slope will initially be more important in altering the speed of the gravity current, but the effect of the wind will become become dominant after a distance of between 10 and 150 lock lengths.

Figure 7.5 shows the density field for two experiments, one with the wind opposing the slope and one with the wind and slope in the same direction. Also shown are the results for the current with only the wind or the slope present and finally the results in the absence of both wind and slope. It can be seen that the direction of the slope makes a difference to the front and back positions of the gravity current, as might be expected. The most noticeable effect of the slope is to alter the head profile. The head at the downslope end of the current
is always much higher and more bluff than the upslope end. The wind has some effect on the head shape, but it is less pronounced than the effect of the slope. With a following wind the head becomes higher and more bluff. With an opposing wind the head becomes thinner and more pointed. As would be expected, the head moves further with a following wind and is slowed down by a head wind.

Figure 7.6 shows the predicted front position for different combinations of slope and head / tail winds. In this case the wind is relatively strong and it is this which most affects the front position, although the slope still plays an important role as well. For these conditions the difference in the distance travelled by the current can be huge. At time $t = 10$, the front position can ranging from $x \approx 3$ for a current travelling upslope against the wind, to $x \approx 10$ for a current travelling downslope with a following wind.

In figure 7.7 the difference between the predicted front position with and without the wind is plotted for currents on various slopes. For a given wind speed, the data collapses very well, except at the lowest wind speed. For all the wind speeds, the difference in the front speed is close to the empirical value of 3/5 suggested by Simpson & Britter (1980).

Near the release point there is some deviation from this value as the current develops. There is a short initial period during which the gravity current forms and its speed adjusts to take account of the applied wind and slope. Since the work of Simpson & Britter (1980) was for a steady gravity current their empirical formula is not strictly applicable to the initial stages of the flow.

The difference in front position is non-dimensionalised on the ambient wind speed, so the error due to the finite grid spacing is largest for the smallest wind speed. This partly explains the larger scatter in the results for low wind speeds. The deviation is particularly bad for a current travelling upslope with a small following wind. When the wind is no longer sufficient to keep the current travelling upslope and the head begins to grow smaller and more dilute the current will become arrested. This changes the dynamics of the head significantly so it is not surprising that the behaviour is different.

The collapse of the data in figure 7.7 suggests that, to a first approximation, the effect of the slope on the front position can be decoupled from the effect of the wind, at least provided the current does not become arrested. For the highest wind speed a slope of $15^\circ$ was used as well as the slope of $10^\circ$. The collapse of the data at this wind speed is seen to be very good, even for the different slopes. Figure 7.8 shows the difference in front position with and without slope for various wind conditions. There are two sets of results, one for the difference between a current going downslope and the same current on the flat (shown in blue) and the other for the difference between a current on the flat and a current going upslope (shown in red). For each set of results the data collapses reasonably well, although towards the end of the
simulation differences are beginning to be seen. The apparent steps in the results are caused by
the discretisation of the front position on the numerical grid. These steps are exacerbated by
the face that the results plotted are the relatively small difference between two larger numbers.
The reasonably good collapse again suggests that the coupling between the wind and the slope
is only weak. This is an important result as, to a first approximation, it allows the two effects
to be considered separately, leading to a great simplification in the problem.

Figure 7.9 is similar to figure 7.7, but for an opposing rather than a following wind. Again
the collapse of the data is seen to be good for the two higher wind speeds. The resolution
of the grid is not sufficient to accurately measure the differences for the lowest wind speed.
Once again the results for an upslope current show the greatest deviation. The effect is more
pronounced for a larger slope since the current is brought to rest soon. Study of the density
fields shows that the time at which the difference in front position deviates from the expected
data collapse corresponds to the time at which the front runs out. At this time the supply
of dense fluid into the head from the flow behind ceases, the density in the head drops and
the head starts to become much shallower. This is illustrated by the sequence of images in
figure 7.10. The images are (a) just before the transition, (b) at the time of the transition
and (c) shortly after the transition. The reduction in the size and density of the head can be
seen. The separation of the upslope head from the following flow is also visible. This run out
is clearly important and it is hoped that further investigation will explain exactly when and
where it occurs.

The results in figure 7.9 are non-dimensionalised using the wind speed at half the release
height as opposed to the wind speed at the release height used in figure 7.7. After an initial
adjustment phase the empirical formula of 3/5 the wind speed (or 3/10 the wind speed at the
release height) seems to provide a good estimate of the difference in speed resulting from the
opposing wind. The adjustment phase is more pronounced for an opposing wind than for the
wind assisted case. The theoretical work of Xu [1992] (described in §2.4 in particular see
figure 2.3) suggests that, for a steady gravity current in a horizontal channel with a linear wind
profile, the change in speed for a current in an opposing wind may be significantly less than
the 3/5 of the applied wind speed suggested by Simpson & Britter [1980]. This may explain
why the results for the opposing wind case collapse better when scaled on the wind speed
at half the release height. For a wind speed of 0.72 the predicted difference in the speed of
the head is 0.42 of the applied wind speed for an opposing flow compared with 0.57 for a
following wind.
(a) Wind from left to right, slope down to the left.

(b) Wind from left to right, slope down to the right.

(c) Channel slopes to the right, no wind.

(d) Wind from left to right, no slope.

(e) No wind or slope.

Figure 7.5: Density of a gravity current in a sloping channel with wind. The slope is 10°. The wind has a linear profile and the wind speed is at height $h_0$ is $0.72(g'_0 h_0)^{1/2}$, where $g'_0$ and $h_0$ are the initial effective gravity and height of the current.
7.4. SUMMARY

Figure 7.6: Front position of a gravity current against time. Results are shown from numerical simulations for different combinations of slope and wind. All results are non-dimensional. The non-dimensional wind speed was 0.72 at the height of the release and the wind profile was linear. The slope was 10°.
Figure 7.7: Difference between front position with and without an assisting wind for a gravity current in a channel, plotted against time. Results are shown from numerical simulations for no slope ($\times$), a downward slope of $10^\circ$ (+), an upward slope of $10^\circ$ (■), a downward slope of $15^\circ$ (○) and an upward slope of $15^\circ$ (△). All results are non-dimensional. The time is non-dimensionalised with $V^{1/4}g^{1/2}$. The difference in front position is non-dimensionalised by the wind speed at the release height multiplied by the timescale $V^{1/4}g^{1/2}$. The wind speeds, non-dimensionalised on $V^{1/4}g^{1/2}$, were 0.72 (blue), 0.36 (red) and 0.18 (green) at the height of the release. The wind profile was linear and the slope was $10^\circ$. The solid line is the empirical formula of $3/5$ the applied wind speed from Simpson & Britter (1980).
Figure 7.8: Difference between front position with and without slope for a gravity current in a channel plotted against time. Results are shown from numerical simulations for different wind conditions. The blue points are the difference between a slope down and no slope. The red points are the difference between no slope and a slope up. All results are non-dimensional. The non-dimensional wind speed was 0.72 at the height of the release and the wind profile was linear. The slope was 10°.
CHAPTER 7. TWO NEW PROBLEMS

Figure 7.9: Difference between front position with and without wind an opposing wind for a gravity current in a channel, plotted against time. Results are shown from numerical simulations for no slope (×), a downward slope of 10° (+), an upward slope of 10° (■), a downward slope of 15° (○) and an upward slope of 15° (△). All results are non-dimensional. The time is non-dimensionalised with \( V^{1/4} g^{1/2} \). The difference in front position is non-dimensionalised by the wind speed at half the release height multiplied by the timescale \( V^{1/4} g^{1/2} \). The wind speeds, non-dimensionalised on \( V^{1/4} g^{1/2} \), were 0.72 (blue), 0.36 (red) and 0.18 (green) at the height of the release. The wind profile was linear and the slope was 10°. The solid line is the empirical formula of 3/5 the applied wind speed from Simpson & Britter (1980).
Figure 7.10: Density of a gravity current in a channel sloping at an angle of 10° to the horizontal. The channel slopes down to the right. A wind of speed 0.36 is blowing from left to right. The images are at non-dimensional times (a) 6.2, (b) 7.2 and (c) 8.1. The transition in the head travelling upslope occurs at about a time of 7.2.
CHAPTER 7. TWO NEW PROBLEMS

7.4 Summary

Two new problems have been considered in this chapter. The first is the instantaneous release of a gravity current on a cone. The second is a study of a gravity current in a sloping channel with an ambient flow.

For the release on a cone some new experiments were performed and an integral model developed. The vorticity–streamfunction model provides good agreement with the experiments. The integral model and the shallow water model are less good at predicting the propagation of the gravity current. While they both get the right trends, the shallow water model over predicts the speed of the gravity current and the integral model fails to capture the details of the current near the release point. Starting the integral model further away from the origin, once the current has developed, would produce better results. As with the model of Tickle (1996) for a current on a uniform slope, the lack of entrainment in the shallow water model is responsible for the over-prediction of the front speed.

A study of the effects of both wind and slope has been conducted using the vorticity–streamfunction model. The propagation of the gravity current is seen to be affected by both the wind and the slope. The slope has the biggest effect on the shape of the head, with the downslope head being much deeper and more blunt than the upslope head. The wind also acts to alter the shape of the head. However, it is seen that, to a first approximation, the effects of slope and wind on the front position can be separated. This means that the problem can be considered in two parts, and previous results with a slope or an applied wind can be used. The numerical model has proved a useful tool for making more detailed quantitative and qualitative predictions about the currents and for determining where the assumption that the wind and slope act independently breaks down.
Chapter 8

Conclusions and future work

8.1 Conclusions

8.1.1 Summary of work

In this thesis the effects of both slope and wind on the motion of a gravity current have been studied. These external influences can play an important role in the development of the gravity current. The role of topography and wind can be particularly relevant to environmental applications, for example the dispersion of a dense gas spillage in the atmosphere. Aspects of this problem have been investigated though a mixture of laboratory experiments, simple integral models and numerical simulations.

In chapter 4 the problem of an instantaneous release of dense fluid on a uniform slope, with no barriers or constraints, was considered. In this chapter there was assumed to be no wind. A new wedge integral model was presented. Results from new laboratory experiments were given and confirmed that, after an initial spreading period, the gravity current developed into a wedge shape which moved down the slope. A detailed comparison was made between the wedge integral model and the experimental results. While the simple integral model does not accurately capture the shape of a gravity current, it does make reasonable quantitative predictions of the front position and the dilution of the current. The present work has shown that the presence of a slope has a significant impact on the spreading of a gravity current, as might be expected. The important role of entrainment was highlighted by the differences between the model of Webber et al. (1992) which include no entrainment and models such as the wedge integral model developed in §4.3.2 which do include entrainment. Entrainment plays an important role in diluting the gravity current and slowing it down. Experimental observations and the wedge integral model suggest that entrainment predominantly occurs in a region near the front of the gravity current. The entrainment coefficient was inferred from
the experimental results and found to be a constant, with a value of about 0.09, over a range of slopes from 0° to 20°. The value of the coefficient is in agreement with previous work on gravity currents on horizontal surfaces and other related flows.

Chapter 5 concentrated on the problem of a dense jet, or forced plume, directed up a slope. From the laboratory experiments, the flow was initially observed to move upslope and behave like a forced plume. As the flow slowed down and spread out it developed into a wider layer of dense fluid that no longer resembled a plume. The dense fluid reached a maximum height upslope then drained back down the slope. The flow was seen to be predominantly two-dimensional with the density difference inhibiting vertical spreading. The fluid moving back down the slope did not flow over the top of the upslope plume, but instead spread down the sides. A plume model was developed for the upward part of the flow and results from it are presented. Entrainment was found to be important in determining how far upslope the flow reached in the plume model. The results from the experiments were compared with simple scalings and with predictions of the plume theory. The initial part of the flow, where the behaviour is plume like, can be reasonably described by the plume model. The spread of the plume and the maximum upslope extent were found to scale on the jet length, \( l_{MF} = (M_0^3/F_0^2)^{1/4} \), based on the initial conditions of the plume. A best fit to the experimental data gave an expression

\[
h_{max} = 0.2080 + 4.180 l_{MF} \sin^{3/4} \theta
\]

for the maximum vertical rise height of the plume. Observations of the downslope draining region in the experiments suggested that viscous forces may be important in that region. The spread of this region was compared with the shallow water model of Bonnecaze & Lister (1999) and the lubrication theory model of Lister (1992). The experimental results for the edge of the draining region appeared to obey the same power law relationship as the lubrication theory model, suggesting that viscous forces were important. The interaction of the upward flow with the downward return flow is also a factor in determining the development of the plume. It can affect the entrainment into the plume and also exert an extra drag on the flow. The plume model described here does not incorporate this interaction.

An extension of the plume model was made to describe releases initially directed at an angle to the upslope direction. Some predictions from the model were given for these tilted plumes and the predictions compared with further laboratory experiments. The agreement was not as good for the tilted plumes as for the plumes directed upslope. However, the model does capture the qualitative features of the flow. Again the jet length was demonstrated to be the important length on which the flow scales.

A numerical model was developed in chapter 6 for a two-dimensional, instantaneous re-
lease gravity current (i.e. a current in a channel or an axisymmetric current). The model allows for the presence of both a slope and an imposed wind. The model (based on the vorticity-streamfunction formulation of the Euler equations) was discussed, along with methods for obtaining numerical solutions to it. The model was compared with previous theoretical and experimental results to validate the code. Despite being two-dimensional and containing no explicit viscous terms, the numerical model provided good qualitative and quantitative agreement with experiments for a variety of channel and axisymmetric flows. Numerical viscosity is present in the code as a result of the discretisation. The model shows Kelvin–Helmholtz type instabilities forming on the interface between the fluids. The billows are larger and more prolonged in the simulations than in experiments due to the two-dimensional nature of the model. Experimentally, the billows break down through a three-dimensional instability. The good agreement between the predictions from simulations and results from previous studies gives confidence in the use of the model to study further aspects of gravity currents.

In chapter 7 the numerical model developed in chapter 6 was used to study two new problems. The first was the instantaneous release of a dense fluid on a cone. New experiments and an integral model developed here were compared with the numerical results for the flow on a cone. The numerical model and the experiments were seen to be in very good agreement. The agreement between the integral model and the experiments was less good, mainly because of the failure to model the initial development of the current. The initial stages can play a larger role for the conical geometry that for the tilted channel because of the rapid of the current near the origin in a radial geometry. After an initial adjustment phase, the integral model provided much better predictions for the speed of the current. Comparison was also made with a shallow water model. As for the model of Tickle (1996) for a uniform slope, the shallow water model over-predicts the front position of the current because of its failure to take entrainment into account.

The second case studied was the flow of a gravity current in a sloping channel with an imposed wind. Both head and tail winds were considered. It was found that, to a first approximation, the effects of wind and slope on a gravity current can be considered separately. This assumption makes the modelling of such a flow considerably easier. The assumption was observed to break down when the gravity current head ran out, i.e. when the head became detached from the flow behind and rapidly diluted. This run out was observed for a current travelling upslope, and was particularly pronounced when the wind was opposing the flow.

8.1.2 Application to the spreading of a cloud of dense gas

As discussed in 8, there has been few large-scale field experiments on the spread of dense gases in the atmosphere. Those studies which have taken place have been predominantly for
axisymmetric releases on a horizontal surface. This study of the effects of slope and wind on a gravity current instead has to focus on the results of laboratory experiments, theoretical models and numerical simulations.

In this thesis work has been presented on gravity currents on several types of slope. The experiments with both instantaneous and continuous releases on a uniform slope provide important details of the spread and mixing of such currents. The work has concentrated on moderate slopes of up to 20°. Most sites on which chemical plants are built have slopes within this range. Knowledge of the maximum upslope extent of a gravity current is useful in assessing the minimum safe distance to leave between a chemical plant and other buildings. Observations of the wedge shape current formed following an instantaneous release on a slope suggest that the region covered by the current will not extend very far upslope or across the slope from the source. A region downslope of the source, with increasing width, will be affected. For the continuous release case the cross-slope extent is also found for a given distance down the slope.

The concentration data obtained from the conductivity probe measurements can be used to estimate peak concentrations and cumulative doses at various distances from the source. Such calculations are important to evaluate the effects on humans of a chemical spillage. The data collected from the experiments on the spread and dilution of the gravity currents are a useful resource for validating and comparing both existing and future models for dense gas dispersion.

Integral models are still frequently used for modelling dense gas dispersion. More sophisticated models often cannot be justified because of the uncertainty in the initial release, the terrain and the atmospheric conditions. After an initial transition period, the integral models for an instantaneous release on a uniform slope and on a cone have been demonstrated to be in fair qualitative and quantitative agreement with the laboratory experiments. In the initial transition period, while the current is forming, the effects of slope can often be neglected. This information about the accuracy of the models is useful in devising more sophisticated integral models to include thermodynamic and atmospheric effects. The results of the numerical simulations in a sloping channel with a wind suggest that, at least in certain situations, the effect of the wind can be incorporated into an integral model by translating the solution with a speed of 60% of the mean applied wind speed (calculated over the initial height of the current.)

Comparison of the experimental results for an instantaneous release on a uniform slope with predictions of the wedge integral model has demonstrated the importance of entrainment. As discussed in the introduction, atmospheric turbulence and convection can increase mixing between the gas cloud and the air. The values for the entrainment deduced from the laboratory experiments may not be appropriate for the atmospheric case. The work described here sug-
suggests that increased entrainment can be expected to slow the movement of the current down
the slope. Increased dilution is useful in reducing the risk of explosion. It can also have ad-
verse effects by increasing the width of a gas cloud, thus enlarging the area affected by it. For
toxic gases, the cumulative dose may not be significantly reduced by increasing the mixing as
a more dilute cloud will move more slowly and take longer to pass a fixed point.

In addition to integral models, shallow water models are also frequently used for modelling
gravity currents. Comparison of the experiments with the work of Webber et al. (1993) and
with the shallow water solutions presented in §7.2.2 have shown that for a variety of sloping
geometries these shallow water models over-predict the spread of a current. To obtain more
accurate predictions it is necessary to include the effects of entrainment.

For axisymmetric or channel geometries the numerical vorticity–streamfunction model
has proved a useful tool. The ability to include the effects of wind allows the model to be used
directly for making predictions in some situations, such as flow down a valley. The study of
the combined effects of wind and slope carried out here has demonstrated that the interaction
between wind and slope is relatively weak, at least until the head of the gravity current runs
out. This is important for developing integral models where correction terms for these effects
are often treated separately. This numerical study suggests that such an assumption is rea-
sonable. The numerical predictions can be tested against existing or future integral models to
assess how accurate these models may be. The numerical model demonstrates that the pres-
ence of wind can significantly affect how far upslope a gas cloud can reach before the supply
of dense fluid from behind ceases. While the cloud may continue upslope after this point,
the head rapidly dilutes and so poses a much reduced threat to people in its path. The model
could be used to indicate combinations of wind and slope which might lead to slow dispersion
and large doses of a toxic gas. Such situations can then be studied in more detail to try and
minimise or avoid any risk in the event of an accident.

Since the interaction between the effects of slope and wind on a gravity current has been
seen to be relatively weak in a channel, it might be supposed that the same could apply to
other geometries such as the instantaneous releases on a cone or an unconfined slope. In the
case considered here the wind is always acting in the same direction or directly against the
slope. On an unconfined slope there is also the possibility of a cross wind. The analysis of the
Thorney Island field trials by Rottman et al. (1985) suggested that the effect of the wind on an
initially axisymmetric release on a horizontal surface could be reasonably approximated by the
same empirical formula given by Simpson & Britter (1979, 1980) for a current in a channel.
Again it might be supposed that the effects of wind and slope can, to a first approximation,
be considered separately. The wedge integral model could be used to predict the downslope
spread, and a speed of 60% of the mean wind speed used to translate the wedge in the across
Releases of a dense gas in more complicated terrains can often be broken down into simpler parts, such as those studied here. For example, consider a release in a valley. Near the top of the valley the width might increase rapidly in the downslope direction. A conical geometry may be most appropriate to model this situation. Further down the width of the valley might become relatively constant and so a sloping channel could be a more suitable model. At the bottom of the valley there may be a plain on which the current will behave like an axisymmetric release on a horizontal surface. Each stage could be modelled separately using the various integral, shallow water and vorticity–streamfunction models discussed in this thesis, and the output from one model used as the initial conditions for the next stage. There will be some adjustment region where one solution alters to the next, possibly with the formation of a hydraulic jump. It is expected that this region will scale on the height of the current, which is generally much less than the horizontal extent of the current, so the effect of the adjustment region is small. This piecewise use of the various models allows a far larger range of problems to be tackled.

8.2 Future work

Further extensions of the numerical model in chapter 6 are planned to deal with a channel with the width and slope being slowly varying functions of the distance, \( x \), from the release. Provided the changes are slow, the flow can still be considered as essentially two-dimensional therefore the vorticity–streamfunction formulation can be applied. It is also hoped to include the effects of wind in the axisymmetric geometry. While a wind blowing in all directions down a hill may not be very realistic, there is a strong likelihood that a wind will be funneled up or down a sloping valley. It is also hoped to alter the numerical model to allow for non-rectangular domains. This would allow more realistic modelling of flows in sloping channels, where the depth of the ambient fluid changes as a result of the slope. The wind profiles used in chapters 6 and 7 have been idealised. In atmospheric applications more complicated winds will occur. The simple cases studied can demonstrate some of the important effects of a background flow, but if necessary the numerical model can also be used to look at more realistic profiles.

The experimental investigation of gravity currents on steep slopes is still an area to be investigated. This was not possible in the current research due to the size of the tank available. Such a study would provide an important link between the study of gravity currents on horizontal surfaces or small slopes and the study of thermals and plumes against a wall. This problem could also be studied with the vorticity–streamfunction model developed in chapter 6.
although here attention has been concentrated on relatively gentle slopes.

An experimental investigation of gravity currents in a sloping channel with a background flow has also yet to be undertaken. These experiments were not possibly in the flume available. Such experiments would be useful in verifying the predictions of the new vorticity–streamfunction model and in clarifying the role of the wind profile in real experiments. The vorticity–streamfunction model cannot be used to investigate the interaction between a gravity current on a uniform slope and the wind since the problem is not two-dimensional. Experiments are needed in this problem, though again they are difficult to realise in the laboratory. Other theoretical approaches will have to be taken to model such a flow.

8.3 Concluding remarks

The class of problems labelled as gravity currents covers a large range of physical phenomena. The horizontal density differences which drives them all provides a common link, but many other competing influences can play a role. The dispersion of a cloud of dense gas in the atmosphere can be affected by many factors such as the background wind, the stability of the boundary layer, the topography and the thermodynamics of the release. These factors combine to make it a complicated problem to model. Basic research into individual effects, for example slope and wind, provides the building blocks from which a more complete understanding of the whole flow can be constructed. This thesis has succeed in adding to our knowledge about some of these effects. Hopefully the results will aid in reducing the risk from an accidental release of dense gas and ensure that a tragedy on the scale of Bhopal never occurs again.
Bibliography


