Mathematical Tripos Part II: Michaelmas Term 2020

Numerical Analysis

1. Course description

Here is an approximate content of the course.

1. **The Poisson equation**
   Finite differences. Accuracy of the five-point scheme. High-order methods. The nine-point scheme.
2. Linear system $A\boldsymbol{u} = \boldsymbol{b}$ arising from the five-point scheme.
   Natural ordering. Gershgorin theorem. Properties of A.
3. Eigenvalues of A. Rate of convergence of the five-point method.
   Hockney method and uncoupled systems.
4. Discrete Fourier transform (DFT). Fast Fourier transform (FFT).
5. **Partial differential equations of evolution**
   The diffusion equation. The Courant number.
   Convergence and stability. The Lax equivalence theorem. Semidiscretization.
6. The Crank-Nicolson scheme. Eigenvalue analysis of stability. Normal matrices.
7. Stability and convergence of the Crank-Nicolson method for the diffusion equation.
   The Euler method for advection equation.
8. Fourier analysis of stability. Parseval identity. Stability for the diffusion equation.
   The advection equation. The leapfrog method. The wave equation.
9. The diffusion equation in two space dimensions. Fourier analysis. The Crank-Nicolson for 2D.
10. Splitting technique. The Crank-Nicolson split version.
    Approximation of the matrix exponential. Splitting of inhomogeneous systems.
11. **Spectral methods**
    Fourier approximation. The Gibbs effect. Approximation of periodic functions.
    Spectral speed of convergence.
12. The algebra of the Fourier expansions. Application to ODEs. The fast Fourier transform (FFT).
13. The Poisson equation. General second-order linear elliptic PDE.
14. The Chebyshev polynomials and Chebyshev methods.
    The algebra of the Chebyshev expansions. Derivatives.
15. Spectral methods for evolutionary PDEs.
16. **Iterative methods for linear algebraic systems**
    Splitting methods. Convergence criterion. Jacobi and Gauss-Seidel methods.
17. Diagonally dominant and positive definite matrices. The Housholder-John theorem.
    Relaxation methods.
18. The damped Jacobi iteration. Multigrid method for the Poisson equation.
19. Minimization of quadratic functional. The conjugate gradient method (CGM).
20. Convergence of the CGM. Krylov subspaces.
21. Properties of Krylov subspaces. Number of iterations in CGM. Preconditioning.
22. **Eigenvalues and eigenvectors**
    The power method. Inverse iteration.
23. Deflation. Algorithms for deflation. Transfomation to the upper Hessenberg form.
24. The QR algorithm. Convergence of the first column and the bottom row.

2. Appropriate books

1. G. H. Golub and C. F. van Loan, Matrix computations, John Hopkins Press, 1996.

2. A. Iserles, A first course in the numerical analysis of differential equations, Cambridge University Press, 2009.

3. K. W. Morton and D. F. Mayers, Numerical solution of partial differential equations: an Introduction, Cambridge University Press, 2005.

⋆ To a large extent, the course follows the book [2] where you can find much more details on each subject.

5. Communication

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