Inverse Problems in Imaging (M16)

Yury Korolev

Inverse problems arise whenever there is a need to infer about quantities of interest from indirectly measured data. Inverse problems are ubiquitous in science; they arise in physics, biology, medicine, engineering, finance and computer science (e.g., in machine learning and computer vision). Many imaging problems, such as reconstruction of medical images (computer tomography, magnetic resonance imaging, positron-emission tomography) and deblurring or denoising of microscopy and astronomy images, are also instances of inverse problems. Inverse problems typically share a feature that makes them challenging to solve in practice: they lack continuous dependence on the data and, therefore, small errors in the measurements can lead to large errors in the reconstructions, making them practically useless. To deal with this issue, special *regularisation* techniques have been developed that overcome the instability by using additional a priori information about the solution, such as smoothness or sparsity in some basis. In this course we will present the mathematical theory of inverse problems and regularisation, from the classical foundations to modern variational regularisation methods, and apply it to some problems in imaging using state-of-the-art numerical algorithms.

Pre-requisites

This course assumes basic knowledge in linear algebra and analysis (e.g. linear analysis or analysis of functions). Additional knowledge in convex analysis is beneficial, but not mandatory.

Literature

1. H. W. Engl, M. Hanke and A. Neubauer. *Regularization of Inverse Problems.* Vol. 375, Springer Science & Business Media, 1996, ISBN: 9780792341574

2. O. Scherzer, M. Grasmair, H. Grossauer, M. Haltmeier and F. Lenzen. *Variational Methods in Imaging.* Applied Mathematical Sciences, Springer New York, 2008, ISBN: 9780387309316

3. A. Chambolle, T. Pock, *An introduction to continuous optimization for imaging*, Acta Numerica, **25**, 161-319 (2016). Also available at

   https://www.cambridge.org/core/journals/acta-numerica/article/an-introduction-to-continuous-optimization-for-imaging/1115AA7E36FC201E811040D11118F67F

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.