Introduction to Nonlinear Spectral Analysis (M16)

Non-Examinable (Graduate Level)

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Spectral filtering methods based on eigenfunctions of a linear operator, such as sines and cosines in the case of the Laplace operator, are a classical tool in signal processing. They can be interpreted as stationary points of a certain Rayleigh quotient which in the case of the Laplace operator is related to the Dirichlet energy $E(u) := \frac{1}{2} \int |\nabla u(x)|^2 \, dx$. In this course, we will see how this perspective can be used to define a eigenfunctions of a *nonlinear* operator via stationary points of a Rayleigh quotient involving an associated convex functional. We will also see how gradient flows and nonlinear power iterations can be used to compute such eigenfunctions, and highlight applications in data science.

Lecture notes are developed jointly with Leon Bungert (University of Bonn).

Prerequisites

This course assumes basic knowledge in linear algebra and analysis (e.g. Linear Analysis or Analysis of Functions). Additional knowledge in convex analysis is beneficial but not mandatory since necessary background will be provided in the lectures.

Literature

1. L. Bungert and M. Burger, *Gradient Flows, Nonlinear Power Methods, and Computation of Nonlinear Eigenfunctions* (2021). Available at https://arxiv.org/abs/2105.08405;

2. L. Bungert, M. Burger, A. Chambolle and M. Novaga. *Nonlinear Spectral Decompositions by Gradient Flows of One-Homogeneous Functionals.* Analysis & PDE, vol. 14, No. 3, pp. 823–860 (2021). Also available at https://arxiv.org/pdf/1901.06979.pdf;

3. G. Gilboa, M. Moeller and M. Burger, *Nonlinear Spectral Analysis via One-Homogeneous Functionals: Overview and Future Prospects.* Journal of Mathematical Imaging and Vision, vol. 56, pp. 300–319 (2016). Also available at https://arxiv.org/abs/1510.01077.