

## Part III: Applications of Differential Geometry to Physics

G W Gibbons: Lent Term 2005

### Contents of the Course

#### I: Differential Forms

- I.1 Grassmann Algebra: Interior Product and Wedge Product
- I.2 Hodge Duality
- I.3 Exterior Differentiation
- I.4 Behaviour under pull-back
- I.5 Stokes's Theorem

#### II: Action of Groups on Manifolds

- II.1 Definition and Elementary Properties of Group Actions
- II.2 Homogeneous Spaces and Co-set spaces.
- II.3 Left and Right Actions on Groups
- II.4 Representations of groups

#### III Geometry of Lie Groups

- III.1 Left and right invariant vector fields
- III.2 Exponential Map
- III.3 Cartan-Maurer Equations
- III.4 Connections and Metrics on Lie Groups
- III.5 Geodesics and Auto-parallels on Lie Groups

#### IV Fibre Bundles

- IV.1 Definition of Fibre bundles
- IV.2 Principal Bundles, Vector Bundles, Associated Bundles
- IV.3 Connections on Bundles, Curvature and Cartan's Equations

#### V Symplectic Geometry and Mechanics

- V.1 Hamiltonian Mechanics
- V.2 Poisson and Symplectic Manifolds
- V.3 Hamiltonian Symmetries, Poisson Brackets and Lie Brackets
- V.4 Moment maps and Hamiltonian Reduction
- V.6 Elementary ideas about Geometric Quantization

Physical applications will be introduced along the way. Handwritten summaries of each lecture will be provided at the beginning of the following lecture. However since these summaries were written for an earlier version they might be slightly 'out of synch'. A transcript of a previous year's lectures was prepared by one of the audience using microsoft word. This is available on my web page

<http://www.damtp.cam.ac.uk/user/gr/members/gwg.html>

to users inside DAMTP.

#### Surgeries

I shall be available in my office (B1.24) from 2.00pm to 3.00pm on most Mondays to answer queries. A sign-up sheet for supervisions will be circulated during lecture 3.