Discrete Symmetries and Gravity

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In this talk I shall begin by recalling some of Andrew's scientific work.

I shall then take this as a springboard to review some questions centring around the the issue of discrete symmetries in gravity and their relation to the signature of spacetime, spinors and spacetime topology.

According to SPIRES Andrew produced

Very well-known papers (100-249) : 4 Well-known papers (50-99) : 4 Known papers (10-49) : 19 Less known papers (1-9) : 16 Unknown papers : 3

Total eligible papers analyzed: 46 Total number of citations: 1436 Average citations per paper: 31

This is a remarkable record for one so young.

Very Well Known Papers

Brane world black holes. A. Chamblin, S.W. Hawking, H.S. Reall (Cambridge U., DAMTP) . DAMTP-1999-133, Sep 1999. 9pp. Published in Phys.Rev.D61:065007,2000 e-Print Archive: hep-th/9909205

Cited 233 times

Supergravity on the brane. A. Chamblin, G.W. Gibbons (Cambridge U., DAMTP) . DAMTP-1999-126, Sep 1999. 4pp. Published in Phys.Rev.Lett.84:1090-1093,2000 e-Print Archive: hep-th/9909130

Cited 133 times

Dynamic dilatonic domain walls. H.A. Chamblin, H.S. Reall (Cambridge U., DAMTP) . DAMTP-1999-35, Mar 1999. 32pp. Published in Nucl.Phys.B562:133-157,1999 e-Print Archive: hep-th/9903225

Cited 205 times

Charged AdS black holes and catastrophic holography. Andrew Chamblin (Cambridge U., DAMTP) , Roberto Emparan (Durham U., Dept. of Math. Basque U., Bilbao) , Clifford V. Johnson (Kentucky U.) , Robert C. Myers (McGill U.) . DAMTP-1999-29, EHU-FT-9902, UK-99-02, MCGILL-99-07, Feb 1999. 19pp. Published in Phys.Rev.D60:064018, e-Print Archive: hep-th/9902170

Cited 119 times

Well known papers

Black hole production at LHC: String balls and black holes from pp and lead-lead collisions. Andrew Chamblin (Los Alamos & Queen Mary, U. of London) , Gouranga C. Nayak (Los Alamos) . Jun 2002. 5pp. Published in Phys.Rev.D66:091901,2002 e-Print Archive: hep-ph/0206060

Cited 60 times

Charged brane world black holes. Andrew Chamblin (MIT, LNS) , Harvey S. Reall (Cambridge U., DAMTP) , Hisa-aki Shinkai (Penn State U.) , Tetsuya Shiromizu (Potsdam, Max Planck Inst. & Tokyo

U. & Tokyo U., RESCEU) . DAMTP-2000-85, CGPG-00-8-1, UTAP-375, Aug 2000. 11pp. Published in Phys.Rev.D63:064015,2001 e-Print Archive: hep-th/0008177

Cited 76 times

Holography, thermodynamics and fluctuations of charged AdS black holes. Andrew Chamblin (Cambridge U., DAMTP) , Roberto Emparan (Durham U., Dept. of Math. & Basque U., Bilbao) , Clifford V. Johnson (Kentucky U.) , Robert C. Myers (McGill U.) . DAMTP-1999-54, EHU-FT-9907, DTP-99-25, UK-99-5, MCGILL-99-15, Apr 1999. 29pp. Published in Phys.Rev.D60:104026,1999 e-Print Archive: hep-th/9904197

Cited 57 times

Large N phases, gravitational instantons and the nuts and bolts of AdS holography. Andrew Chamblin (Cambridge U.) , Roberto Emparan (Durham U., Dept. of Math.) , Clifford V. Johnson (Kentucky U.) , Robert C. Myers (McGill U.) . Aug 1998. 20pp. Published in Phys.Rev.D59:064010,1999 e-Print Archive: hep-th/9808177

Cited 56 times

In Minkowski spacetime a great deal of interest centres on Parity P and Time reversal T which are isometries reversing space or time orientation respectively. In an even dimensional spacetime, their composition $P \circ T = T \circ P$ or total inversion I reverses all spacetime coordinates

$$I \rightarrow: x^{\mu} \rightarrow -x^{\mu} . \tag{1}$$

According to 't Hooft and Nobbenhuis,(gr-qc/0602076) total inversion I has a square root

$$\sqrt{I} \rightarrow: x^{\mu} \rightarrow i x^{\mu} . \tag{2}$$

This has the effect of reversing the spacetime signature

$$\eta_{\mu\nu} \rightarrow -\eta_{\mu\nu} \tag{3}$$

In quantum mechanics we usually represent P and T by unitary and anti-unitary operators $\hat{P}$. $\hat{T}$ on the quantum mechanical Hilbert space $\mathcal{H}_{\rm qm}$ and if spinors are present

$$\begin{aligned} \hat{P}^2 &= \pm 1 && (4) \\ \hat{T}^2 &= \pm 1 && (5) \\ \hat{P}\hat{T} &= \pm \hat{T}\hat{P}\,. && (6) \end{aligned}$$

There are eight covers of the Bosonic viergruppe $\equiv Z_2 \times Z_2$ generated by P, T and two of these are Cliffordian, i.e.isomorphic to that generated by the gamma matrices

$$\begin{aligned} \gamma_0^2 &= \pm 1 && (7) \\ \gamma_1^2 &= \mp 1 && (8) \\ \gamma_0\gamma_1 &= -\gamma_1\gamma_0\,. && (9) \end{aligned}$$

Andrew studied the resulting superselection rules in the general case.

A distinction, clearly brought out in Andrew's paper, is between the Racah and the Wigner approach to discrete symmetries for fermions. The Racah approach is basically to use linear (in fact Cliffordian) representions which are complex linear.

On the another hand Wigner (followed by almost all textbooks) used what he called co-representations which contain anti-linear and hence (non-Cliffordian) elements.

The distinction is in fact between an action of the space of classical solutions of the fermionic equations of motion and on the first quantized Hilbert space one constructs from it

In what follows we shall mainly be concerned with the former.

A futher source of confusion is that charge conjugation is a unitary operator in quantum mechanics but is often represented anti-linearly on the space of solutions of the Dirac equation.

Some thought will reveal that what is going on here is is that people use more than one complex structure on the same space and don't let on (or may not even being aware of the fact) when they do it.

In the case of C it would appear that then use of complex numbers quantum mechanically and to describe an $O(2)$ symmetry is not helpful.

A general curved spacetime $\{M, g\}$ may fail to be space, time, or spacetime orientable , but we may always pass to a suitable covering spacetime $\tilde{M}$ which is. A spacetime which is not space, time, or spacetime orientable may thus be regarded as a quotient, e.g.

$$M = \tilde{M}/T \tag{10}$$

This induces an action on the Hibert space

$$\mathcal{H}_{qm}[M] = \mathcal{H}_{qm}[\tilde{M}]/\hat{T}\,. \tag{11}$$

In the case of time reversal, although we may have started with conventional complex quantum mechanics we have ended up with real quantum mechanics

An absence of a time orientation does not allow globally consistent canonical quantization of quantum fields. (GWG, B Kay) The antipodal map is an isometry of both de-Sitter spacetime dS and Anti-de-Sitter spacetime AdS. In all dimensions, the first case it reverses time orientation and in the second space orientation in odd spactime dimensions.

Thus the antipodal identification is quantum-mechanically permissable for AdS, but not for deS.

Similar statements hold for the elliptic interpretation of black holes or branes.

In the 1950's and 1960's Goldhaber, Stannard, Klein, Alfven, Okun and others, speculated that in addition to the conventional matter we see, the universe also contains mirror matter or shadow matter with some opposite discrete quantum numbers to our own and which interacts with standard matter model only via gravitational interactions.

In fact Stannard speculated about the existence of Faustian matter which is identical to our matter but with the opposite time sense.

Observations of dark matter tend to rule out this possility, but something like this works in brane scenarios such as the Horava-Witten model and its descendents in which our world and the shadow world have many opposite properties, Perhaps this idea makes contact with the recently introduced idea of ghost-branes of

T. Okuda and T. Takayanagi, Ghost D-branes *JHEP* **0603** (2006) 062 [arXiv:hep-th/0601024].

Goldhaber's idea (Science 124 (1956) 218-219) was that the Big-Bang was related to an initial explosion in which matter went one way and anti-matter the other.

In modern terms one might imagine a gravitational instanton with a separating hypersurface of symmetry (a real tunnelling geometry).

An example would be the Reissner-Nordstrom -de-Sitter instanton studied by Mellor and Moss and others. This has a region $I \times S^2$, bounded by two throats from one of which electric flux emerges and down the other of which it disappears. The separation is caused by the cosmological constant which destablises the vacuum.

In this model there is complete symmetry between the two receeding counter-worlds. They might be identified, or one might follow Stannard (Nature 211 (1966) 693-695) and Schulman (Phys Rev Letts 83 (1999) 5419) and others and imagine the that they have the opposite arrow of time.

In fact setting up initial data $\{\Sigma, h_{ij}, K_{ij}\}$ in general relativity with regions in which their gravitational arrows of time differ is easy. One picks Σ to admit an involutive isometry f of the initial 3-metric hij, $f_\star h_{ij} = h_{ij}$ under which the second fundamental form K_{ij} is taken to be odd $f_\star K_{ij} = -K_{ij}$. Including matter is also easy.

In current brane scenarios, particularly those related to the Horava-Witten model, these ideas come into their own. Shadow matter resides on a near by brane. It may have opposite quantum numbers, including discrete symmetries asscociated with P and T. One may also envisage Faustian branes.

Recently ghost branes have been introduced into String Theory with the opposite sign for the Dirac-Nambu-Goto action. (that is negative tension). Open strings stretching between such p ordinary branes and q ghost branes give rise to the gauge group $SU(p, q)$.

In the absence of gravity there is a complete symmetry between positive and negative branes.

When gravity is taken into account, this ceases to be true. This is curiously reminiscent of Penrose's idea that Quantum Gravity and CPT may be incompatible.

In Minkowski spacetime, P and T lie outside the identity component of the isometry group $Isom_0(M,g) \equiv E_0(n-1,1)$, but not of the identity component of the diffeomorphism group $Diff_0(M) \equiv Diff_0(R^n)$.

In a general or space or time-orientable spacetime there need be *no* diffeomorphism in the identity component $Diff_0(M)$ which reverses space-orientation or, more unexpectedly, which reverses time orientation.

Such spacetimes have an *intrinsic handedness* or an *intrinsic arrow of time*.

Andrew and I found examples using higher dimensional generalisations of Taub-NUT spacetime.

Reversal of signature

$$g_{\mu\nu} \rightarrow -g_{\mu\nu} \tag{12}$$

induces a change of sign of the cosmological constant and and of mass terms

$$\Lambda \rightarrow -\Lambda\,, \qquad m \rightarrow im\,. \tag{13}$$

It also changes the Clifford algebras, in general * *over the reals*

$$\text{Cliff}(n-1,1;R) \not\equiv \text{Cliff}(1,n-1;R) \tag{14}$$

For example Cliff$(0,1) \equiv C$, Cliff$(1,0) \equiv R \oplus R$, Cliff$(0,2) \equiv H$, Cliff$(1,1) \equiv$ Mat$_2(R)$, where Mat$_2(R)$ is the algebra of 2×2 real matrices.

*I use a convention in which the Clifford algebra determines the signature and *vice-versa* via $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}$ and positive precedes negative

The fact that signature reversal is not a symmetry of nature encourages the speculation that one signature is preferred *

It is a striking fact that classical supergravity, both in four dimensions and in eleven dimensions, makes essential use of Majorana spinors. These are most conveneniently dealt with in a really real representation, using the facts that

$$\mathrm{Cliff}(3,1) \equiv \mathrm{Mat}_4(R)\,, \qquad \mathrm{Cliff}(1,3) \equiv \mathrm{Mat}_2(H) \tag{15}$$

$$\mathrm{Cliff}(10,1) \equiv \mathrm{Mat}_{32}(R) \oplus \mathrm{Mat}_{32}(R)\,, \tag{16}$$

$$32 = 8 \times 4 \tag{17}$$

*Duff and Kalkkinen prefer a more democratric attitude in which only dimensions which are indifferent to signature reversal are favoured. But this appears to favour the wrong dimension(s)

In fact one may argue that classical physics, incuding spinors, need only use real numbers, while quantum mechanics requires complex numbers in an essential way.

Thus the procedure of second quantization may be viewed as converting

- BOSONS: A Real symplectic vector space $\{V, \omega\}$ to a complex Hermtian vector space $\mathcal{H}_{\mathrm{qm}}$

- FERMIONS: A real orthogonal vector space $\{V, g\}$ to a complex Hermtian vector space $\mathcal{H}_{\mathrm{qm}}$

.

This requires a choice of complex structure, the complex structure of quantum mecahnics

$$i_{\mathsf{qm}} : V \to V\,, \qquad \text{s.t.} \qquad i_{\mathsf{qm}}^2 = -1\,. \tag{18}$$

Experience in Quantum Cosmology suggests that this may only be possible in an approximate fashion.

If so,Quantum Gravity might entail a breakdown of quantum mechanics to a less restrictive formalism, real quantum mechanics, which emcompasses the conventional complex formulation as a special case, valid in special circumstances

Another piece of circumstantial evidence which appears to favour this view point is that for an $so(10)$ GUT, all the fermions fit into a really real 32-dimensional representation of $Spin(10)$ subject to the constraint

$$\gamma_{11}\psi = \psi^M \,, \tag{19}$$

where ψ^M denotes Majorana conjugate.

This works because in part because

$$\mathrm{Cliff}(10; R) \equiv \Lambda^\star(R^{32}) \,. \tag{20}$$

Tantalisingly , the M-theory Clifford algebra is reducible

$$\mathrm{Cliff}(10,1) \equiv \mathrm{Mat}_{32}(R) \oplus \mathrm{Mat}_{32}(R)\,. \tag{21}$$

One must therefore make a choice:

$$\gamma_0\gamma_1\gamma_2\cdots\gamma_{10} = \pm 1? \tag{22}$$

Which is a sort of orientation convention.

Perhaps, as Paul Townsend has speculated, there are two inequivalent local theories which combine together in some larger whole.

Obstructions to spinors and pinors

De-Witt and Carlip pointed out that while the second Stiefel Whitney class $w_2(M) \in H^2(M, Z_2)$ is the obstruction to spinors for both Lorentzian signatures, the obstruction to pinors is signature dependent. The situation is simplest for Cliffordian pinors but Andrew worked out the obstruction for non-Cliffordian pinors and (with Lloyd Alty) for Kleinian signature (i.e signature (p, p))

He also applied this theory to M-branes wrapped both in space *and* time.

Andrew and I found an interesting application of this theory to Lorentzian universe which are born from nothing. Richard Gott and John Friedman have proposed this as an alternative to Hartle and Hawking's No Boundary * proposal.

In its simplest version the universe $\{M, g\} \equiv dS_4/Z_2$, where Z_2 acts as the antipodal map. M is compact with a a single spacelike S^3 boundary†. However the metric is not time-orientable.

*i.e. one boundary

†i.e. no kinks

As noted above, this makes it impossible to quantise (BOSONIC) quantum fields globally. But what about FERMIONS? One must use pinors. However now the obstructions cut in. If one wants to have really real Majorana pinors one must use signature $(3,1)$ *.

*positive before negative

However, for signature $(3,1)$, pinors (called by Friedman in this context "sinors ") are obstructed.

Thus we have at least two reasons for rejecting Lorentzian One Boundary scenerios on global consistency grounds.

On the other hand Andrew and I studied the production of non-space orientable black hole spacetimes via non-orientable instantons.

It could have been the case that these instantons failed to have a spinor structure (as in the example of the Page metric which is a real tunneling geometry studied by GWG and H-J Pohle).

In our case we found that the instantons admitted pinor structures and these could be compatible with the pinor structures found in nature.