Lorentz-violating extension of QED

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Based on
J. A. and A. Vergou, PRD 2011
J. A. and N. Mavromatos, PRD 2011

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3. One-loop properties
4. Beyond one-loop
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1 Model

Bare Lagrangian with massless fermions

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}\left(1 - \frac{\Delta}{M^2}\right)F_{\mu\nu} + \overline{\psi}(i\ \not\partial - e\ \not A)\psi$$

Some properties:

- Higher order space derivatives only → no ghost excitation
  Introduction of mass scale M → dynamical mass expected $m_{dyn} \propto M$
- M = regulator for graphs with internal photon line
  Dim. reg. ($d = 4 - \epsilon$) for graphs without internal photon line
  → Counterterms including $1/\epsilon$ and $\ln(M/m_{dyn})$
- Anisotropy → different renormalization for space and time derivatives
  → Effective light cone for each particle species

2 Fermion dynamical mass

Schwinger-Dyson equation for fermion propagator
$\longrightarrow$ Non-perturbative self consistent equation
Assume

$$G = i\frac{\not{p} + m_{dyn}}{p^2 - m_{dyn}^2}$$

$\longrightarrow$ non-vanishing solution for m_{dyn}:

$$m_{dyn} \simeq M \exp\left(-\frac{2\pi}{(4+\zeta)\alpha}\right) \quad << M$$

Comments:

- Non-analyticity: no expansion for small α;
- Gauge dependence: pinch technique
  (J. Cornwall and J. Papavassiliou, PRD 1989)
  $\longrightarrow$ Feynman gauge ($\zeta = 0$) for physical processes;
- Magnitude: $\zeta = 0 \longrightarrow m_{dyn} << m_{e^-}$ if $M = M_{Plank}$
  Magnification in brane world model
  (N. Mavromatos, PRD 2011)

3 One-loop properties

- Gauge invariance and speed of light
  Same vaccum polarization as in QED $\rightarrow$ no modification to speed of light
  Ward identity satisfied $\rightarrow$ no modification to running coupling constant

- Dressed fermion kinetic terms: $i\overline{\psi}\left((1+Z_0)\partial_0\gamma^0 - (1+Z_1)\vec{\partial}\cdot\vec{\gamma}\right)\psi$, with

$$
\begin{aligned}
Z_0 &= -\frac{\alpha}{2\pi}\left(\ln\left(\frac{1}{\mu}\right) + 4\ln 2 - 2\right) + \mathcal{O}(\mu^2\ln(1/\mu)) \\
Z_1 &= -\frac{\alpha}{2\pi}\left(\ln\left(\frac{1}{\mu}\right) + \frac{50}{9} - \frac{20}{3}\ln 2\right) + \mathcal{O}(\mu^2\ln(1/\mu))
\end{aligned}
$$

  where $\mu = m_{dyn}/M$.

- Effective light cone for fermions (after renormalization)

$$
v^2 \equiv v_\phi v_g = 1 - \frac{2\alpha}{\pi}\left(\frac{34}{9} - \frac{16}{3}\ln 2\right) + \mathcal{O}(\alpha^2)\ < 1
$$

  IR dispertion relation $\omega^2 = m_{dyn}^2 + v^2p^2$
  $v \neq$ speed of light, because $m_{dyn} \neq 0$

4 Beyond one-loop

- Dressed Maxwell term

$$2(1+\hbar X+\hbar^2 Y_0+\cdots)F_{0i}F^{0i}+\ (1+\hbar X+\hbar^2 Y_1+\cdots)F_{ij}F^{ij}$$

Rescaling $t\ \rightarrow\ t/\kappa$ and $A_0\ \rightarrow\ \kappa A_0$, where

$$\kappa=\sqrt{\frac{1+\hbar X+\hbar^2 Y_1}{1+\hbar X+\hbar^2 Y_0}}=1+\mathcal{O}(\hbar^2)$$

$\rightarrow$ usual term $F_{\mu\nu}F^{\mu\nu}$ $\rightarrow$ gauge invariance and speed of light not violated

- Fermion kinetic term after rescaling by κ

$$\kappa\,\overline{\psi}\,(1+\hbar Z_0+\hbar^2\tilde{Z}_0+\cdots)\,(i\partial_0-eA_0)\,\gamma^0\,\psi$$
$$+\ \ \overline{\psi}\,(1+\hbar Z_1+\hbar^2\tilde{Z}_1+\cdots)\left(i\vec{\partial}-e\vec{A}\right)\cdot\vec{\gamma}\,\psi$$

$\rightarrow$ κ contributes to higher order correction to fermions effective light cone.

- Running coupling constant: different from QED at 2 loops and above

5 Alternative model and vector mass generation

(J. A. and N. Mavromatos, to appear)

- Two fermions $\Psi = (\psi, \chi)$, two vectors (A_μ, B_μ)

  $$\mathcal{L} = \frac{1}{4}F_{\mu\nu}\left(1 - \frac{\Delta}{M^2}\right)F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}\left(1 - \frac{\Delta}{M^2}\right)G^{\mu\nu} + \overline{\Psi}\left(i\ \partial\!\!\!/ + g_A\ A\!\!\!/ + g_B\ B\!\!\!/\tau_2\right)\Psi$$

  $$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \rightarrow \quad \text{flavour mixing current}$$

- Fermion self energy

  $$\Sigma = m_1 + m_2\tau_3 \quad \text{with} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Ward identity $g_B^{-1}k^\mu\Gamma_\mu^B(p,k) = G^{-1}(p)\tau_2 - \tau_2 G^{-1}(p-k)$
  $m_2 \neq 0 \rightarrow \{G^{-1}, \tau_2\} \neq 0 \rightarrow$ vector dynamical mass
  (Jackiw and Johnson, PRD 1973; Cornwall and Norton PRD 1973)

- Mass hierarchy $m_\chi << m_\psi << m_B$ if $g_A \simeq g_B$