OSCILLONS AFTER INFLATION

1) 1002.3380 (WITH D. SHIROKOFF)
2) 1006.3075 (MA)
3) 1009.2505 (MA, EASTHER AND FINKEL)
4) 1106.3335 (MA, EASTHER, FINKEL, HERTZBERG & FLAUGHER)

MUSTAFA AMIN (MIT)

SUPPORTED BY A PAPPALARDO FELLOWSHIP
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what does the universe look like at the end of inflation?
INFLATON FRAGMENTATION INTO OSCILLONS

• observationally consistent

• theoretically motivated

• dominate the energy density of the post inflationary universe

• weak couplings to other fields

\[ \sim \frac{d_H}{4} \]
SYNOPSIS

- end of inflation: simple scenarios
- fragmentation → clumps
  - motivation
  - clumps? oscillons
  - conditions for emergence
  - energy fraction is oscillons
- Consequences
END OF INFLATION

$\phi$ inflation

$V(\phi)$
END OF INFLATION

\[ V(\phi) \]

\[ \phi \]

inflation

end: oscillatory regime
END OF INFLATION

$\phi$ inflation

end: oscillatory regime

$V(\phi)$

$\chi, \psi$
daughter fields

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• shape of the potential

• how does it couple to other fields
SCENARIO I

\[ V \sim \frac{1}{2} m_\varphi^2 \varphi^2 + g^2 \varphi^2 \chi^2 + h \varphi \bar{\psi} \psi + \ldots \]

perturbative

decay rate \ll H

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SCENARIO II

\[ V \sim \frac{1}{2} m_\varphi^2 \varphi^2 + g^2 \varphi^2 \chi^2 + h \varphi \bar{\psi} \psi + \ldots \]

resonant

decay rate >> H

Movie: courtesy of R. Easther

Trachen & Brandenberger (1990), Kofman, Linde, Starobinsky et. al (1994) ...
SCENARIO II

\[ V \sim \frac{1}{2} m_{\phi}^2 \phi^2 + g^2 \phi^2 \chi^2 + h \phi \bar{\psi} \psi + \ldots \]

Movie: courtesy of R. Easther

Trachen & Brandenberger (1990), Kofman, Linde, Starobinsky et. al (1994) ...
SCENARIO III

\[ V \sim \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4} \varphi^4 + \frac{g^2}{6m^2} \varphi^6 + \ldots + h \varphi \bar{\psi} \psi \]

perturbative

decay rate \(<< H\)

MA 2010
MA, Finkel, Easther 2010
MA, Easther, Finkel, Flauger, Hertzberg 2011

Also see: McDonald & Broadhead, Rajantie & Copeland, Hindmarsh & Salmi, Gleiser et. al.

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MOTIVATION:
SHALLOW POTENTIALS

$$V(\varphi) \propto \varphi^{2\alpha}$$

$$r \approx \frac{8\alpha}{N}$$

$$n_s \approx 1 - \frac{\alpha + 1}{N}$$
MOTIVATION:
SHALLOW POTENTIALS

- monodromy (Silverstein & Westphal) $\alpha = 1/3$
- axion monodromy (McAllister, Silverstein & Westphal) $\alpha = 1/2$
- supergravity (Kallosh & Linde) $\alpha < 1$
- generic flattening of potentials (Dong et. al) $\alpha < 1$

\[ V(\varphi) \propto \varphi^{2\alpha} \]
\[ V''(\varphi) \sim m^2 \]
\[ \varphi \sim M \]
LUMPS?

(1) oscillatory (2) spatially localized (3) very long lived

Bogolubsky & Makhankov 1976, Gleiser 1994, Copeland et al. 1995, ...
LUMPS?

(1) oscillatory (2) spatially localized (3) very long lived

necessary:

\[ V(\varphi) - \frac{1}{2}m^2 \varphi^2 < 0 \]

for some range of \( \varphi \)

satisfied if \( \alpha < 1 \)

Bogolubsky & Makhankov 1976, Gleiser 1994, Copeland et al. 1995, ...
STABILITY

• linear stability analysis: short and long wavelength (MA & Shirokoff 2010)

• effects of expansion (Farhi et. al 2008)

• rate of energy loss by radiation, classical and quantum (Segur and Kruskal 1987, Hertzberg 2010)

• For a restricted class of potentials
HOW DO THEY FORM?

SELF RESONANCE!

MA (2010)
HOW DO THEY FORM?

SELF RESONANCE!

$t = 0 \, m^{-1}, \, a = 1.$

MA (2010)
REQUIRES
EFFICIENT GROWTH

- growth rate $\mu_k$
- expansion rate $H$

$\mu_k \gg H$

$\mu_k$ depends on the parameters, and can be calculated easily

MA (2010)
PARAMETERIZED MODEL

- matches axion monodromy (McAllister, Silverstein & Westphal) $\alpha = 1/2$
- good fit to other models: (e.g., monodromy and super gravity models)

\[ V(\varphi) = \frac{m^2 M^2}{2\alpha} \left[ \left( 1 + \frac{\varphi^2}{M^2} \right)^\alpha - 1 \right] \]
WMAP constraints

\[ V(\varphi) = \frac{m^2 M^2}{2\alpha} \left[ \left( 1 + \frac{\varphi^2}{M^2} \right)^\alpha - 1 \right] \]

\[ \Delta^2_R = \frac{1}{96\pi^2 \alpha^3} \left( \frac{m}{m_{pl}} \right)^2 \left( \frac{M}{m_{pl}} \right)^{2-2\alpha} (4\alpha N)^{1+\alpha} = 2.42 \times 10^{-9} \]

Inflaton fragmentation as a function of 2 params.

scale where potential changes shape

asymptotic slope
SIMULATIONS

- pseudo spectral code \texttt{pspectre} :256^3 \cite{Easther,Finkel,Roth 2008}
- initial conditions at end of inflation (small zero point fluctuations)
- fraction of energy in oscillons (overdense by a few)
- universe increases in size by a factor of a few
energy fraction: $\gg 50\%$

\[
\frac{E_{\text{osc}}}{E_{\text{tot}}} = f
\]

(inverse) scale where potential changes shape

asymptotic slope

\[
\beta = \frac{m_{\text{pl}}}{M} \gg 1
\]

\[
\alpha \lesssim 1
\]
Floquet Analysis: Expanding Universe

\[ \delta \varphi_k \approx \frac{\delta \varphi_k(t_i)}{a^{3/2}(t)} \exp \left[ \int dt \mu_k(t) \right] \]

\[ = \frac{\delta \varphi_k(a_i)}{a^{3/2}} \exp \left[ \int d\ln a \frac{\mu_k(a)}{H(a)} \right] \]
CONDITION FOR EMERGENCE

\[
\left[ \frac{|\Re(\mu_k)|}{H} \right]_{\text{max}} \approx f(\alpha) \beta
\]

\[
\left[ \frac{|\Re(\mu_k)|}{H} \right]_{\text{max}} \gtrsim 10
\]

(inverse)scale where potential changes shape
asymptotic slope

\( \beta = \frac{m_{\text{pl}}}{M} \gg 1 \)

\( \alpha \lesssim 1 \)
A HEURISTIC GUIDE ...

necessary

emergence

\( \mu_k \gg H \)

stability

cosmologically relevant

\[ \Gamma \ll H \ll m \]
WHAT TO DO WITH THEM?
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- thermalization \textit{(Gleiser)}, enhanced decay rates \textit{(McDonald 2003)}
WHAT TO DO WITH THEM?

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• thermalization *(Gleiser)*, enhanced decay rates *(McDonald 2003)*

• gravitational effects:
What to do with them?

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- gravitational effects:
  - early universe structure formation
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  - black holes ?
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- thermalization *(Gleiser)*, enhanced decay rates *(McDonald 2003)*

- gravitational effects:
  - early universe structure formation
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  - expansion history and influence on inflationary observables
WHAT TO DO WITH THEM?

• thermalization (Gleiser), enhanced decay rates (McDonald 2003)

• gravitational effects:
  - early universe structure formation
  - black holes ?
  - expansion history and influence on inflationary observables
  - g-waves (probably not)
SUMMARY
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- oscillons can "emerge naturally" in well motivated inflationary models.
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• oscillons can *emerge naturally* in well motivated inflationary models.

• dominate the post inflationary energy density
SUMMARY

• oscillons can **emerge naturally** in well motivated inflationary models.
• **dominate** the post inflationary energy density
• conditions for **existence, emergence** (and stability)
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  • include gravity (black holes?, g waves?)
oscillons can *emerge naturally* in well motivated inflationary models.

*dominate* the post inflationary *energy density*

*conditions for existence, emergence* (and stability)

**To Do:**

- couplings to other fields
- include gravity (black holes?, g waves?)
- implications for reheating?
GENERIC EMERGENCE

Farhi et. al 2008
gravitational waves

adapted from Abott et. al. 2007

fraction of energy density in g-waves (per logarithmic frequency interval)

\[ \Omega_{gw}(f) \]

\[ f \text{ [Hz]} \]

BBN

pulsar timing

LISA

Adv. LIGO

(p)reheating

adapted from Abott et. al. 2007

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GRAVITATIONAL WAVES

fraction of energy density in g-waves (per logarithmic frequency interval)

\[ \Omega_{gw}(f) \]

\( f \) [Hz]

adapted from Abott et. al. 2007

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Gravitational waves

\[ \Omega_{gw}(f) \]

Fraction of energy density in g-waves (per logarithmic frequency interval)

\[ 10^{-18}, 10^{-14}, 10^{-10}, 10^{-6}, 10^{-2}, 10^2, 10^6, 10^{10} \]

\[ f [\text{Hz}] \]

Adapted from Abott et. al. 2007
COUPLINGS:
SELF & DAUGHTER FIELDS

smooth
perturbative couplings
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COUPLINGS:
SELF & DAUGHTER FIELDS

- **smooth**
  - perturbative couplings

- **lumpy**
  - self-resonant

- **messy**
  - parametric resonance with other daughter fields

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\[ \partial_t^2 \varphi - \partial_x^2 \varphi + V'(\varphi) = 0 \]

localized periodic solution: \[ \varphi(t, x) \sim \Phi(x) \cos \omega t \]
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localization: \( \omega^2 < m^2 \)
WHY?

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\[ [(m^2 - \omega^2)\varphi] + [-\partial_x^2 \varphi] + [V'(\varphi) - m^2 \varphi] \sim 0 \]

frequency curvature non-linearity
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frequency \ curvature \ non-linearity

+ve
WHY?

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\[
\begin{align*}
[(m^2 - \omega^2)\varphi] + [-\partial_x^2\varphi] + [V'(\varphi) - m^2\varphi] &\sim 0 \\
frequency &\quad curvature &\quad non-linearity
\end{align*}
\]

+ve +ve

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WHY?

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\[ [(m^2 - \omega^2)\varphi] + [-\partial_x^2 \varphi] + [V'(\varphi) - m^2 \varphi] \sim 0 \]

frequency  curvature  non-linearity

+ve  +ve

Need, at least somewhere  \( V'(\varphi) - m^2 \varphi < 0 \)
EXAMPLE

\[ V(\varphi) = \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4} \varphi^4 + \frac{g^2}{6m^2} \varphi^6 \]

\[ \lambda^2 \ll g^2 \]

quasi-stable, approximately single frequency solutions exist in the model.
\[ \partial_\varrho^2 \Phi_\alpha + \frac{2}{\varrho} \partial_\varrho \Phi_\alpha - \alpha^2 \Phi_\alpha + \frac{3}{4} \Phi_\alpha^3 - \frac{5}{8} \Phi_\alpha^5 = 0 \]
HOW TO SOLVE

\[ \Box \varphi = V'(\varphi) \]

\[ \varrho = (\lambda/g)mr \]

\[ \omega^2 = m^2 \left[ 1 - (\lambda/g)^2 \alpha^2 \right] \]

\[ \varphi(t, r) \approx \frac{m}{\sqrt{\lambda}} \left[ \left( \frac{\lambda}{g} \right) \Phi_\alpha(\varrho) \cos \omega t + O[\lambda/g^3] \right] \]

can be done with amplitude as a small parameter as well: \( \epsilon \sim \lambda/g \)
SOLUTIONS

$t = 1 \text{m}^{-1}$

\[ y \quad (\text{m}^{-1}) \quad x \quad (\text{m}^{-1}) \]

Thanks to Antony Speranza for movies (MIT)

MA & D. Shirokoff (2010)
Thanks to Antony Speranza for movies (MIT)
WHAT ABOUT STABILITY?
WHAT ABOUT STABILITY?
WHAT ABOUT STABILITY?

perturbations
LINEAR STABILITY ANALYSIS

• *collapse* instability? \( \text{wavelength} \sim \text{width} \)

• *Floquet* instability? \( \text{wavelength} \ll \text{width} \)
COLLAPSE INSTABILITY?

\[ N = \frac{\lambda^2}{g m^2} \int \varphi^2(r) d^3r \]

\[ \alpha^2 = \frac{g^2}{\lambda^2 m^2} (m^2 - \omega^2) \]

\[ dE/d\omega < 0 \]

\[ \sim \text{energy} \]

\[ \sim \text{frequency shift} \]

Fodor et al. (2008) for related numerical evidence.

MA & D. Shirokoff (2010)
FLOQUET INSTABILITY ?
FLOQUET INSTABILITY ?
Floquet Instability?

\[ \mu_{max} \sim \begin{cases} 
(\lambda/g)^2 m, & \text{if } (\lambda/g)^2 m > 1/2re \\
0 & \text{otherwise}
\end{cases} \]
Floquet Instability?

\[ \mu_{\text{max}} \sim \begin{cases} (\lambda/g)^2 m, & \text{if } (\lambda/g)^2 m > 1/2r_e \\ 0 & \text{otherwise} \end{cases} \]

narrow band instability

Hertzberg (2010)
LINEAR STABILITY

collapse

floquet

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OUTGOING RADIATION

- **classical**: Kruskal and Segur (1987)
OUTGOING RADIATION

• classical: *Kruskal and Segur* (1987)

radiative tail
DECAY RATES

\[ \Gamma_{\text{classical}} \sim m \frac{\lambda}{g} e^{-b \frac{g}{\lambda}} \]

\[ \Gamma_{\text{quantum}} \sim m \frac{\lambda^3}{g^2} \]

EXPANDING BACKGROUND

\[ \partial_t^2 \varphi - \nabla^2 \varphi + 3H \partial_t \varphi + m^2 \varphi - \lambda \varphi^3 + \frac{g^2}{m^2} \varphi^5 = 0 \]

• **stretching** instability?

• **radiative** tail
INCLUDING EXPANSION

- loss of energy

\[ \Gamma_H \sim m e^{-m H^{-1}(\alpha \lambda/g)^2} \]

Farhi et. al (2007)
MA & D. Shirokoff (2010)
LONG LIVED IF ...

\[ \Gamma \sim \Gamma(\text{expansion}) + \Gamma(\text{radiation}) \]

\[ \Gamma \ll H \ll m \]

possible for inflaton (and axions)
POWER SPECTRUM
Number Density

\[ n_{\text{osc}} a^3 \sim \left( \frac{k_{nl}}{2\pi} \right)^3 \approx \beta^{-3/5} \left( \frac{\lambda \, m}{g \, 2\pi} \right)^3 \]

\[ \frac{\mu_k}{H} \sim \beta = \left[ \left( \frac{m_{pl}}{m} \right) \frac{\lambda^{3/2}}{g} \right] \]

MA (2010)
NUMERICAL SIMULATIONS

- verify individual characteristics
- confirm predictions for number densities and determine energy fraction
- more realistic inflationary potentials where analytics are difficult (e.g., monodromy models)
ROBUSTNESS OF NUMERICS

- **defrost**
  - $(N = 1024, L = 400)$

- **PSpectRe**
  - $(N = 256, L = 200, 400, 600)$

- **PSpectRe** $(N = 384, L = 400)$
WIDTH-HEIGHT

\[ a = 1.74 \]

\[ \frac{\lambda m}{r_e} \]

\[ \text{width: } 2 \frac{g}{\rho_{\text{core}}} \]

\[ \text{core density: } \frac{g^2}{\lambda m^4} \rho_{\text{core}} \]
\( \frac{\mu_k}{H} \sim \beta \equiv \left[ \left( \frac{m_{pl}}{m} \right) \frac{\lambda^{3/2}}{g} \right] \)

\( n_{osc} a^3 \sim \beta^{-3/5} \left( \frac{\lambda m}{g 2\pi} \right)^3 \)

MA, Easther & Finkel (2010)
COLLISIONS
COLLISIONS

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THE ALL IMPORTANT SIGN

$\Phi$

$x (m^{-1})$

$y (m^{-1})$

$t = 1m^{-1}$

$t = 1m^{-1}$
THE ALL IMPORTANT SIGN
INCLUDING GRAVITY
\[ f_0 \sim f_e a_e \]
\[ \sim \sqrt{H_i H_0} \]
\[ \sim 10^{-4} \left( \frac{k_B T}{\text{TeV}} \right) \text{Hz} \]

\[ f_e \sim H_i \]
\[ a_e \sim (H_0/H_i)^{1/2} \]