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Reproducing ν effects on the matter power spectrum through a DFG

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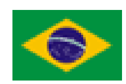

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Outline

- Motivations
- Assumptions
- Cosmological SM linear perturbation theory
- DFG approach for CDM and HDM components
- Results: Density Fluctuation, δ, and Fluid Velocity Divergence, θ
- Boltzmann equation solutions for a DFG
- Analytical *versus* Numerical Analysis
- Results: Matter Power Spectrum $P(k)$
- Conclusions

MOTIVATIONS

Continuous search for the effective role of COSMOLOGICAL BACKGROUND NEUTRINOS on the (MATTER POWER SPECTRUM of the) Universe.

- **Inclusive contribution of some *extra* DFG component to the matter power spectrum;**

- **The DFG test-fluid can *mimic* the massive neutrino cosmological behaviour at several dominant cosmological backgrounds ($\gamma \rightarrow$ (D)M $\rightarrow$ DE ... etc);**

- **The DFG approach allows for (analytically) quantifying the smooth transition between ultra-relativistic (UR) and non-relativistic (NR) thermodynamic regimes for neutrinos;**

- **Analytical Calculations (Cutoff in the multipole expansion) are ratified by limits of the Numerical Results (Boltzmann Equation).**

ASSUMPTIONS

- TEST FLUID APPROACH - The hypothesis of a tiny contribution from a degenerate Fermi gas (DFG) test-fluid to some dominant cosmological scenario;
- DFG CHEMICAL POTENTIAL - Eventual *collision terms* are parameterized by the DFG chemical potential;
- PERFECT FLUID APPROACH (**Analytical**) - Bianchi Identities for dominating background scenarios;
- DFG approach reproduces a UR to NR ANALYTICAL TRANSITION.
- RESULTS FROM CAMB CODE (**Analytical**).

COSMO SM LINEAR PERTURBATION THEORY

- FRW flat universe with averaged energy density, $\bar{\rho}(\eta)$, and averaged pressure, $\bar{P}(\eta)$, is described in terms of the scale factor, $a(\eta)$, through the following components of the Einstein equation,

$$
\begin{aligned}
\left(\frac{da/d\eta}{a}\right)^2 &= \frac{8\pi}{3}Ga^2\bar{\rho}\,, \\
\frac{d}{d\eta}\left(\frac{da/d\eta}{a}\right) &= -\frac{4\pi}{3}Ga^2(\bar{\rho}+3\bar{P})\,,
\end{aligned}
\tag{1}
$$

  where η is the conformal time, G is the Newtonian constant, and we have set the light velocity equals to unity, i. e. $c = 1$.

- ANALYTICAL APPROACH - **Perfect Fluid Approach** the stress-energy tensor T^{μ}_{ν} at a comoving frame is given by

$$
T^{\mu\nu} = Pg^{\mu\nu} + (\rho+P)U^{\mu}U^{\nu}\,, \tag{2}
$$

  where U is the 4-velocity of the fluid particles, $g_{\mu\nu}$ is the metric tensor, and energy density and pressure are decomposed into averaged and perturbative values as $\rho = \bar{\rho}+\delta\rho$ and $P = \bar{P}+\delta P$.

- **Bianchi identities** provide us with the constraints on T^{μ}_{ν} in the Fourier k-space.
  - Continuity Equation,

$$\dot{\delta} = -(1+\omega)(\theta - 3\dot{\phi}) - 3\frac{\dot{a}}{a}(1+\delta)(1+\omega) - \frac{\dot{\bar{\rho}}}{\bar{\rho}}(1+\delta)\,, \tag{3}$$

  - Euler equation,

$$\dot{\theta} = \frac{k^2\,\delta}{1+\omega}\frac{\delta P}{\delta\rho} - \sigma\,k^2 - 4\frac{\dot{a}}{a}\theta + \psi\,k^2 - \left(\frac{\dot{\bar{\rho}} + \dot{\bar{P}}}{\bar{\rho}}\right)\frac{\theta}{1+\omega}\,, \tag{4}$$

with

$$a\frac{\partial\bar{\rho}}{\partial a} + 3\bar{\rho}\,(1+\omega) = 0\,, \tag{5}$$

where $\omega = \bar{P}/\bar{\rho}$, and θ, δ and σ are, respectively, the density fluctuation, the fluid velocity divergence, and the shear stress defined by

$$\delta \equiv \delta\rho/\bar{\rho}\,, \quad (\bar{\rho}+\bar{P})\theta \equiv ik^j\delta T^0_j\,, \quad \text{and} \quad (\bar{\rho}+\bar{P})\sigma \equiv -\left(\hat{k}^i\hat{k}_j - \frac{1}{3}\delta^i_j\right)\Sigma^j_i\,, \tag{6}$$

- We have to solve the (Einstein Equation) four equations for the linear perturbations in the k-space in terms of scalar metric perturbations ϕ and ψ

$$8\pi a^2 G T^0_0 = -3\left(\frac{\dot{a}}{a}\right)^2 + 6\frac{\dot{a}}{a}\dot{\phi} + 6\left(\frac{\dot{a}}{a}\right)^2 \psi + 2k^2\phi\,, \tag{7}$$

$$4\pi G a^2(\bar{\rho} + \bar{P})\theta = k^2\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right)\,, \tag{8}$$

$$\frac{4}{3}\pi G a^2 T^i_i = -\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 + \frac{k^2}{3}(\phi - \psi) + \ddot{\phi} + \frac{\dot{a}}{a}(\dot{\psi} + 2\dot{\phi}) + 2\psi\frac{\ddot{a}}{a} - \psi\left(\frac{\dot{a}}{a}\right)^2\,, \tag{9}$$

$$12\pi G a^2(\bar{\rho} + \bar{P})\sigma = k^2(\phi - \psi)\,. \tag{10}$$

DFG FLUID APPROACH

- Expanding the distribution function, $f(x^i, p_j(p, n_j), \eta)$ up to first-order in the temperature perturbation, δT, with $f|_{\bar{T}} = \bar{f}(p, \bar{T}(\eta))$, one has

$$f(x^i, p_j, \eta) \approx f|_{\bar{T}} + \left.\frac{\partial f}{\partial T}\right|_{\bar{T}} \delta T(x^i, n_j, \eta) \equiv \bar{f}(p, \eta)\left[1 + \Psi(x^i, p_j, \eta)\right], \qquad (11)$$

and the Boltzmann equation in the Fourier k-space for the longitudinal gauge becomes

$$\frac{\partial \bar{f}}{\partial \eta}(1+\Psi) + \dot{\Psi}\bar{f} + i\frac{q}{\varepsilon}(\vec{k}\cdot\hat{n})\Psi\bar{f} + \left(q\dot{\phi} - i\varepsilon\,(\vec{k}\cdot\hat{n})\psi\right)\frac{\partial \bar{f}}{\partial q} = \left(\frac{\partial f}{\partial \eta}\right)_C, \qquad (12)$$

where energy and momentum are respectively rewritten in terms of the (pseudo)comoving energy, $\varepsilon = aE = a\,(p^2 + m^2)^{1/2} = (q^2 + a^2 m^2)^{1/2}$, and of the comoving momentum, $q = p\,a$.

- DFG distribution function

$$\bar{f}(p,\eta) = g_d \left(\exp\left[\frac{E-\mu}{T}\right] + 1 \right)^{-1} \approx \left\{ \begin{array}{lll} g_d & \text{for} & E < \mu \\ 0 & \text{for} & E > \mu \end{array} \right. , \qquad (13)$$

where g_d is the number of spin degrees of freedom, μ is the chemical potential (that for lower temperatures approximates the Fermi energy $\mu \to E_F$ as $T \to 0$), and $\hbar = k_B = 1$.

- Pressure and Energy Density for a DFG

$$\begin{aligned} \bar{P}_d = & \quad g_d \frac{m^4}{16\pi^2} \left[\frac{\sqrt{1+\chi^2}}{\chi^4} \left(\tfrac{2}{3} - \chi^2\right) + \ln \frac{1+\sqrt{1+\chi^2}}{x} \right], \\ \bar{\rho}_d = & \quad g_d \frac{m^4}{16\pi^2} \left[\frac{\sqrt{1+\chi^2}}{\chi^4} (2+\chi^2) - \ln \frac{1+\sqrt{1+\chi^2}}{\chi} \right], \end{aligned} \qquad (14)$$

where $\chi \equiv (m/q_F)a$, m is the particle mass, and q_F is the comoving Fermi momentum.

- Test fluid approach - Continuity and Euler equations

$$\frac{\partial \delta\rho_d}{\partial\eta} = -\frac{m^4}{3\pi^2}\frac{\sqrt{1+\chi^2}}{\chi^4}\left(\theta_d - 3\dot{\phi}\right) - \frac{\dot{a}}{a}\,\delta\rho_d\frac{4+3\chi^2}{1+\chi^2}, \tag{15}$$

$$\dot{\theta}_d + \sigma_d k^2 + \frac{\dot{a}}{a}\frac{\theta_d}{1+\chi^2} - \phi\, k^2 = \delta\rho_d\frac{k^2\chi^4\pi^2}{m^4(1+\chi^2)^{3/2}}. \tag{16}$$

Analytical results for the above equations in case of:

A. Radiation-to-matter transitory regime at very large scales.

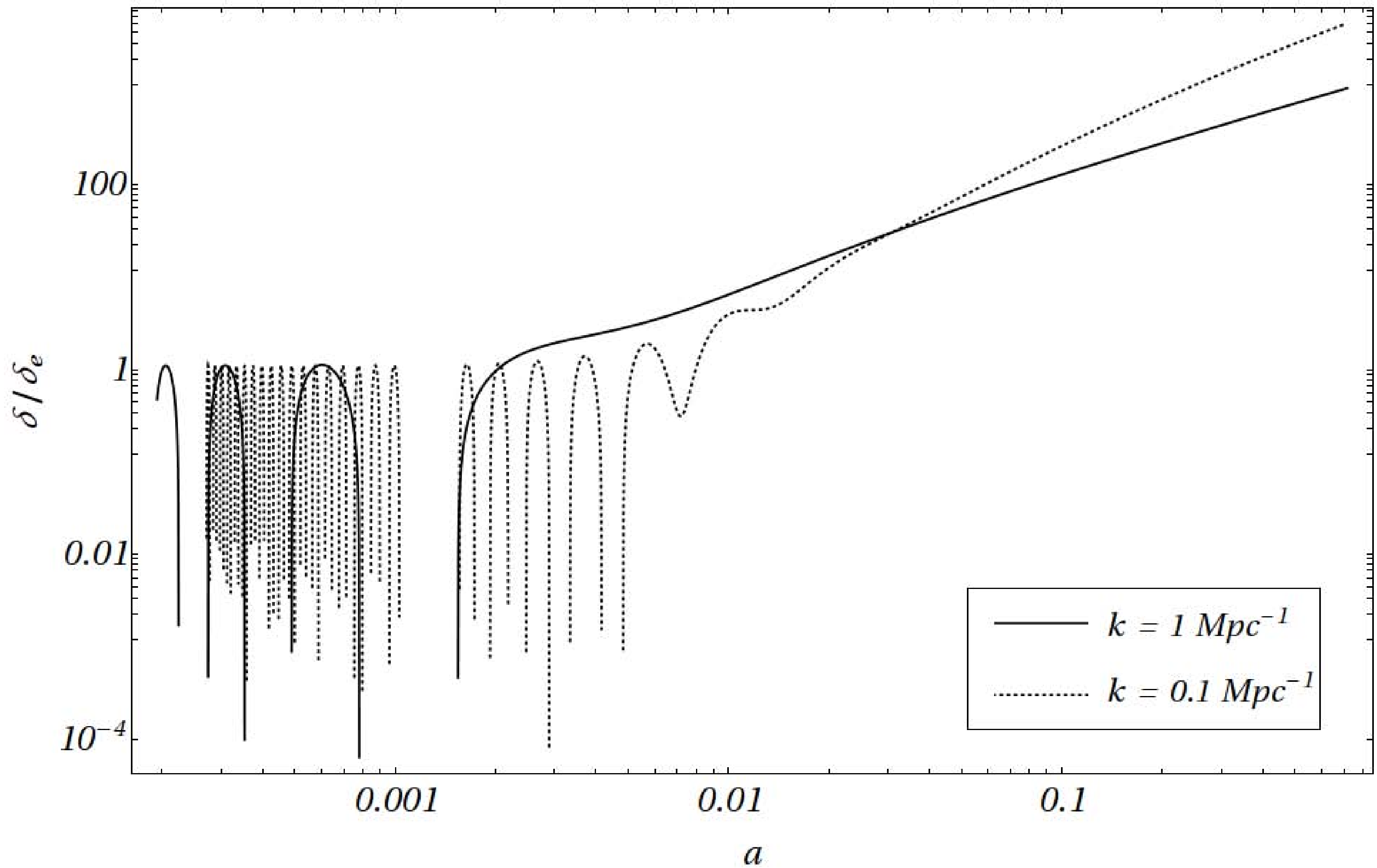
B. Radiation-dominated era.

C. Matter-dominated era.

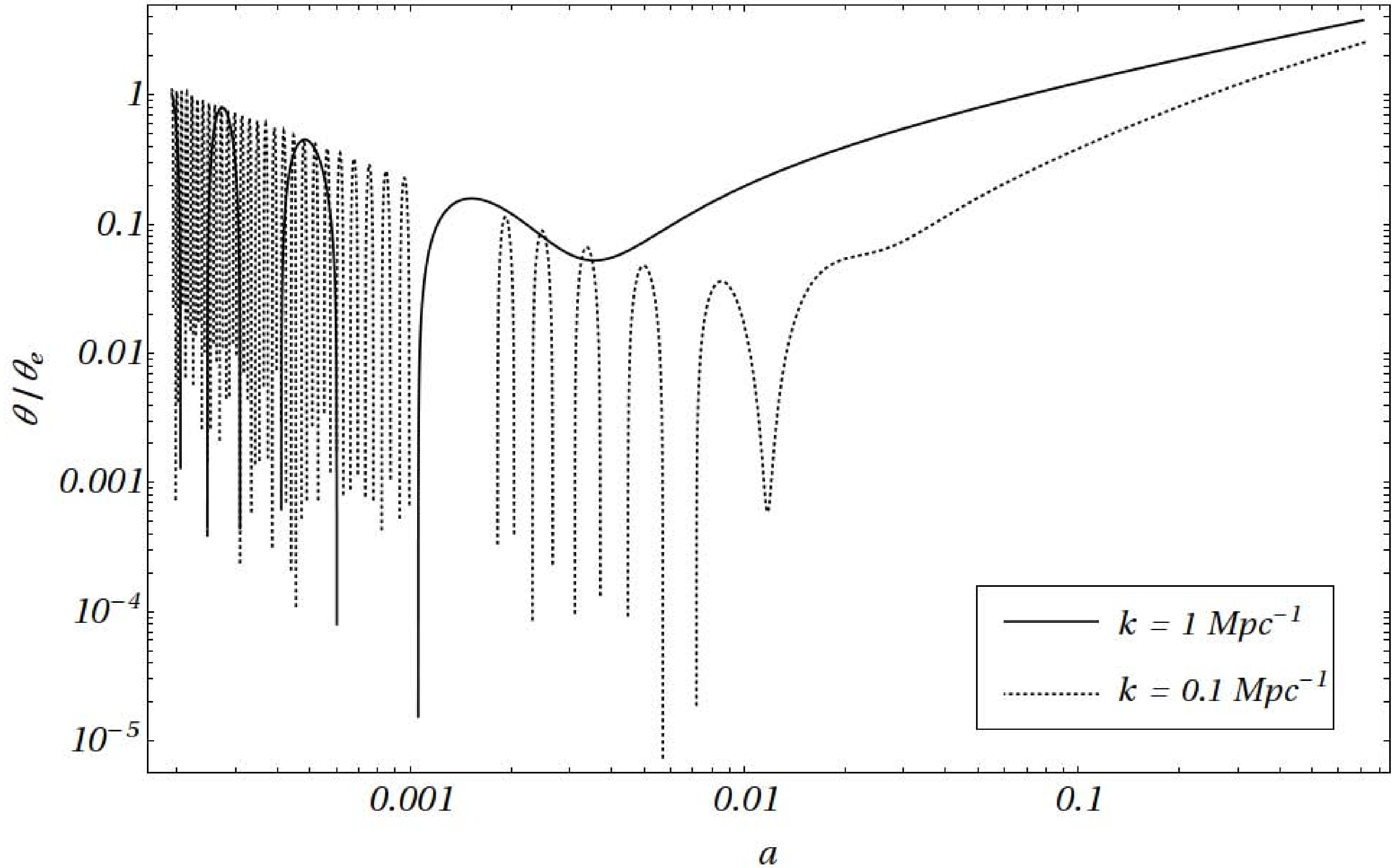
D. ΛCDM present era.

For the equations please see the references at the end!!!

DFG Density Fluctuation, δ



DFG Fluid Velocity Divergence, θ



BOLTZMANN EQUATION SOLUTIONS

- **The simplest way to consistently include the chemical potential** - Zeroth-order Boltzmann equation for a non-interacting fluid

$$0 = \frac{\mathrm{d}\bar{f}}{\mathrm{d}\eta} = \dot{a}\frac{\partial \bar{f}}{\partial a} \propto \frac{\partial}{\partial a}\left(\frac{E-\mu}{\bar{T}}\right), \tag{17}$$

from which the last equality sets the constraint among the energy, E, the temperature, T, and the chemical potential, μ, that results in

$$\left.\frac{\partial f}{\partial T}\right|_{\bar{T}} = \frac{aE}{p}\frac{\mu - E}{\bar{T}}\frac{\partial \bar{f}}{\partial q} = \frac{\varepsilon}{q}\frac{\nu - \varepsilon}{\bar{T}}\frac{\partial \bar{f}}{\partial q}, \tag{18}$$

with $\nu = a\mu$, a kind of comoving parametrization of the chemical potential.

It allows one to identify the first-order perturbation coefficient, Ψ, as

$$\Psi(x^i, n_j, \eta) = \frac{\varepsilon(\nu - \varepsilon)}{q^2}\frac{\partial \ln \bar{f}}{\partial \ln q}\Delta(x^i, n_j, \eta), \qquad \Delta \equiv \delta T/\bar{T}. \tag{19}$$

- The Boltzmann equation can be reduced to

$$\frac{\partial}{\partial \eta}\left(\frac{\varepsilon(\nu-\varepsilon)}{q}\frac{\partial \bar{f}}{\partial q}\Delta\right)+i(\vec{k}\cdot\hat{n})(\nu-\varepsilon)\frac{\partial \bar{f}}{\partial q}\Delta+\left(q\dot{\phi}-i(\vec{k}\cdot\hat{n})\varepsilon\,\phi\right)\frac{\partial \bar{f}}{\partial q}=0, \qquad (20)$$

where $\Delta(x^i, n_j, \eta)$ is read from $\mathscr{D}(k^i, n_j, \eta)$ in the Fourier k-space as

$$\Delta(x^i, n_j, \eta) = \frac{1}{(2\pi)^3}\int dk^3 \exp\left[ik_i x^i\right]\mathscr{D}(k^i, n_j, \eta). \qquad (21)$$

Performing the P_l-Legendre expansion with respect to $\hat{k}\cdot\hat{n} = \cos\varphi$, one obtains

$$\mathscr{D}(k^i, n_j, \eta) = \sum_{l=0}^{\infty}(-i)^l(2l+1)\,\Delta_l(k,\eta)\,P_l(\cos(\varphi)), \qquad (22)$$

$$\Delta_0 = \frac{1}{4\pi}\int d\Omega\,\mathscr{D},\ \ \Delta_1 = \frac{i}{4\pi}\int d\Omega\cos\varphi\,\mathscr{D}\ \text{ and }\ \Delta_2 = -\frac{3}{8\pi}\int d\Omega\left(\cos^2\varphi - \frac{1}{3}\right)\mathscr{D}, \qquad (23)$$

for the first three multipole cofficients.

- Evolution equations for the multipole coefficients, $\Delta_l(k,\eta)$,

$$
\begin{aligned}
\frac{\partial}{\partial\eta}\left(\frac{\varepsilon\,(\nu-\varepsilon)}{q}\frac{\partial\bar{f}}{\partial q}\Delta_0\right)+(\nu-\varepsilon)\frac{\partial\bar{f}}{\partial q}k\,\Delta_1+q\frac{\partial\bar{f}}{\partial q}\dot{\phi} &= 0\,,\\
\frac{\partial}{\partial\eta}\left(\frac{\varepsilon\,(\nu-\varepsilon)}{q}\frac{\partial\bar{f}}{\partial q}\Delta_1\right)-(\nu-\varepsilon)\frac{\partial\bar{f}}{\partial q}\frac{k}{3}\left(\Delta_0-2\Delta_2\right)+\varepsilon\frac{\partial\bar{f}}{\partial q}\frac{k}{3}\phi &= 0\,,\\
\frac{\partial}{\partial\eta}\left(\frac{\varepsilon\,(\nu-\varepsilon)}{q}\frac{\partial\bar{f}}{\partial q}\Delta_l\right)+(\nu-\varepsilon)\,\frac{\partial\bar{f}}{\partial q}\frac{k}{2l+1}\left[(l+1)\,\Delta_{l+1}-l\,\Delta_{l-1}\right] &= 0\,, \qquad \text{for} \qquad l\geq 2\,.
\end{aligned}
\tag{24}
$$

- Perturbations for the stress-energy tensor can thus be rewritten as

$$
\begin{aligned}
\delta\rho_d &= \frac{\Delta_0}{2\pi^2a^4}\int dq\,q\varepsilon^2\,(\nu-\varepsilon)\,\frac{\partial\bar{f}}{\partial q}\,,\\
\delta P_d &= \frac{\Delta_0}{6\pi^2a^4}\int dq\,q^3(\nu-\varepsilon)\frac{\partial\bar{f}}{\partial q}\,,\\
\theta_d(\bar{\rho}_d+\bar{P}_d) &= \frac{\Delta_1}{2\pi^2a^4}\int dq\,q^2\,\varepsilon\,(\nu-\varepsilon)\frac{\partial\bar{f}}{\partial q}\,,\\
\sigma_d(\bar{\rho}_d+\bar{P}_d) &= -\frac{\Delta_2}{3\pi^2a^4}\int dq\,q^3\,(\nu-\varepsilon)\,\frac{\partial\bar{f}}{\partial q}\,,
\end{aligned}
\tag{25}
$$

and one can resolve the system of coupled equations and find the time dependence of the DFG perturbations.

Through the cutoff of the σ (l-poles, $l \leq 2$) terms ABOVE EQUATIONS reproduce the previous ANALYTICAL results for:

A. Ultra-relativistic DFG during the radiation-dominated era;

B. Non relativistic DFG during the matter-dominated era;
(NUMERICAL x ANAlLITICAL consistency)

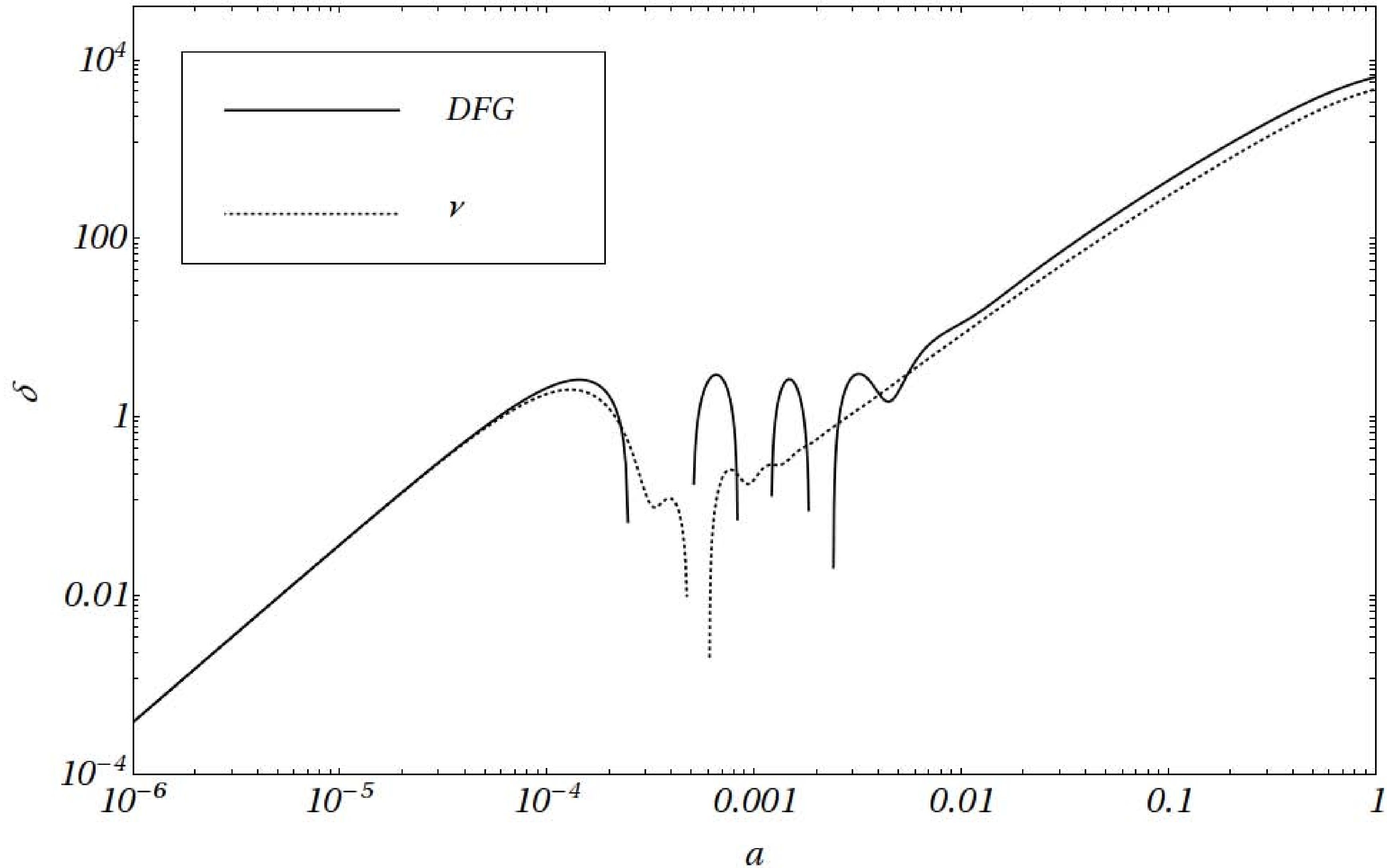
NEUTRINOS - ν

It is assumed the same initial and final averaged densities for both fluids DFG and ν

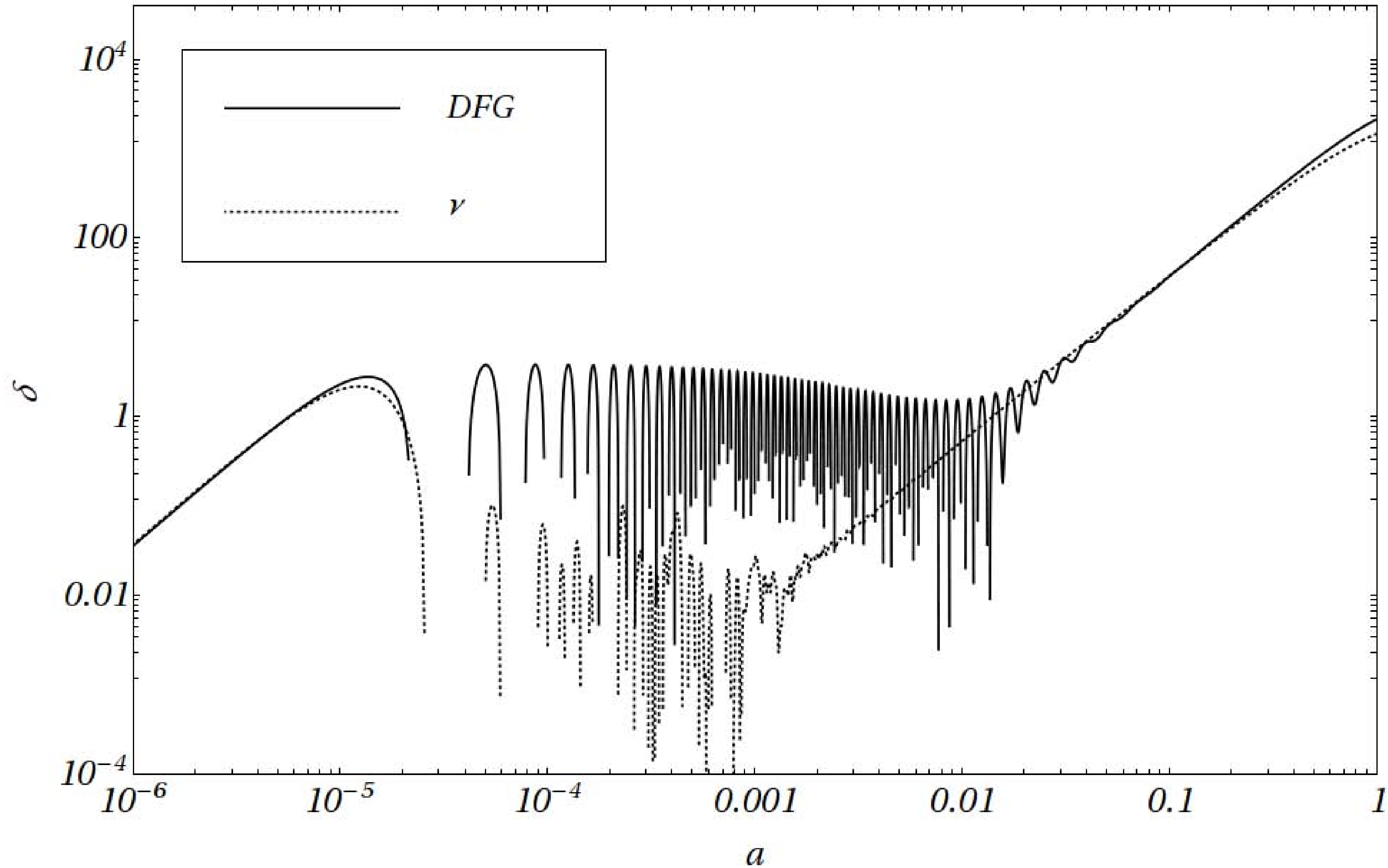
Ω	without ν	massless ν	massive ν	DFG
Ω_Λ	0.73	0.73	0.73	0.73
Ω_γ	4.6E-5	4.6E-5	4.6E-5	4.6E-5
Ω_b	0.0425	0.0425	0.0425	0.0425
Ω_c	0.2+r	0.2	0.2	0.2
Ω_r	0	r	0	0
Ω_ν	0	0	r	0
Ω_d	0	0	0	r

Table 1: Cosmic inventory corresponding to the matter power spectrum obtained. It is assumed a flat universe with$\Sigma_i \Omega_i = 1$ that results in $r = 0.027454$

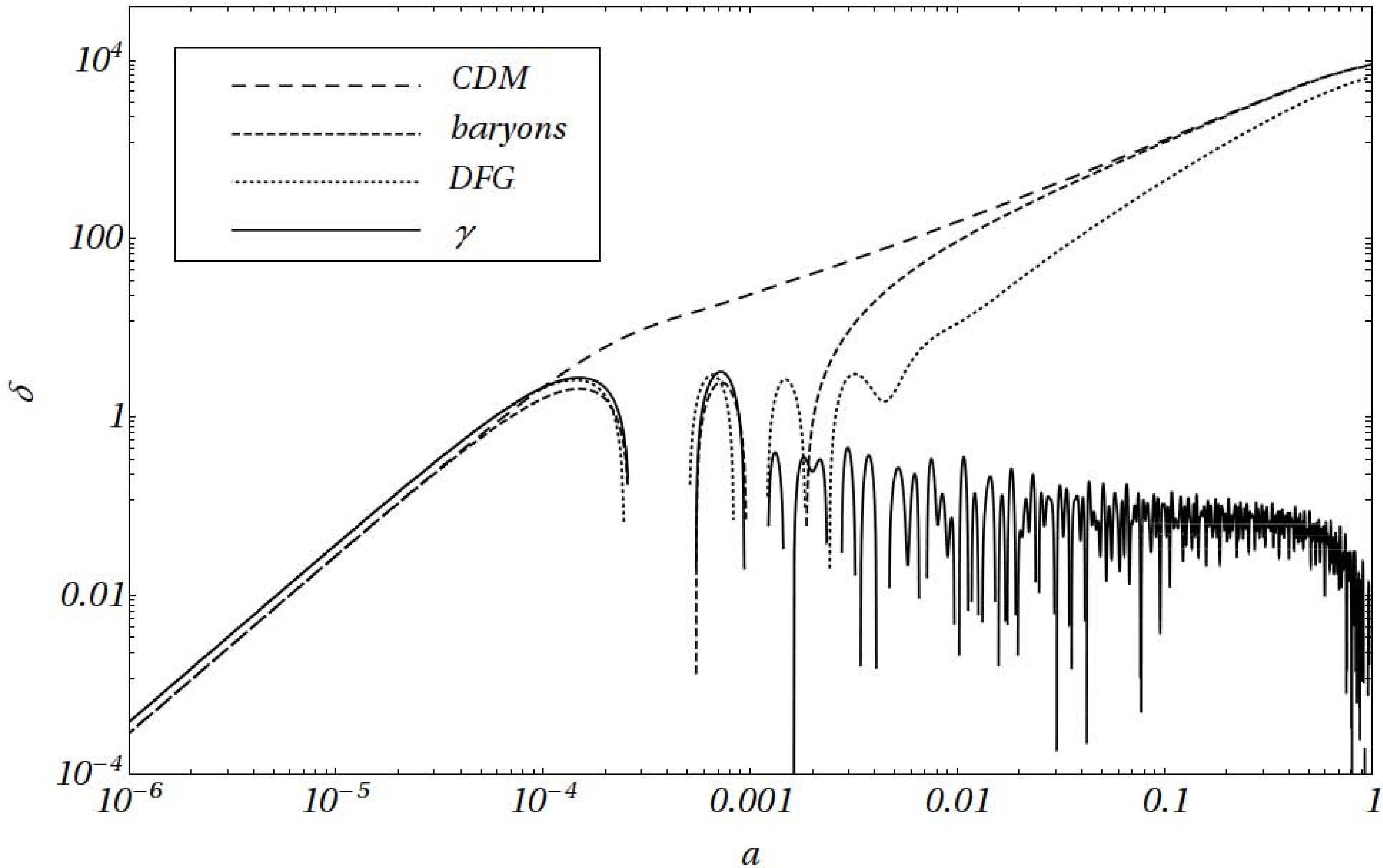
ANALYTICAL *versus* NUMERICAL CALCULATIONS ($k = 0.1\,Mpc^{-1}$)



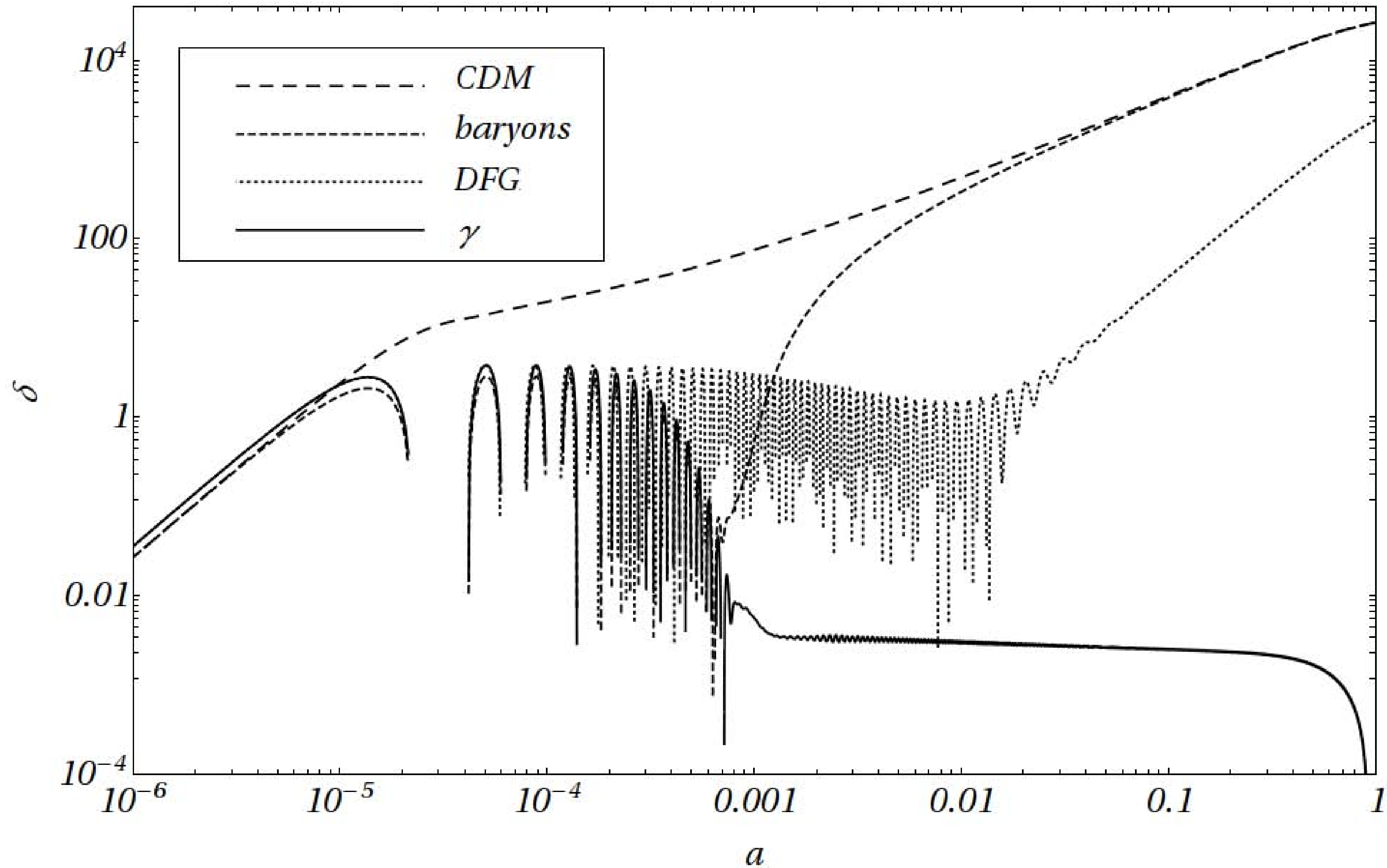
ANALYTICAL *versus* NUMERICAL CALCULATIONS ($k = 1 Mpc^{-1}$)



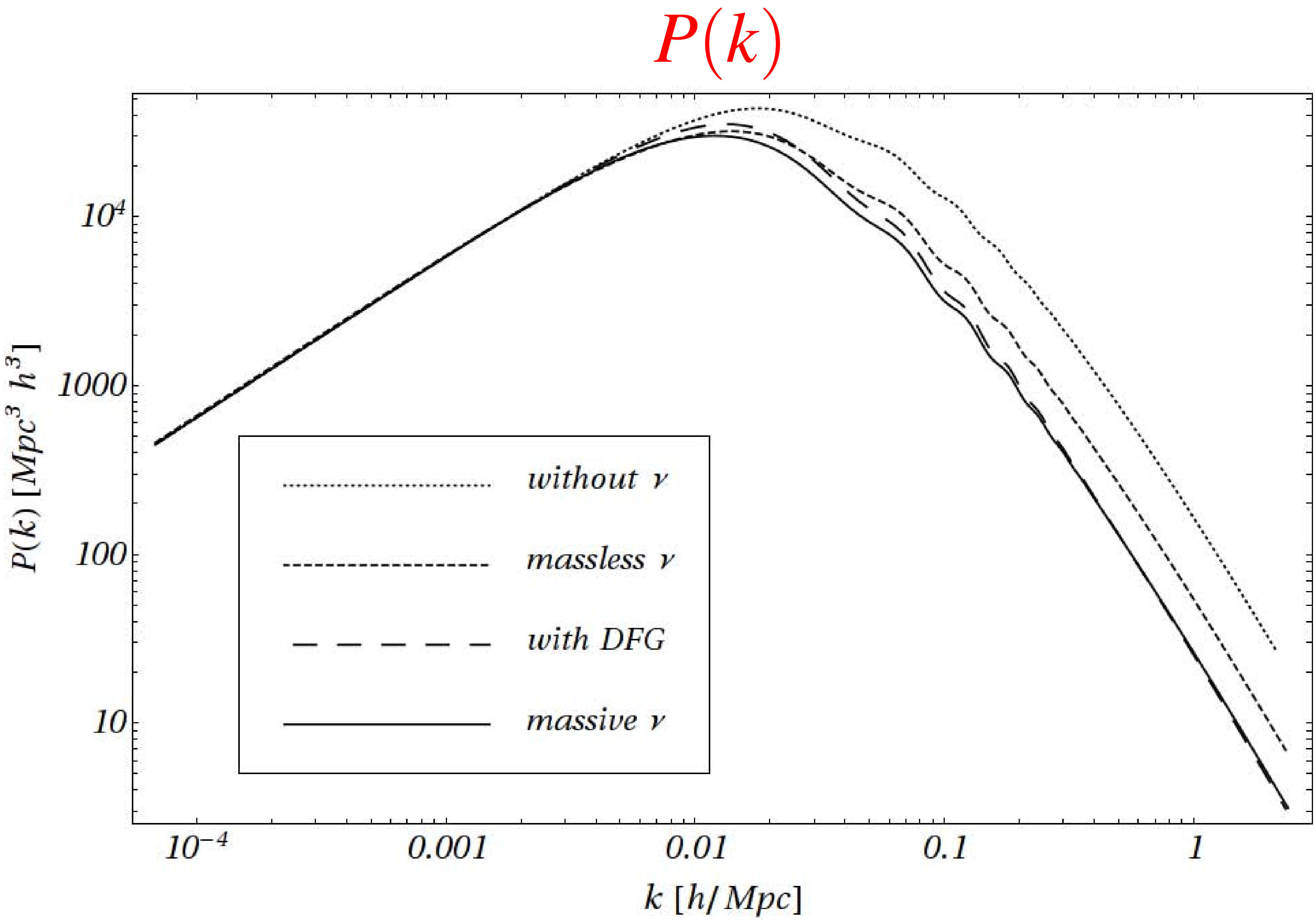
NUMERICAL ANALYSIS ($k = 0.1 Mpc^{-1}$)



NUMERICAL ANALYSIS ($k = 1 Mpc^{-1}$)



Matter Power Spectrum



CONCLUSIONS

- **The global evolution of DFG perturbations are NUMERICALLY and ANALYTICALLY consistent.**

- **Massive Neutrinos should become non-relativistic earlier than a DFG since $m_\nu^{DFG} < m_\nu$ for the same averaged densities.** Meanwhile, the density contrast for the DFG and for massive neutrinos are very similar during the non-relativistic regime, even though it does not reach the growing rate of the CDM density contrast.

- **Lessons concerning the formation of large scale structures of a DFG are depicted, and consequent deviations from standard ΛCDM predictions for the matter power spectrum (with and without neutrinos) are quantified.**

- **The effects on the power spectrum for large and small scales ($k\eta \ll 1$ and $k\eta \gg 1$) are minimal if changing massive neutrinos by DFG.** The region of intermediate scales are softly open to theoretical speculations (Formation of galaxy overdense regions of DFG).

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