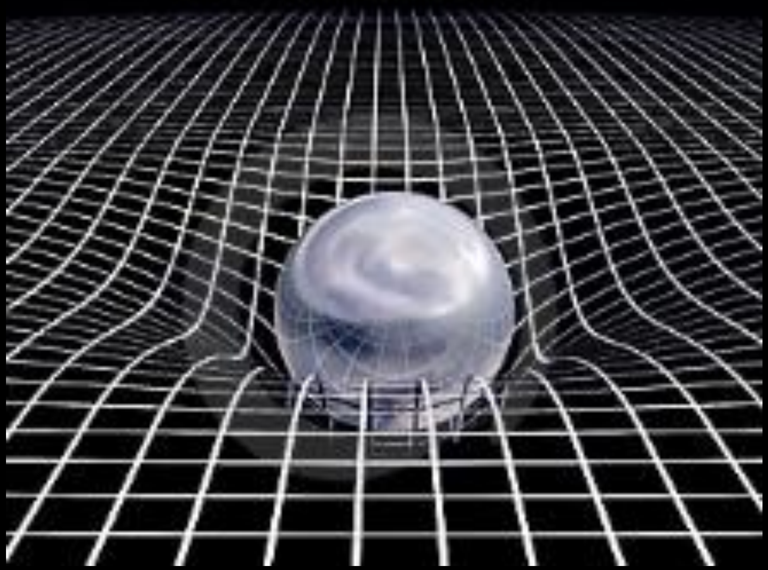

Cosmology from Brane Backreaction

Higher codimension branes and their bulk interactions

w Leo van Nierop

Outline

- Motivation
  - Extra-dimensional cosmology
- Setup
  - A 6D example
- Calculation
  - Maximally symmetric geometries
  - FRW geometries

Motivation

- Few honest-to-God extra-dimensional cosmologies exist
  - work in 4D effective theory
  - work with moving branes in static backgrounds

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- Few honest-to-God extra dimensional cosmologies exist
  - work in 4D effective theory
  - work with moving branes in static backgrounds
- What do extra dimensions do during inflation?
- Brane back-reaction can be important
  - RS models include codimension-1 back-reaction

Setup

- 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2}[R + (\partial\phi)^2] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

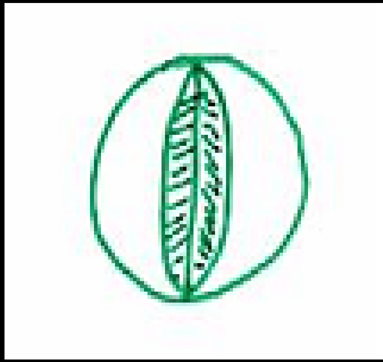
- Two specific cases
  - 6D axion: $a = 0$ and $V = \Lambda$
  - 6D supergravity: $a = 1$ and $V = \frac{2g_R^2}{\kappa^4} e^{\phi}$

Setup

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m\, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-a\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-a\phi_0} \qquad \phi = \phi_0$$



Setup

- Simple solution (including back-reaction)

$$ds^2 = \hat{g}_{mn} dx^m\, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-a\phi_0}$$

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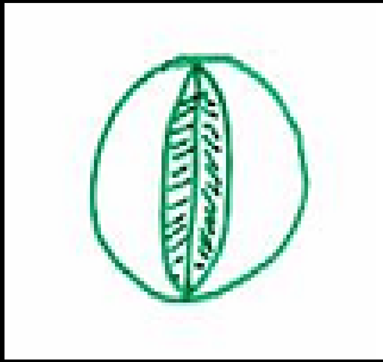

$$1 - \alpha = \frac{\kappa^2 T}{2\pi}$$

Setup

- Simple solution (non-SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m \, dx^n + dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2$$

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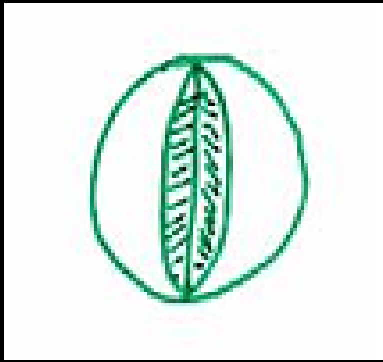

Field equations

$$\frac{2}{L^2} = \kappa^2\left(\frac{3Q^2}{2} + \Lambda\right)$$

$$\hat{R} = \kappa^2(Q^2 - 2\Lambda)$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q$$

Setup

- Simple solution (non-SUSY case)

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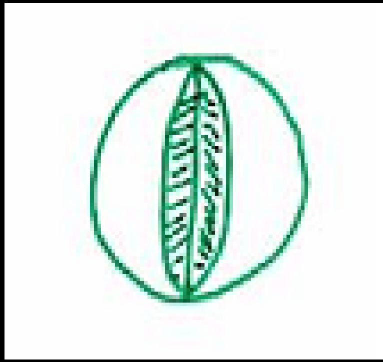

$$\mathrm{Q} = \frac{n}{2\alpha g L^2} \qquad \hat{R} = \kappa^2(Q^2 - 2\Lambda)$$

$$\frac{1}{L^2} = \frac{8\alpha^2 g^2}{3n^2\kappa^2}\left[1 \mp \sqrt{1 - \left(\frac{3n^2\kappa^4\Lambda}{8\alpha^2 g^2}\right)}\right]$$

Setup

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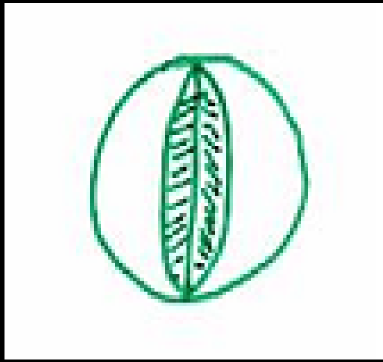


Tune $\Lambda = \frac{Q^2}{2}$ so $\hat{R} = 0$

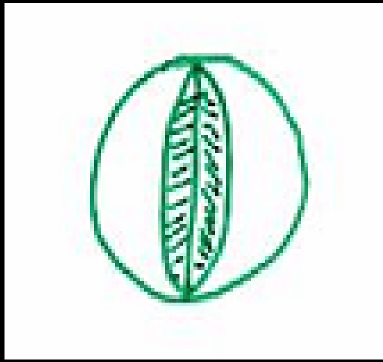
If $T \to T + \delta T$ then $\hat{R} \to -\frac{\kappa^2 \rho}{\pi \alpha L^2}$ where $\rho = 2\,\delta T$

Setup

- Simple solution (SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m \, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-\phi_0} \qquad \phi = \phi_0$$



Field equations

$$\frac{2g_R^2}{\kappa^2} = \frac{\kappa^2 Q^2}{2}$$

$$\kappa^2 Q^2 L^2 = 1 \qquad \hat{R} = 0$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

Setup

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

Obstructs T to δT

Setup

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

Obstructs T to δT

- On other hand, general argument:

$$\rho = \int dV\, L_{bulk} = -\frac{1}{2\kappa^2}\int dV\ \partial^2\phi = \oint dS\, n\cdot\partial\phi\ \propto\ \frac{\partial T}{\partial\phi}$$

Setup

- Resolution: subdominant effects in the brane action are important for flux quantization

$$\text{if } L_b = T_b(\phi) + \Phi_b(\phi) \; {}^*F + \ldots$$

$$\frac{n}{g} = \int F + \frac{1}{2\pi} \sum_b \Phi_b \, e^{\phi}$$

- New function Φ has interpretation as brane-localized flux

Calculation

- More general solutions

$$ds^2 = e^{2W}\hat{g}_{mn}dx^m\,dx^n + dr^2 + e^{2B}d\theta^2$$

$$F_{r\theta} = Qe^{B-4W} \qquad\qquad \phi = \phi(r)$$

Calculation

- Perturb brane properties

$$T \to T + \delta T(\phi)$$

- To evade time-dependence add current

$$\Delta L_{bulk} = J\phi \qquad \text{or} \qquad \Delta L_{bulk} = J$$

- Find general solution to linearized equations

$$\kappa^2 J L^2 \ll 1$$

Calculation

- Sample solutions

$$\delta W = W_0 + W_1 \cos\left(\frac{r}{L}\right)$$

$$\delta\phi = \phi_0 + \phi_1 \ln\left(\frac{1-\cos(r/L)}{\sin(r/L)}\right) - \kappa^2 J L^2 \ln\left[\sin\left(\frac{r}{L}\right)\right]$$

and so on

Calculation

- Brane-bulk boundary conditions:

$$(e^B \phi')_b = \frac{\kappa^2}{2\pi}\left(\frac{\partial L_b}{\partial \phi}\right)$$

$$(e^B W')_b = \frac{\kappa^2}{4\pi}\left(\frac{\partial L_b}{\partial g_{\theta\theta}}\right) = U_b$$

$$(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi}\left[\left(\frac{\partial L_b}{\partial \phi} + \frac{3}{2}\frac{\partial L_b}{\partial g_{\theta\theta}}\right)\right]$$

Constraint: $4U_b[2 - 2L_b - 3U_b] - \left(\frac{\partial L_b}{\partial \phi}\right)^2 = 0$

Calculation

- Non-SUSY result:

$$V_{eff}(\phi) = \phi \int \frac{d\phi}{\phi^2} \left[\frac{\pi \alpha L^2 \hat{R}(\phi)}{\kappa^2} \right]$$

$$\left[\frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi_*} = 0$$

$$\rho = \left[\sum_b \delta T_b - 2 Q \delta \Phi_b \right]_{\phi_*}$$

Calculation

- SUSY result:

$$\left[\delta T_b - 2Q\delta\Phi_b + \frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*} = 0$$

ie Einstein frame potential: $V = U(\phi)e^{2\phi}$

Calculation

- SUSY result:

$$\left[\delta T_b - 2Q\delta\Phi_b + \frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*} = 0$$

$$\rho = [\delta T_b - 2Q\delta\Phi_b] = \left[-\frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*}$$

Calculation

- Three intriguing choices:

Case 1: scale invariant:

if δT independent of ϕ and $\delta\Phi = Ce^{-\phi}$ then $V(\phi) = Ae^{2\phi}$

Calculation

- Three intriguing choices:

Case 1: scale invariant:

if δT independent of ϕ and $\delta\Phi = Ce^{-\phi}$ then $V(\phi) = Ae^{2\phi}$

Case 2: exponentially large volume:

$\delta T_b = A + B\,(\phi + v)^2$ with $v \sim 50$ then $r = Le^{-\phi/2} \gg L$

Calculation

- Three intriguing choices:

Case 3: parametrically small vacuum energy:

δT_b and $\delta \Phi_b$ both independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

Calculation

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Case 3: parametrically small vacuum energy:

δT_b and $\delta \Phi_b$ both independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

Brane action independent of ϕ stable against brane loops
Bulk loops generate corrections of order $e^{2\phi} = \left({}^{1}/{}_{r}\right)^4$

Higher-dimensional inflation

- What about cosmological solutions?
  - Must generalize brane matching conditions to case where on-brane geometry is not maximally symmetric
  - Must solve the higher-dimensional field equations exactly

Higher-dimensional inflation

- What about cosmological solutions?
  - Must generalize brane matching conditions to case where on-brane geometry is not maximally symmetric
  - Must solve the higher-dimensional field equations exactly
- For supersymmetric system exact time-dependent scaling solutions are known
  - Can these be matched to sensible brane physics to see how brane properties control bulk fields?

Higher-dimensional inflation

- 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2}[R + (\partial\phi)^2] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

- Brane-localized inflaton, χ

$$L_{b1} = T_1 + e^{-\phi}[\,(\partial\chi)^2 + V_0 + V_1 e^{\lambda\chi} + \cdots]$$

$$L_{b2} = T_2$$

Higher-dimensional inflation

Tolley, CB, de Rham

- Exact time-dependent solution $e^{-\phi} = (H_0 \tau)^{c+2}$

$$ds^2 = (H_0 \tau)^c \left[g_{mn} dx^m dx^n + \tau^2 \left(g_{ij} dx^i dx^j \right) \right]$$

- FRW time in 4D Einstein frame $dt = \mp (H_0 \tau)^{c+1} d\tau$

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- If c = -2 then $a(t) = e^{H_0 t}$ and r constant
- 4D de Sitter geometry: evades no-go results due to near-brane asymptotics

Higher-dimensional inflation

Tolley, CB, de Rham

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$$ds^2 = (H_0\tau)^c\left[g_{mn}dx^m dx^n + \tau^2\left(g_{ij}dx^i dx^j\right)\right]$$

- FRW time in 4D Einstein frame $dt = \mp(H_0\tau)^{c+1}d\tau$

- If $c \neq -2$ then $a(t) = (H_0 t)^p$ and $r(t) = (H_0 t)^{1/2}$ with $p = (c+1)/(c+2)$

accelerated expansion if $p > 1$ and so $c < -2$

Higher-dimensional inflation

- Connection to brane inflaton, and how does it end?
  - Add inflaton χ evolution to the equations

$$L_b = T + e^{-\phi}\left[\,(\partial\chi)^2 + V_0 + V_1 e^{\lambda\chi} + \cdots\right]$$

$$\chi = \chi_0 + \chi_1 \ln(H_0 \tau)$$

Then $c + 2 = -\lambda\chi_1$ controls the slow roll

and $H_0{}^2 = \lambda V_1/[\chi_1(3 + 2\lambda\chi_1)]$

Conclusions

- Relatively little is known about explicitly higher-dimensional cosmology
  - Higher-dimensional inflation with evolving x-dims

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- Relatively little is known about explicitly higher-dimensional cosmology
  - Higher-dimensional inflation with evolving x-dims
- Branes and brane back-reaction can have important implications for low-energy theory
  - Little explored beyond codimension 1
  - Different parametric dependences in energy: unusual stability to quantum corrections

Conclusions

- Relatively little is known about explicitly higher-dimensional cosmology
  - Higher-dimensional inflation with evolving x-dims
- Branes and brane back-reaction can have important implications
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Potentially wide-ranging observational implications for Dark Energy cosmology, the LHC and elsewhere…