Effective Field Theory for Inflation

The effective theory approach to calculations of higher order correlators of curvature perturbations

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based on work with N.Bartolo, S.Matarrese, A.Riotto.
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+ work in progress..

Pascos 2011, Cambridge

Motivations

The power spectrum from the simplest models of standard single-field slow-roll inflation already nicely accounts for CMB and LSS observations;

higher order because

- A detailed analysis of the amplitude and shape of higher order correlators (e.g. bispectrum,trispectrum) for each inflationary mechanism is instrumental in discriminating among all the allowed scenarios.
- In studying interactions one is probing the dynamics of inflation.
- *The timing is quite good.*
  New experiments provide a better sensitivity to deviations from Gaussian statistics: Planck et al. present us with the opportunity to test this zoo of possibilities.

The effective approach

- The effective theory approach offers several advantages:

  • A unifying perspective on inflation in that the effective Lagrangian comprises many known inflationary models (standard, Ghost, DBI, etc)

  • Symmetries and properties of the action emerge that would not otherwise be manifest

  • Considerable calculational advantages in the decoupling regime.

Ref: [Cheung et al. JHEP 0803 (2008) 014]

The Lagrangian of the theory

The background solution is $\phi_0(t)$. It identifies a privileged spacetime slicing [1]. Perturbations around the background:

$$\phi(t,\vec{x}) = \phi_0(t) + \delta\phi(t,\vec{x})$$

Choose the priviledged slicing to coincide with surfaces of constant t, it amounts to:

$$\delta\phi(t,\vec{x}) = 0 \qquad\qquad \textit{unitary/comoving gauge.}$$

The scalar fluctuation is eaten by the metric, i.e. no explicit scalar fluctuations.

Lagrangian only space diffs invariant now.

Ref: [Cheung et al.]

Once the most general Lagrangian around a given FRW background is written, use the Stuckelberg trick to restore full rep. inv. and make the scalar explicit.

[1] **The geometry of this slicing is described by the extrinsic curvature of surfaces at constant time we will employ later on.**

Decoupling

For sufficiently high energy, E_{mix}, the dynamics of the scalar π decouples.

If $H > E_{mix}$, $\Rightarrow$ the dynamics of the scalar is well described by:

$$S_2^{\pi} = \int d^4x\sqrt{-g}\Big[M_{\rm Pl}^2\dot{H}(\partial_\mu\pi)^2 + 2M_2(t)^4\dot{\pi}^2 - \bar{M}_1(t)^3 H\left(3\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{2\,a^2}\right) - \frac{\bar{M}_2(t)^2}{2}\frac{(\partial_i^2\pi)(\partial_j^2\pi)}{a^4} - \frac{\bar{M}_3(t)^2}{2}\frac{(\partial_i^2\pi)(\partial_j^2\pi)}{a^4}\Big] \quad (1)$$

At the quadratic level we can see:

• $\bar{M}_1 = \bar{M}_2 = \bar{M}_3 = 0 = M_2$ $\rightarrow$ *standard single-field slow-roll inflation with unitary sound speed*

• $M_2 \neq 0$ opens up the gate for $c_s < 1$ models which are often associated to large higher order correlators (DBI inflation&Co)

• $\dot{H} \rightarrow 0$, $\bar{M}_1 = 0$ gives back Ghost inflation

These correspondences continue at third and fourth order.

e.o.m.

In Fourier space with $u(\tau, \vec{k}) = a(\tau)\pi(\tau, \vec{k})$ and τ conformal time, the e.o.m. for the scalar π is:

$$u_k^{''} - \frac{2}{\tau^2}u_k + \alpha_0 k^2 u_k + \beta_0 k^4 \tau^2 u_k = 0 \tag{2}$$

see also [Senatore, Phys.Rev. D71 (2005)]

$$\alpha_0 = \frac{-M_{Pl}^2\dot{H} - \bar{M}_1^3 H/2}{-M_{Pl}^2\dot{H} + 2M_2^4 - 3\bar{M}_1^3 H} \qquad \beta_0 = \frac{(\bar{M}_2^2 + \bar{M}_3^2)H^2}{-M_{Pl}^2\dot{H} + 2M_2^4 - 3\bar{M}_1^3 H} \tag{3}$$

Note that:

- in general single-field slow-roll $\alpha_0 = c_s^2$; $\beta_0 = 0$
- ghost inflation corresponds to $\alpha_0 = 0$; $\beta_0 \neq 0$
- for $\alpha_0 \neq 0$; $\beta_0 \neq 0$ we are in a more general playground, the solution to the e.o.m. would interpolate between known and possibly new inflationary models.

We solve this e.o.m. for the first time.

Solution

$$\pi_k(\tau) = \frac{-i\,H\,e^{\frac{1}{2}i\sqrt{\beta_0}k^2\tau^2}\Gamma\left[\frac{5}{4}-\frac{i\alpha_0}{4\sqrt{\beta_0}}\right]Hyp\mathcal{G}(-\frac{1}{4}-i\frac{\alpha_0}{4\sqrt{\beta_0}},-\frac{1}{2},-i\sqrt{\beta_0}k^2\tau^2)}{\sqrt{2(M_P^2\epsilon H^2+2M_2^4-3\bar{M}_1^3H)}\,k^{3/2}\,(\alpha_0+\sqrt{\beta_0})^{3/4}\Gamma(\frac{5}{4}+\frac{\alpha_0}{4\alpha_0-4i\sqrt{\beta_0}})}$$

It gives back Ghost Inflation ($\alpha_0=0$) and $P(X,\phi)$ ($\beta_0\to 0$).

From here, using $\zeta=-H\pi$, the power spectrum:

$$P_\zeta = \frac{(\alpha_0+\sqrt{\beta_0})^{-3/2}H^4}{16\pi(M_{Pl}^2\epsilon H^2+2M_2^4-3\bar{M}_1^3H)\,|\Gamma(\frac{5}{4}+\frac{\alpha_0}{4\alpha_0-4i\sqrt{\beta_0}})|^2}, \qquad (4)$$

Here, schematically, its tilt is:

$$n_s-1=\Theta_\epsilon(1)\times\epsilon+\Theta_{\epsilon_1}(1)\times\epsilon_1+\Theta_s(1)\times s+\Theta_{\epsilon_0}(1)\times\epsilon_0+\Theta_\eta(1)\times\eta-\dot{\Gamma}/(H\Gamma)$$

In the paper we also give the explicit expression for the running.

One can put mild bounds on the new, generalized slow-roll parameters from the value of the power spectrum itself and from the requirement that they are not so large as to produce too large a value for f_{NL}.

Higher order correlators

Following the same algorythm that gave us $S_\pi^{(2)}$ one obtains:

usual quadratic terms

$M_2 \neq 0 \iff c_s \neq 1$

general s.-f. s.-r. inflation interactions (DBI&Co)

$$
\begin{aligned}
S_\pi^{(3)} = \int d^4x\sqrt{-g}\Big[& M_{\rm Pl}^2\dot H(\partial_\mu\pi)^2 + M_2(t)^4\left(2\dot\pi^2 - 2\dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3(t)^4\dot\pi^3 \\
& -\frac{\bar M_1(t)^3}{2}\left(\frac{-2H(\partial_i\pi)^2}{a^2} + \frac{(\partial_i\pi)^2\partial_j^2\pi}{a^4}\right) - \frac{\bar M_2(t)^2}{2}\left(\frac{(\partial_i^2\pi)(\partial_j^2\pi) + H(\partial_i^2\pi)(\partial_j\pi)^2 + 2\dot\pi\partial_i^2\partial_j\pi\partial_j\pi}{a^4}\right) \\
& -\frac{\bar M_3(t)^2}{2}\left(\frac{(\partial_i^2\pi)(\partial_j^2\pi) + 2H(\partial_i\pi)^2\partial_i^2\pi + 2\dot\pi\partial_{ij}^2\pi\partial_j\pi}{a^4}\right) - \frac{2}{3}\bar M_4(t)^3\frac{1}{a^2}\dot\pi^2\partial_i^2\pi + \frac{\bar M_5(t)^2}{3}\frac{\dot\pi}{a^4}(\partial_i^2\pi)^2 \\
& +\frac{\bar M_6(t)^2}{3}\frac{\dot\pi}{a^4}(\partial_{ij}\pi)^2 - \frac{\bar M_7(t)}{3!}\frac{(\partial_i^2\pi)^3}{a^6} - \frac{\bar M_8(t)}{3!}\frac{\partial_i^2\pi}{a^6}(\partial_{jk}\pi)^2 - \frac{\bar M_9(t)}{3!}\frac{1}{a^6}\partial_{ij}\pi\partial_{jk}\pi\partial_{ki}\pi\Big] .
\end{aligned}
$$

Ghost inflation

Novel curvature-generated terms. Their amplitude can be significant; their shape has interesting, distinctive features

N.B. $\alpha_0 \ll 1$ *and* $\beta_0 \ll 1$ put mild bounds on $M_2, \bar M_1, \bar M_2, \bar M_3$,.
Other parameters first appearing at higher orders bounded by loop corrections to the spectrum and $\mathbf{M}_n \ll \mathcal{M}$, with $\mathcal{M}$ mass of the underlying theory.

Perspective

• We are going to look at higher order correlators of all non trivial interaction terms. All the terms are driven by the M_n coefficients and at this stage we do not worry about their including higher derivative operators which would spoil the e.o.m. in the sense of Ostrogradski
(more rigorous treatment work in progress with X. Chen et al).

• A larger value of some of the the M_n's might be more natural for example, but in the spirit of the effective theory we consider all the interactions which would provide a distinctive signature in the form of higher correlators.
The specific weight of each coefficient is known once one specifies the model, imposes symmetries etc...

Bispectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \equiv (2\pi)^3 \delta^{(3)}(\vec{k_1} + \vec{k_2} + \vec{k_3}) \times f_{NL} \times F(k_1, k_2, k_3)$$

f_{NL} is the amplitude
$F(k_1, k_2, k_3)$ accounts for the shape of the 3-point function.
All results obtained using the IN-IN formalism.

f_{NL}

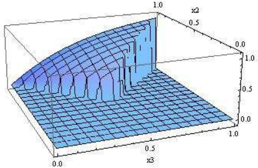
We calculate it for a grid of values of the α_0, β_0 parameters:

Benchmarks	1	2	3	4	5	6
α_0	10^{-2}	0	$0.5 \cdot 10^{-2}$	$2 \cdot 10^{-7}$	10^{-4}	10^{-6}
β_0	0	$0.5 \cdot 10^{-4}$	$0.25 \cdot 10^{-4}$	$5 \cdot 10^{-5}$	0	0

(1)K-inflation wavefunction (2)Ghost (3) Intermediate A (4) Int. B
(5)K-inflation wf with very small generalized speed of sound (6) even smaller

Equilateral shape as a consistency check

- $M_2^4\,\dot{\pi}(\nabla\pi)^2$, typical interaction term, its wf is known to peak on the equilateral configuration $k_1 = k_2 = k_3$. $F(1, x_2, x_3)x_2^2 x_3^2$ is plotted ($x_3 = k_3/k_1,\ x_2 = k_2/k_1$)



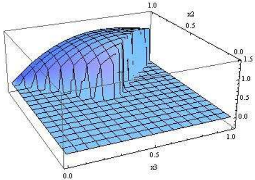


Figure: $P(X,\phi) \uparrow$ w.f. Ghost $\nearrow$ w.f. No news, just obtaining what we should

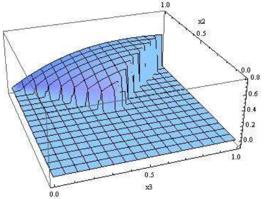


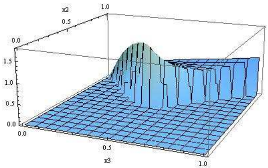
Figure: interpolating solution $\uparrow$, still equilateral shape

News!

- $\bar{M}_6^2\, \dot{\pi}(\partial_{ij}\pi)^2/a^4$, extr. curvature-generated term we consider for the first time.

benchmarks	1	2	3	4	5	6
$f_{\rm NL}^{\bar{M}_6}$	$10^4\ \gamma_7$	$4\cdot 10^3\ \gamma_7$	$5\cdot 10^3\ \gamma_7$	$4\cdot 10^3\ \gamma_7$	$10^8\ \gamma_7$	$10^{12}\ \gamma_7$

$$\gamma_7 = (\bar{M}_6^2 H^2)/(M_{\rm Pl}^2 \epsilon H^2 + 2M_2^4 - 3\bar{M}_1^3 H).$$



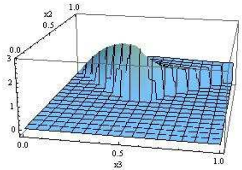
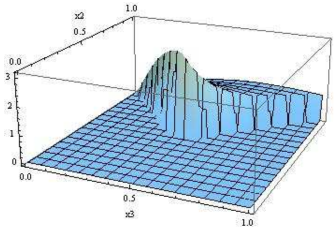


Figure: $P(X,\phi)\uparrow$ Interpolating solution $\uparrow$

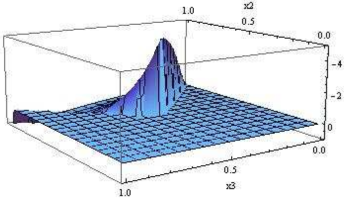
Here a shape function of a single, independent interaction operator peaks on a flat ($k_1 = 2k_2 = 2k_3$) configuration. See also [Senatore et al. JCAP 1001 (2010) 028] where this shape was first obtained by considering two operators and [Creminelli et al. JCAP 1102 (2011) 006] where this shape was obtained in the context of galileon models.

More news!

- $\bar{M}_8\,\partial_{i^2}\pi\,(\partial_{jk}\pi)^2/a^6$, another new extr. curvature-generated term. *Flat* shape.



- $\bar{M}_9\,\partial_{ij}\pi\partial_{jk}\pi\partial_{ki}\pi/a^6$, extr. curvature term. A slightly different but still *flat* shape.



In the papers we also look at the running of $f_{NL}^{\bar{M}_6}$ and find that it can be dominated by generalized slow roll parameters other than ϵ, η, s.

Four-point function contributions: diagrams

Figure: Scalar exchange diagram $\nwarrow$. Contact interaction diagram $\uparrow$

- s.e.: third-order interaction terms integrated twice over time
- c.i.: fourth-order interactions integrated once

Symmetries

Quite a number of interactions ⇒ ordering principle/selection criterium would be useful.
Symmetries come handy!
Symmetries are also useful if one is looking for small bispectrum & large trispectrum examples.

$$\mathbf{S1}: \quad \pi \to -\pi; \qquad \mathbf{S2}: \quad \pi \to -\pi \ \ and \ \ t \to -t. \tag{6}$$

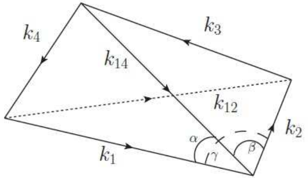
Table 1

Coefficients	M_2	M_3	M_4	$\bar{M}_1$	$\bar{M}_2$	$\bar{M}_3$	$\bar{M}_4$	$\bar{M}_5$	$\bar{M}_6$	$\bar{M}_7$	$\bar{M}_8$	$\bar{M}_9$
S1	X	X	✓	X	X	X	X	X	X	X	X	X
S2	✓	✓	✓	X	X	X	X	✓	✓	X	X	X
Coefficients	$\bar{M}_{10}$	$\bar{M}_{11}$	$\bar{M}_{12}$	$\bar{M}_{13}$	$\bar{M}_{14}$	$\bar{M}_{15}$	$\bar{N}_1$	$\bar{N}_2$	$\bar{N}_3$	$\bar{N}_4$	$\bar{N}_5$	/
S1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
S2	X	✓	✓	X	X	X	✓	✓	✓	✓	✓	

The Coefficients marked with " ✓" in correspondence of a given symmetry **S** are **S**-invariant, those marked with "X" violate the **S** symmetry.

Will be studying the interactions driven by the parameters in red.

Six variables ⇒ need to pick a specific configuration



- equilateral: $k_1 = k_2 = k_3 = k_4$, plot $k_{12} \equiv |\vec{k_1} + \vec{k_2}|$ and $k_{14} \equiv |\vec{k_1} + \vec{k_4}|$
- folded: $k_{12} \to 0, \quad k_1 = k_2, \quad k_3 = k_4$, plot k_{14}/k_1 and k_4/k_1
- specialized planar limit: $k_1 = k_3 = k_{14}, \quad k_{12} = f(k_1, k_2, k_4)$, plot k_2/k_1 and k_4/k_1
- near the double squeezed limit:
  $k_3 = k_4 = k_{12} \quad k_2 = g(k_1, k_3, k_4, k_{14}, k_{12}) \quad k_{12} \to 0$, plot k_{14} and k_4

Trispectrum shape functions

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle \propto (2\pi)^9 P_\zeta^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \Pi_{i=1}^4 \frac{1}{k_i^3} \mathcal{T}(k_1, k_2, k_3, k_4, k_{12}, k_{14})$$

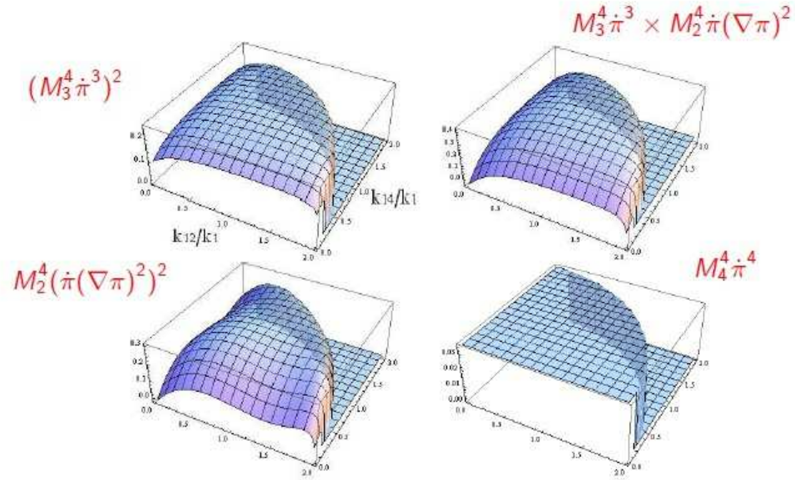
$$\zeta = -H\pi$$

Plotted quantities

- Equilateral $\longrightarrow$ $\mathcal{T}$
- Folded $\longrightarrow$ $\mathcal{T}$
- Specialized planar limit $\longrightarrow$ $\mathcal{T}$
- Near double squeezed limit $\longrightarrow$ $\mathcal{T}\left(\Pi_{i=1}^4 \frac{1}{k_i}\right)$

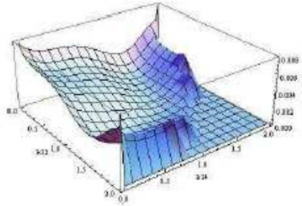
Shapes literature

Equilateral conf. for $P(X, \phi)$ models, [Chen et al. JCAP 0908 (2009) 008]

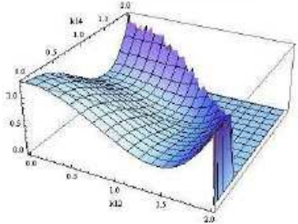


Our equilateral findings

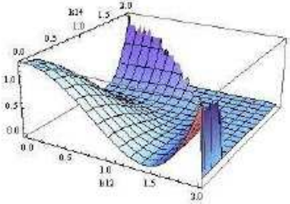
$(\bar{M}_6 \dot{\pi} (\nabla \pi)^2)^2$



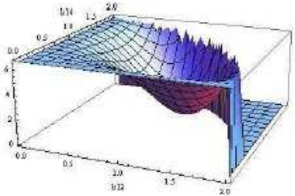
$M_2^4 (\nabla \pi)^4$ *(for M2, see also Huang '10 Izumi et al '10)*



$\bar{N}_3\, \partial_\rho^2 \pi\, \partial_{ij}\pi\, \partial_{jk}\pi\, \partial_{ki}\pi \;\; / a^8$

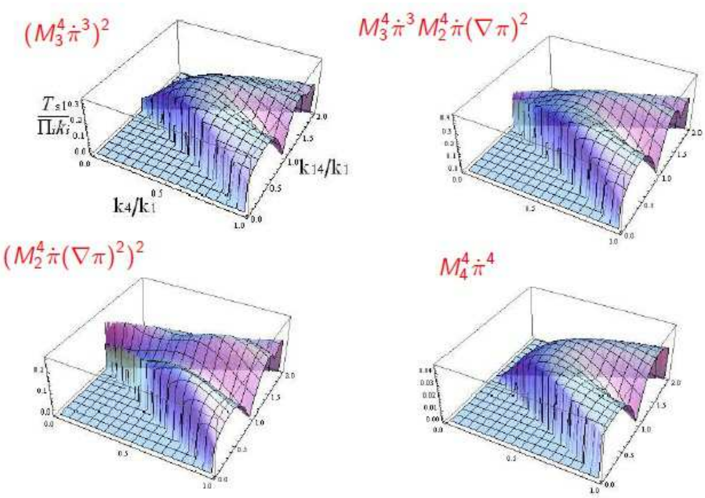


$\bar{M}_6^2 (\partial_k \pi)^2 (\partial_{ij}\pi)^2 / a^4$



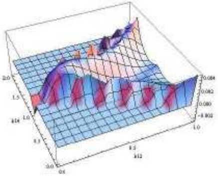
Shape Literature

Double squeezed limit for $P(X, \phi)$ models, [Chen et al. '09]

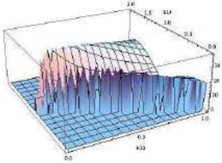


Our double-squeezed findings

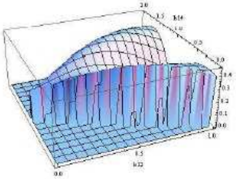
$(\bar{M}_6 \dot{\pi} (\nabla \pi)^2)^2$



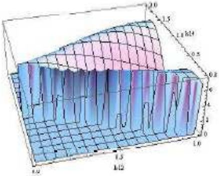
$M_2^4 (\nabla \pi)^4$



$M_3^4 \dot{\pi}^3 M_2^4 \dot{\pi} (\nabla \pi)^2$



$\bar{M}_6^2 (\partial_k \pi)^2 (\partial_{ij} \pi)^2 / a^4$



Compare with $P(X,\phi)$ models: here the double squeezed limit **cannot** remove degeneracies.

One cannot tell if an interaction is third or fourth order by looking at the plot in d.s. configuration

Summary and Future Work

Message

Extrinsic curvature-generated terms should be taken into account because:

- Significantly enlarge the useful parameter space spanned by the effective theory
- Can produce large NG and present interesting distinctive features in the form of their shape functions

Future Work

- Non Bunch-Davies vacua
- Relax the shift symmetry requirement
- Specially for the trispectrum, focus on concrete and well grounded models spanned by the effective theory.

Explicit expressions II

$$\Theta_0 = \frac{6M_0^2}{\left(2{M_0}^2 + \sqrt{2}\,(2H^2M_P^2\epsilon + {M_1}^4)\,\sqrt{\frac{1}{H^2M_P^2\epsilon + 2{M_2}^4 + 3{M_1}^4}}\right)} .$$

$$\epsilon_0 = \frac{\dot{M}_0}{HM_0}\ ; \qquad \epsilon_1 = \frac{\dot{M}_1}{HM_1}\ ; \qquad \epsilon_\Gamma = \frac{\dot{\Gamma}}{H\Gamma}\ . \tag{7}$$

$$\epsilon_2 \equiv \frac{\dot{M}_2}{HM_2} = \frac{\eta}{2} - \epsilon + \frac{s}{2(c_s^2 - 1)}\ . \tag{8}$$

$$M_0^4 \equiv \bar{M}_0^2 H^2\ ; \quad M_1^4 \equiv \bar{M}_1^3 H\ . \tag{9}$$

WMAP 7 bounds on f_{NL}, (95% CL)

$$
\begin{aligned}
-10 \leq f_{NL}^{\mathrm{local}} \leq 74 \ ; \\
-214 \leq f_{NL}^{\mathrm{equilateral}} \leq 266 \ ; \\
-410 \leq f_{NL}^{\mathrm{orthogonal}} \leq 6 \ .
\end{aligned}
\tag{10}
$$

How to get to the π Lagrangian in detail

Whenever one breaks time reparametrization invariance there appears automatically a preferred slicing of spacetime described by a function $\tilde{t}(x)$. This function is such that if, for example, the breaking is realized by a scalar function of time $\phi_0(t)$ then whenever $\tilde{t}$ is constant so is $\phi_0(t)$.

Unitary gauge consists in requiring that $\tilde{t}(x)$ coincides with t so as to make the additional scalar degree of freedom $\tilde{t}$ not to appear explicitly in the Lagrangian.

$$1 \tag{11}$$

Unperturbed space diffs invariant Lagrangian

In Creminelli et al. '07 it is shown that the most general unperturbed Lagrangian in unitary gauge has the form:

$$S = \int d^4x\sqrt{-g}F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t) \tag{12}$$

with the requirement that all the remaining free indices in F are upper '0'. Here $K_{\mu\nu}$ is the extrinsic curvature of surfaces at constant time. Take the unit vector perpendicular to the surface of constant $\tilde{t}$,

$$n_\mu = \frac{\partial_\mu \tilde{t}}{\sqrt{-g^{\mu\nu}\partial_\mu \tilde{t}\partial_\nu \tilde{t}}}$$

and consider the induced metric on the aforementioned surfaces, $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$. $K_{\mu\nu}$ is then given by

$$K_{\mu\nu} = h^\sigma_\mu \nabla_\sigma n_\nu$$

Expanding around FRW

The most generic theory (with broken time diffs) around a FRW background:

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{2}M_{Pl}^2 R + M_{Pl}^2 \dot{H} g^{00} - M_{Pl}^2(3H^2 + \dot{H}) + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 - \frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big]. \quad (13)$$

The dots stand for terms starting at third and fourth order in perturbations.

This is all good but one eventually wants to see explicitly the scalar degree of freedom rather than working with the metric; something which is readily obtained using the *Stueckelberg* trick.

Making the scalar d.o.f. explicit again

Consider a simplified version of the action written above:

$$S = \int d^4x \, \sqrt{-g} \left[A(t) + B(t) g^{00}(x) \right] \; .$$

Upon $t \to \widetilde{t} = t + \xi^0(x)$, $\vec{x} \to \widetilde{\vec{x}} = \vec{x}$; one gets:

$$S = \int d^4x \, \sqrt{-\widetilde{g}(\widetilde{x}(x))} \left| \frac{\partial \widetilde{x}}{\partial x} \right| \left[A(t) + B(t) \frac{\partial x^0}{\partial \widetilde{x}^\mu} \frac{\partial x^0}{\partial \widetilde{x}^\nu} \widetilde{g}^{\mu\nu}(\widetilde{x}(x)) \right] \; .$$

which can be written as:

$$S = \int d^4\widetilde{x} \, \sqrt{-\widetilde{g}(\widetilde{x})} \left[A(\widetilde{t} - \xi^0(x(\widetilde{x}))) + B(\widetilde{t} - \xi^0(x(\widetilde{x}))) \frac{\partial(\widetilde{t} - \xi^0(x(\widetilde{x})))}{\partial \widetilde{x}^\mu} \frac{\partial(\widetilde{t} - \xi^0(x(\widetilde{x})))}{\partial \widetilde{x}^\nu} \widetilde{g}^{\mu\nu}(\widetilde{x}) \right]$$

Now, promote $\xi^0(x)$ to a field, $\xi^0(x) \to -\pi(x)$, whose transformation under time reparametrization reads:

$$\pi(x) \to \widetilde{\pi}(\widetilde{x}(x)) = \pi(x) - \xi^0(x);$$

It's easy to verify that the action below is now invariant under full diffs:

$$S = \int d^4x \, \sqrt{-g(x)} \left[A(t + \pi(x)) + B(t + \pi(x)) \frac{\partial(t + \pi(x))}{\partial x^\mu} \frac{\partial(t + \pi(x))}{\partial x^\nu} g^{\mu\nu}(x) \right] .$$

Pay back time I

The scalar degree of freedom is now explicit in the time dependence of the coefficients and the metric transformation.
This choice pays off in that it is expected, from standard gauge theory, that the dynamics of the scalar π decouples from the one of the metric above some energy range. Consider one second order sample term in the action:

$$S = \int \sqrt{-g}[\ldots + \frac{M_2(t)^4}{2!}(1+g^{00})^2] \to \int \sqrt{-g}[\ldots + \frac{M_2(t+\pi)^4}{2!}(1+\frac{\partial(t+\pi(x))}{\partial x^\mu}\frac{\partial(t+\pi(x))}{\partial x^\nu}g^{\mu\nu}(x))^2]$$

$$= \int d^4x\sqrt{-g}\left[\ldots + \frac{M_2(t+\pi)^4}{2!}\left((1+\dot\pi)^2 g^{00} + 2(1+\dot\pi)g^{0i}\partial_i\pi + g^{ij}\partial_i\pi\partial_j\pi + 1\right)^2\right],$$

$$= \int d^4x\sqrt{-g}\left[\ldots + \frac{M_2(t+\pi)^4}{2!}\left((1+\dot\pi)^2(g^{00}_{(0)} + g^{00}_{(1)} + ..) + 2(1+\dot\pi)\partial_i\pi(g^{0i}_{(0)} + g^{0i}_{(1)}..) + ..\right)\right]$$

Second order fluctuations: the π-terms contain always one or two more derivatives than the g-ones. For sufficiently high energies, the derivatives (think in Fourier space) will provide the π flucuations with a much bigger weight.

Pay back time II

If one sets the working regime (H, in our cosmological scenario) above the so called mixing energy, which can be read off from the canonically normalized coefficients of the kinetic quadratic terms in the action, calculations are greatly simplified. One now is concerned only with S_π. At second order:

$$S_2 = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2\dot{H}(\partial_\mu\pi)^2 + 2M_2(t)^4\dot{\pi}^2 - \bar{M}_1(t)^3 H\left(3\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{2\,a^2}\right)\right.$$
$$\left. - \frac{\bar{M}_2(t)^2}{2}\frac{(\partial_i^2\pi)(\partial_j^2\pi)}{a^4} - \frac{\bar{M}_3(t)^2}{2}\frac{(\partial_i^2\pi)(\partial_j^2\pi)}{a^4}\right] \qquad (14)$$

Already at the quadratic level we see different inflationary mechanisms captured:

- $\bar{M}_1 = \bar{M}_2 = \bar{M}_3 = 0 = M_2 \;\rightarrow$ *standard single-field slow-roll inflation with unitary sound speed*

- $M_2 \neq 0$ opens up the gate for $c_s < 1$ models which are often associated to large higher correlators (DBI inflation&Co)

- $\dot{H} \rightarrow 0, \bar{M}_1 = 0$ gives back Ghost inflation

These clear-cut correspondences continue at third and fourth order.

Schwinger-Keldysh (*IN-IN*) formalism

Main formula:

$$\langle\Omega|\Theta(t)|\Omega\rangle = \left\langle 0 \left| \bar{T}\left(e^{i\int_0^t H_I(t')dt'}\right)\Theta_I(t)\, T\left(e^{-i\int_0^t H_I(t')dt'}\right)\right| 0\right\rangle$$

$\Theta(t)$ = field operator
$|\Omega\rangle$ = vacuum of the interaction theory
$T, \bar{T}$ =time-ordering, anti-time-ordering operators
H_I = interaction Hamiltonian

• All the fields in the interaction picture so free-field operator expansion.

• Just like in usual QFT, only here correlations between observables are at the same spacetime points and there's the anti-time order operator as well.

Fourth order, Trispectrum

$$
\begin{aligned}
\mathcal{S}_4 = \int d^4x \sqrt{-g} \Bigg[& \frac{1}{2!} M_2(t)^4 \frac{(\partial_i \pi)^4}{a^4} + 2 M_3(t)^4 \frac{\dot{\pi}^2 (\partial_i \pi)^2}{a^2} + \frac{2}{3} M_4(t)^4 \dot{\pi}^4 \\
& - \frac{\bar{M}_1(t)^3}{4} \left(\frac{H (\partial_i \pi)^4}{a^4} - \frac{2 \dot{\pi} (\partial_i \pi)^2 \partial_j^2 \pi}{a^4} \right) - \frac{\bar{M}_2(t)^2}{2} \left(\frac{(\partial_j \pi)^2 (\partial_i^2 \pi)^2}{a^6} + \frac{2 \partial_k^2 \pi \partial_i \pi \partial_{ij} \pi \partial_j \pi}{a^6} \right) \\
& - \frac{\bar{M}_3(t)^2}{2} \left(\frac{(\partial_{ij} \pi)^2 (\partial_k \pi)^2}{a^6} + \frac{2 \partial_i \pi \partial_{ij} \pi \partial_{jk} \pi \partial_k \pi}{a^6} \right) + \frac{2}{3} \bar{M}_4(t)^3 \, \frac{\dot{\pi} (\partial_i \pi)^2 \partial_j^2 \pi}{a^4} \\
& - \frac{\bar{M}_6(t)^2}{3!} \frac{(\partial_k \pi)^2 (\partial_{ij} \pi)^2}{a^6} - \frac{\bar{M}_7(t)}{3!} \left(\frac{3}{2} \frac{(\partial_i^2 \pi)^2 H (\partial_j \pi)^2}{a^6} + \frac{6 \, \dot{\pi} \partial_k^2 \pi (\partial_j \partial_i^2 \pi) \partial_j \pi}{a^6} \right) \\
& - \frac{\bar{M}_8(t)}{3!} \left(\frac{H (\partial_i \pi)^2 (\partial_j^2 \pi)^2}{a^6} + \frac{H (\partial_i \pi)^2 (\partial_{jk} \pi)^2}{2a^6} - \frac{2 \, H \partial_k^2 \pi \partial_i \pi \partial_{ij} \pi \partial_j \pi}{a^6} + \frac{2 \dot{\pi} \partial_k^2 \pi \partial_i^2 \partial_j \pi \partial_j \pi}{a^6} \right. \\
& \qquad \left. + \frac{2 \dot{\pi} \partial_k^2 \partial_i \pi \partial_{ij} \pi \partial_j \pi}{a^6} + \frac{2 \dot{\pi} \partial_{ij} \partial_{ijk} \partial_k \pi}{a^6} \right) - \frac{\bar{M}_5(t)^2}{3!} \frac{(\partial_i \pi)^2 (\partial_j^2 \pi)^2}{a^6} \\
& - \frac{\bar{M}_9(t)}{2} \left(\frac{H \partial_k^2 \pi (\partial_{ij} \pi)^2}{2a^6} - \frac{H \partial_i \pi \partial_{ij} \pi \partial_{jk} \pi \partial_k \pi}{a^6} + \frac{\dot{\pi} \partial_{ij} \pi \partial_{ijk} \pi \partial_k \pi}{a^6} + \frac{\dot{\pi} \partial_i^2 \partial_j \pi \partial_{jk} \pi \partial_k \pi}{a^6} \right) \\
& + \frac{\bar{M}_{10}^3(t)}{3} \frac{\dot{\pi}^3 \partial_i^2 \pi}{a^2} - \frac{\bar{M}_{11}^2(t)}{3!} \frac{\dot{\pi}^2 (\partial_i^2 \pi)^2}{a^4} - \frac{\bar{M}_{12}^2(t)}{3!} \frac{\dot{\pi}^2 (\partial_{ij} \pi)^2}{a^4} + \frac{\bar{M}_{13}(t)}{4!} \frac{2 \, \dot{\pi}}{a^6} (\partial_i^2 \pi)^3 \\
& + \frac{\bar{M}_{14}(t)}{4!} \frac{2 \, \dot{\pi} \partial_k^2 \pi (\partial_{ij} \pi)^2}{a^6} + \frac{\bar{M}_{15}(t)}{4!} \frac{2 \, \dot{\pi} \partial_{ij} \pi \partial_{jk} \pi \partial_{ki} \pi}{a^6} - \frac{\bar{N}_1(t)}{4!} \frac{(\partial_i^2 \pi)^4}{a^8} - \frac{\bar{N}_2(t)}{4!} \frac{(\partial_k^2 \pi)^2 (\partial_{ij} \pi)^2}{a^8} \\
& - \frac{\bar{N}_3(t)}{4!} \frac{\partial_\rho^2 \pi \partial_{ij} \pi \partial_{jk} \pi \partial_{ki} \pi}{a^8} - \frac{\bar{N}_4(t)}{4!} \frac{(\partial_{ij} \pi)^4}{a^8} - \frac{\bar{N}_5(t)}{4!} \frac{\partial_{ij} \pi \partial_{jk} \pi \partial_{k\rho} \pi \partial_{\rho i} \pi}{a^8} \Bigg]
\end{aligned}
$$

Running

Want to consider the running of the f_{NL} contribution generated by the $\bar{M}_6$-driven interaction term: this coefficient regulates operators that produce interesting bispectrum and trispectrum shape-functions.

$$n_{NG} \equiv \frac{d\,\ln|f_{NL}^{\bar{M}_6}(k)|}{d\ln k} \simeq \frac{1}{Hf_{NL}}\frac{d\,f_{NL}^{\bar{M}_6}}{dt} = \Theta(\epsilon,\eta,s,\epsilon_1,\epsilon_0) + 2\epsilon_6 \qquad (15)$$

$$\epsilon_6 = \frac{\dot{\bar{M}}_6}{H\bar{M}_6}$$

There is a window for the parameter ϵ_6 to be larger than the usual slow roll parameters without having a small $\bar{M}_6$ spoil the importance of $f_{NL}^{\bar{M}_6}$

$$\frac{\bar{M}_6^2 H^2}{M_2^4(\alpha_0+\sqrt{\beta_0})} > 1\,; \qquad \frac{\bar{M}_6^2 H^2}{M_3^4(\alpha_0+\sqrt{\beta_0})^2} > 1\,. \qquad (16)$$

and without having a large $\dot{\bar{M}}_6$ give too large a correction to the power spectrum.

$$\frac{M_2^4}{\bar{M}_6^2 H^2} > \epsilon_6 \gg \epsilon\,; \qquad \frac{(\alpha_0+\sqrt{\beta_0})M_3^4}{\bar{M}_6^2 H^2} > \epsilon_6 \gg \epsilon\,; \qquad \frac{(\alpha_0+\sqrt{\beta_0})^2 M_4^4}{\bar{M}_6^2 H^2} > \epsilon_6 \gg \epsilon\,. \quad (17)$$