Phases of holographic matter

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July 2011 – PASCOS (DAMTP)

.

Based on various works with:
Chris Herzog, Diego Hofman, Gary Horowitz,
Pavel Petrov, Joe Polchinski, Eva Silverstein,
Alireza Tavanfar, David Tong, David Vegh

A less compressed, but also not too long version of what I'm about to say can be found in

arXiv:1106.4324 [hep-th]

Very brief condensed matter motivation

- QED in 2+1 dimensions at finite chemical potential μ looks simple:

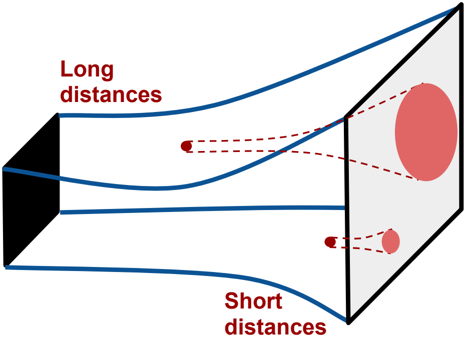
$$\mathcal{L} \sim F^2 + \overline{\Psi} \left[\Gamma \cdot (\partial + A) + \Gamma^0 \mu \right] \Psi \,.$$

- Chemical potential $\Rightarrow$ Fermions form a Fermi surface.
- Low energy physics is strongly interacting [e.g. S-S. Lee '09].
- Difficult/impossible to answer basic questions:
  - At what temperature is there a superconducting instability?
  - What do correlators look like at low energies?

Holography digested

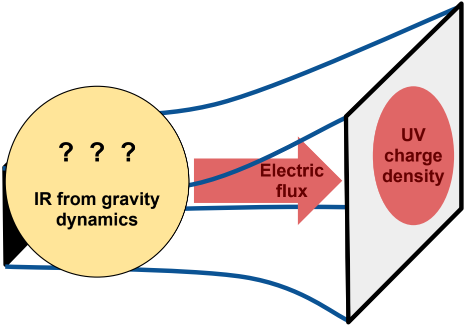
- In certain quantum field theories the locality of the renormalisation group flow can be usefully geometrised.

- Append a dynamical extra dimension to the field theory.



Holographic framework for finite density physics

- Change the problem, keep basic ingredients:
  - $SU(N)$ gauge fields coupled to matter with global symmetries.
- Holographically:
  - Finite chemical potential $\Rightarrow$ charge density $\Rightarrow$ flux at infinity.
  - Low energy dynamics $\Rightarrow$ Solve the geometry in the interior.

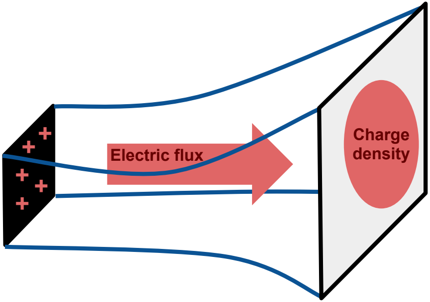


Zero temperature, nonzero densities: I

- Simplest possible framework: Einstein-Maxwell

$$\mathcal{L} \sim \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F^2 \,.$$

- Answer: Extremal back hole. $AdS_4 \to AdS_2 \times \mathbb{R}^2$. [c.f. Gibbons '85]

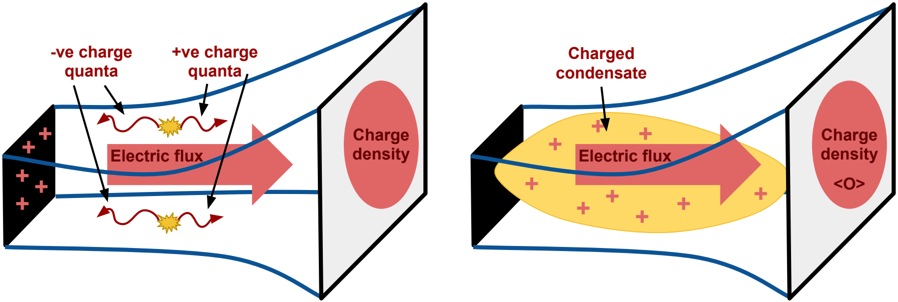


- $AdS_2 \times \mathbb{R}^2$ robust under higher derivative corrections. [e.g. Sen '05]

Superradiant/Schwinger-like instabilities: charged bosons

[Gubser; Hartnoll-Herzog-Horowitz '08]

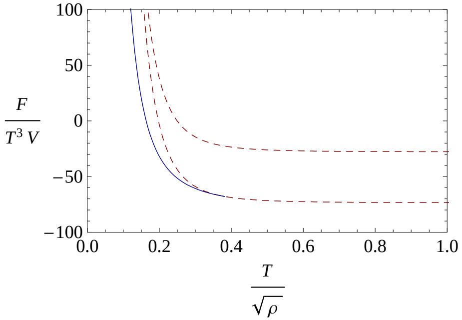
- Electric fields can destabilize scalar fields: $m^2_{\text{eff.}} \sim m^2 + g^{tt} A_t A_t$.
- Pair production + charge neutralisation of black hole.



Heating up holographic superconductors

[Hartnoll-Herzog-Horowitz '08]

- Heating up the black hole (away from extremality)
  - $\Rightarrow$ Make the AdS box smaller.
  - $\Rightarrow$ Eventually stabilise the tachyon.
- Condensate goes to zero at T_C. Second order transition.



Superradiant/Schwinger-like instabilities: charged fermions

[Hartnoll-Polchinski-Silverstein-Tong '09; Hartnoll-Tavanfar '10]

- Fermions obey Pauli exclusion.

  $\Rightarrow$ Do not coherently occupy ground state.

  $\Rightarrow$ ~~Einstein-Maxwell-Dirac~~.

- With a background A_t, fermions try to build up a Fermi sea.

- Difficult problem: self-consistently
  - Occupy all Dirac eigenstates in spacetime with energy $E < \mu$.
  - Backreact the energy-momentum tensor of these eigenstates.

- Simplifies in a large-charge/large mass limit.

  [Thomas-Fermi '27, Oppenheimer-Volkov '39]

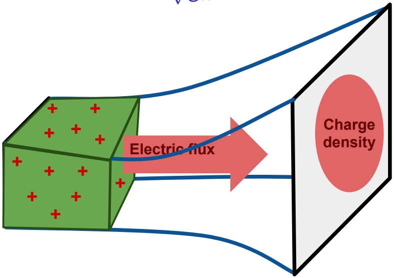
Electron stars

[Hartnoll-Tavanfar '10; Hartnoll-Hofman-Tavanfar '11]

- Fluid description valid when

$$e^2 \sim \frac{\kappa}{L} \ll 1 \,, \qquad L^2 m^2 \sim \frac{L^2 e^2}{\kappa^2} \gg 1 \,.$$

- Solve Einstein-Maxwell-charged ideal fluid equations.
- Local chemical potential: $\mu_{\text{loc.}} = \frac{A_t}{\sqrt{g_{tt}}}$.



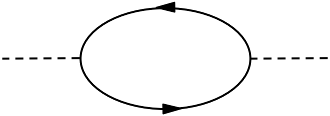
Electron stars: IR scaling

[Hartnoll-Tavanfar '10; Hartnoll-Hofman-Tavanfar '11]

- The IR geometry of the star is Lifshitz [c.f. Kachru, Liu, Mulligan]

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \,.$$

- This is good: Field theory computations also suggest the emergence of an IR dynamical critical exponent $z > 1$. 'Landau damping'.

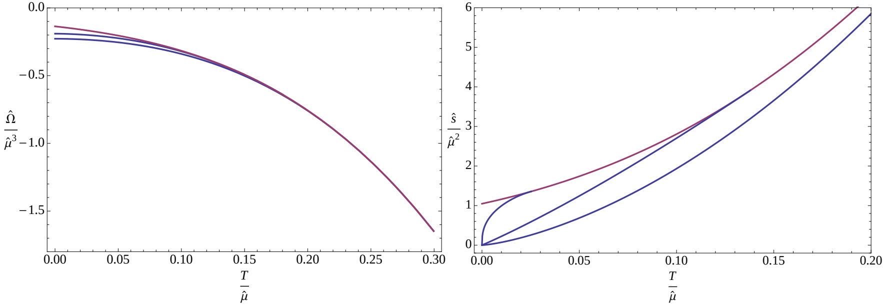


$$\langle A_x A_x \rangle(\omega, k) \sim \frac{1}{k^2 + \frac{|\omega|}{|k|}} \,, \qquad (z = 3)$$

Electron star birth: a third order transition

[Hartnoll-Petrov '11 (also Thorlacius et al. '11)]

- Free energy and entropy:



- Why?

$$\Delta\Omega \sim p(r)\Delta r \sim (\delta\mu)^{5/2}\Delta r \sim (\delta\mu)^3 \sim (T - T_C)^3 \,.$$

(There is a cancellation in $-p = \epsilon - \mu\sigma \sim (\delta\mu)^{5/2}$.)

The Luttinger count, fractionalisation and horizons

[Hartnoll-Hofman-Vegh '11 (also Iqbal, Liu, Mezei '11)]

- In the fluid limit – many closely spaced Fermi surfaces for fermion ψ.
- These satisfy a 'Luttinger count'

$$\text{Charge in fermion fluid} = \sum \text{Fermi surface volumes of } \psi \,.$$

- Therefore

$$\text{Flux from horizon} = \langle J^t \rangle - \sum \text{Fermi surface volumes of } \psi \,.$$

- Flux from horizon is 'missing charge' in field theory Luttinger count.
- Characteristic of 'Fractionalised Fermi liquids'. c.f. $\psi = \mathrm{Tr}\Psi\Phi$.

Conclusions

- Difficult questions in condensed matter theory motivate finite charge density holography.
- Extremal AdS black holes not especially robust in the presence of charged matter.
- Bosons $\Rightarrow$ Holographic superconductors.
- Fermions $\Rightarrow$ Electron stars.
- Emergent Lifshitz geometry in IR at zero temperature.
  - Landau damped gauge fields?
- Dichotomy: Flux from horizon vs. Flux from matter.