Instability in a matter bounce

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On the Instability of the Lee-Wick Bounce

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Outline :

1)What is a bouncing universe ? Why?

2)How? Lee-Wick scalar fields and radiation

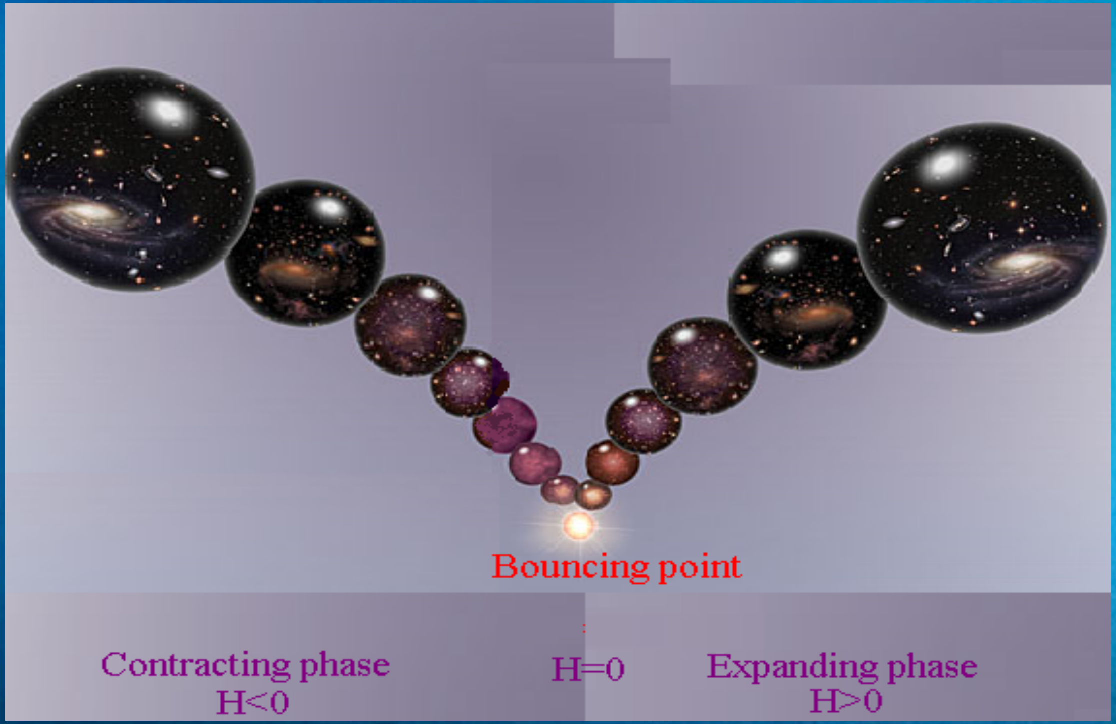
- goal and assumptions
- dynamics

3)Analytical results

4)Numerical results

5)Conclusion

1)Non singular bounces



Conditions to get a bounce :

At the bouncing point:

- $H=0 \Rightarrow \sum \rho i = 0$
- $\partial t\, H > 0$

$\Rightarrow \rho + p < 0$

$\Rightarrow w = p/\rho < -1$

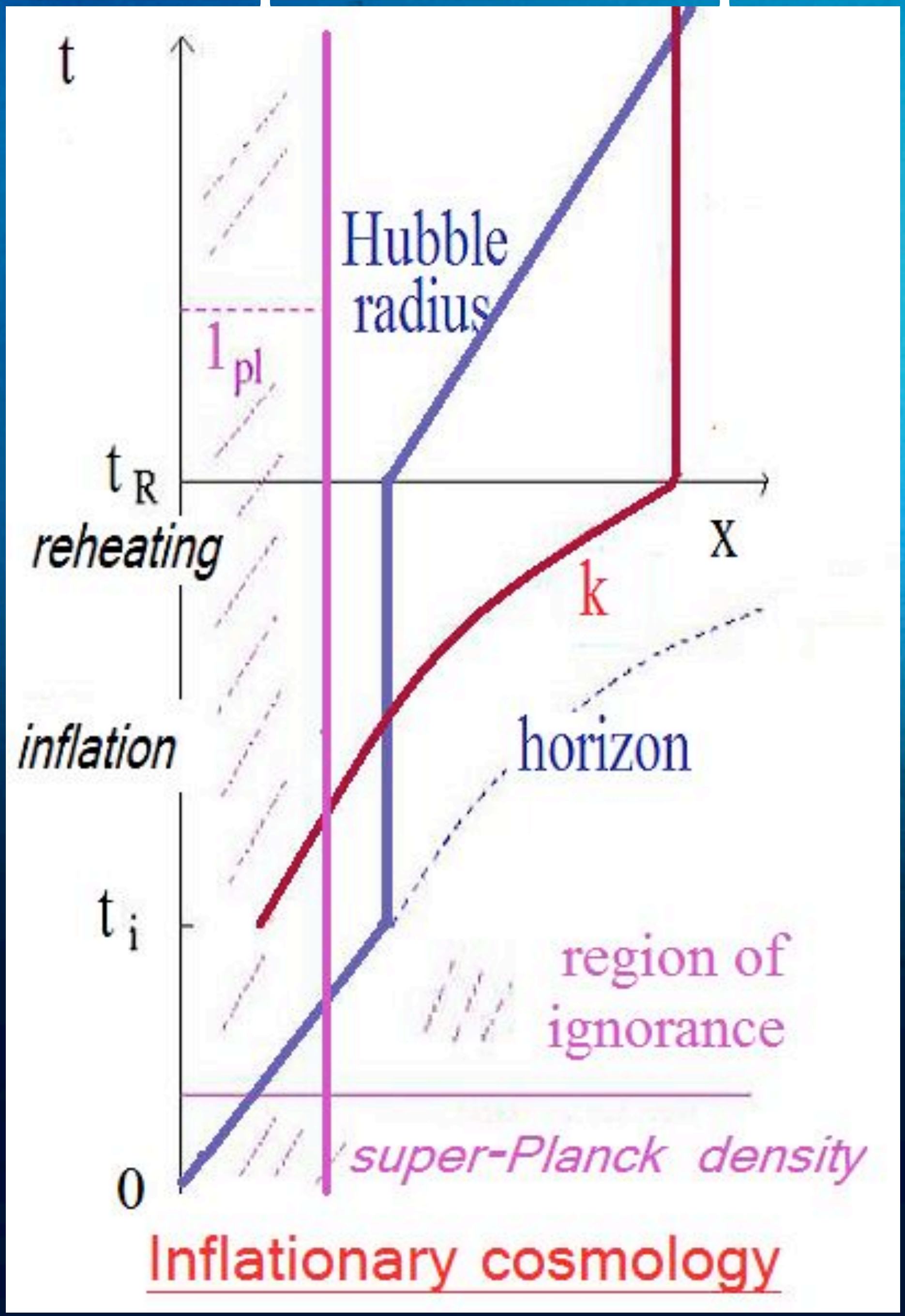
Non-singular bounces may be alternatively investigated using effective field theory techniques, introducing matter fields which violate the null energy condition : $w < -1$

Motivation :

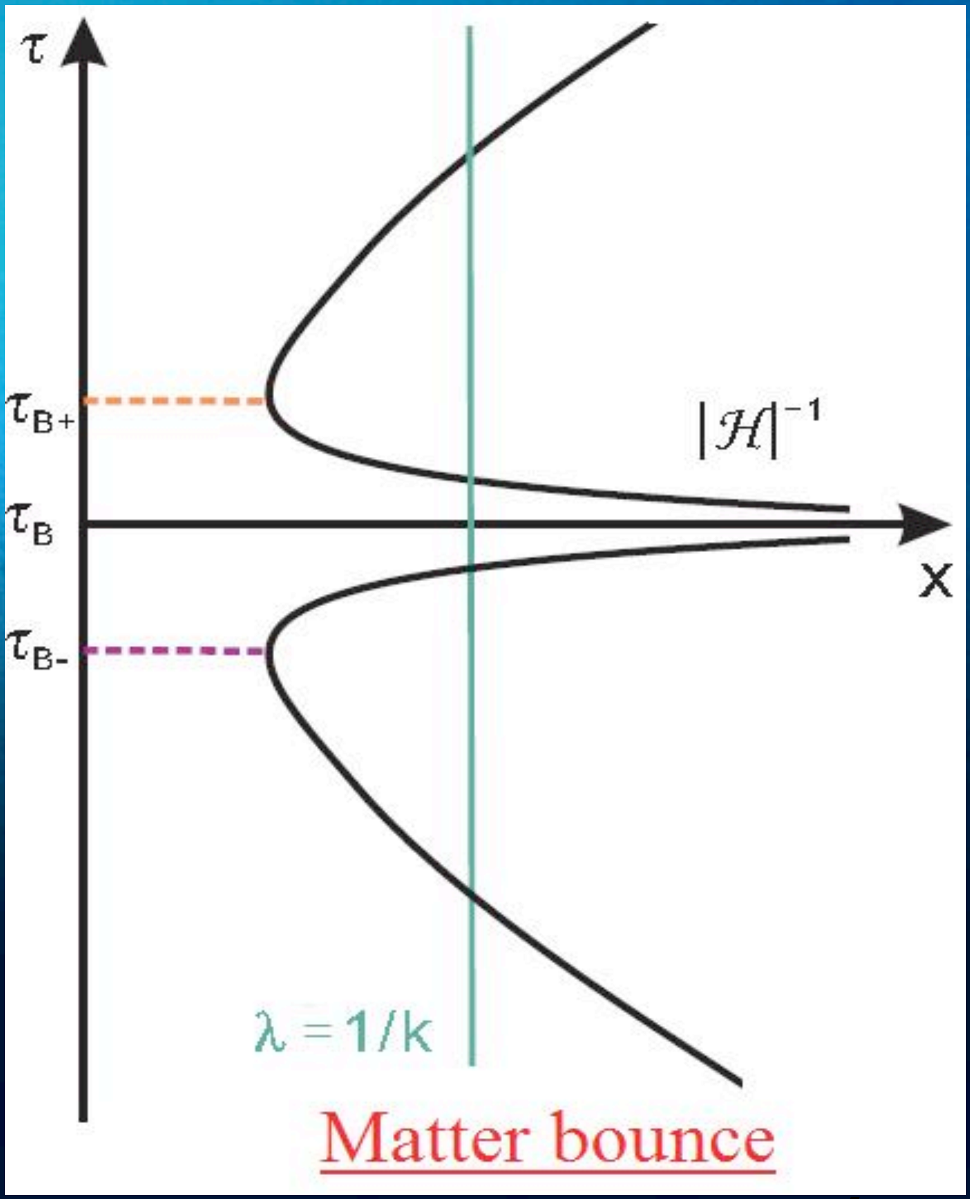
1. Avoid the big bang singularity
2. Avoid Transplanckian problem (see next slide)
3. Can get a scale invariant power spectrum for scalar metric fluctuations.

Examples : Horava-Lifshitz bounce, ghost condensation... cyclic : Ekpyrotic scenario

Transplanckian problem



Comoving coordinates



2)Lee-wick model : A toy model

- Higher derivative Lagrangian
- *T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969);Phys. Rev. D 2, 1033 (1970).*
- *B. Grinstein, D. O'Connell, and M. B. Wise, Phys. Rev. D77, 025012 (2008).*
- Field redefinition and diagonalize mixing term
- Extra coupling term to allow draining of energy from the radiation sector
- **On the Instability of the Lee-Wick Bounce** *Johanna Karouby, Taotao Qiu, Robert Brandenberger* ***arXiv:1104.3193***

We choose m<<M<<M $_{Pl}$

Problem : ghost instability

→ we use this work as a toy model !

Lee wick Lagrangian :

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2M^2}(\partial^2\phi)^2 - \frac{1}{2}m^2\phi^2 - V(\phi) -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - f(\phi, \partial^2\phi, F_{\mu\nu}F^{\mu\nu})$$

Field redefinition :

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}M^2\phi_2^2 -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - f(\phi_1, \phi_2, F_{\mu\nu}F^{\mu\nu})$$

Choice of coupling :

$$f(\phi_1, \phi_2, F_{\mu\nu}F^{\mu\nu}) = -\frac{1}{4}(c\phi_1^2 + d\phi_2^2)F_{\mu\nu}F^{\mu\nu}$$

Goal and assumptions

- Friedmann equation :

  $H^2 = \rho_{tot} = \rho_{\varphi 1} + \rho_{\varphi 2} + \rho_{rad} + \rho_c$

•**Goal** : H=0 ⇨ $\rho_{tot}=0$ + AVOID Big Crunch

•**Assumptions**: _$\rho_{mat} = \rho_{\varphi 1}$ dominates initially.
_Perfect fluid
_ Neglect metric fluctuation and backreaction on the scalar and gauge field perturbation

- Previous try : Pure Lee-Wick radiation : no bounce unless extreme fine tuning.

A Radiation Bounce from the Lee-Wick Construction?
J ohanna Karouby , Robert BrandenbergerPhys.Rev.D82:063532,2010.

- Coupling term may help us to get a bounce allowing to drain energy from the radiation sector.

Dynamics

1.Born approximation :

$$\phi_1(t,z) = \phi_1^{(0)}(t) + \epsilon\phi_1^{(1)}(t,z) + \epsilon^2\phi_1^{(2)}(t)$$
$$\phi_2(t,z) = \phi_2^{(0)}(t) + \epsilon\phi_2^{(1)}(t,z) + \epsilon^2\phi_2^{(2)}(t)$$

where $\varepsilon \ll 1$

Gauge field : A1(k, t) = f(t)cos(k.z) ≡ γ(k, t) ⟶ **Breaks isotropy**

2.EOM :

$$\Box\phi_1 - (m^2 - \frac{c}{2}F^2)\phi_1 = 0\ ,$$
$$\Box\phi_2 - (M^2 + \frac{d}{2}F^2)\phi_2 = 0\ ,$$
$$(1 - c\phi_1^2 - d\phi_2^2)(\partial_\nu F^{\mu\nu} + 3HF^{\mu 0})$$
$$-2(c\phi_1\partial_\nu\phi_1 + d\phi_2\partial_\nu\phi_2)F^{\mu\nu} = 0$$

3. Energy density

$$\rho = \frac{1}{2}(\dot\phi_1^2 + \frac{k^2}{a^2}\phi_1^2 + m^2\phi_1^2) - \frac{1}{2}(\dot\phi_2^2 + \frac{k^2}{a^2}\phi_2^2 + M^2\phi_2^2)$$
$$+(1 - c\phi_1^2 - d\phi_2^2)(\frac{F^2}{4} + F_{0\lambda}F_0^\lambda)$$

Parametrization of the scale factor : $a(t)=a_0\, t^{p} = \eta^{p/(1-p)}$

η :conformal time and $p=2 / [3\,(1+w)]$

Rescaled fields : $u=a(\eta)\,\varphi(\eta)$

3)Analytical Results

A.Background

Far away from the bounce :

a.|η| >> m^{-1} , M^{-1}

Damped(-anti) Oscillations

$\eta^{3p/[2(1-p)]} \cos(\eta^{1/(1-p)} \ldots)$

$$u_{1,2}^{(0)''} + (a_0^2 m^2 \eta^{\frac{2p}{1-p}} - \frac{p(2p-1)}{(1-p)^2\eta^2})u_{1,2}^{(0)} = 0$$

Solution :

$$u_{1,2}^{(0)} \sim \sqrt{|\eta|} H_{\pm\frac{1-3p}{2}}((1-p)am|\eta|)$$

Close to the bounce :

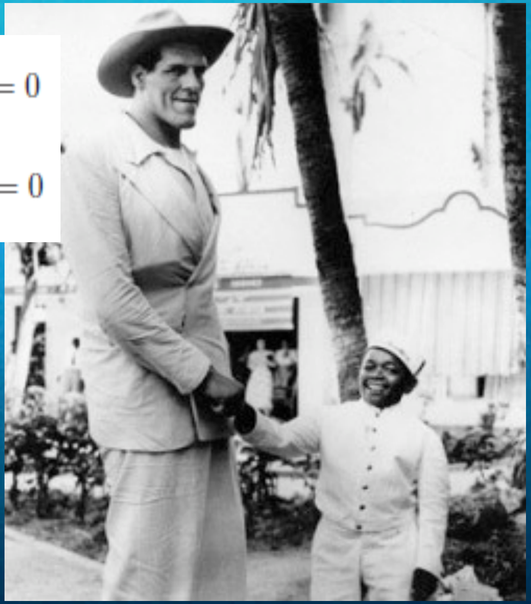
a.|η| << m^{-1} , M^{-1}

No oscillations

$\eta^{(1-3p)/(1-p)}$ ...+constant

B.1st order

$$\begin{cases} u_1^{(1)''} + (k^2 + a_0^2 m^2 \eta^{\frac{2p}{1-p}} - \frac{p(2p-1)}{(1-p)^2\eta^2})u_1^{(1)} = 0 \\ u_2^{(1)''} + (k^2 + a_0^2 M^2 \eta^{\frac{2p}{1-p}} - \frac{p(2p-1)}{(1-p)^2\eta^2})u_2^{(1)} = 0 \end{cases}$$



Large wavelengths (k|η|<<1)

Short wavelengths (k|η| >>1)

- Gauge field :

$$\gamma'' + k^2\gamma - \frac{2(c\phi_1^{(0)}\phi_1^{(0)'} + d\phi_2^{(0)}\phi_2^{(0)'})}{1 - c{\phi_1^{(0)}}^2 - d{\phi_2^{(0)}}^2}\gamma' = 0$$

Solution :

$$\gamma \simeq \gamma_0 + \delta\gamma$$

$$\delta\gamma \sim C_2|\eta|^{\frac{3-7p}{1-p}} , \quad p > \frac{1}{3}$$

$$\delta\gamma \sim C_3 \cos(k|\eta| + \theta_{\delta\gamma}) . \quad p < \frac{1}{3}$$

C.Second order

$$u_{1,2}^{(2)''} + a^2 m^2 u_{1,2}^{(2)} = c\frac{(k^2\gamma^2 - \gamma'^2)}{a^2} u_1^{(0)}$$

→ Energy density up to order 2

4)Numerical results :

ρ for 1 mode

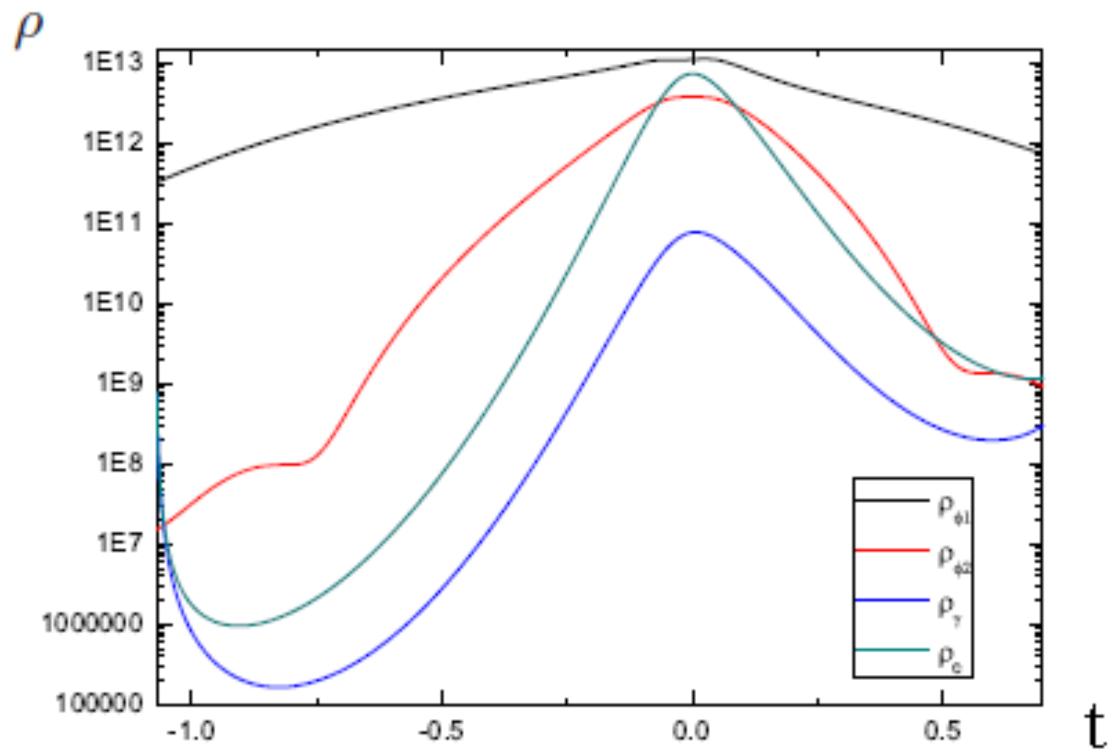


FIG. 3: Energy densities of ϕ_1, ϕ_2 and γ in the system with parameters chosen as in Fig. 1. The curves from up to down are: ρ_{ϕ_1} (black), ρ_{ϕ_2} (red), ρ_c (dark cyan) and ρ_γ (blue), respectively. The variables are also normalized with the mass scale $m_{rec} = 10^{-6} m_{Pl}$.

W

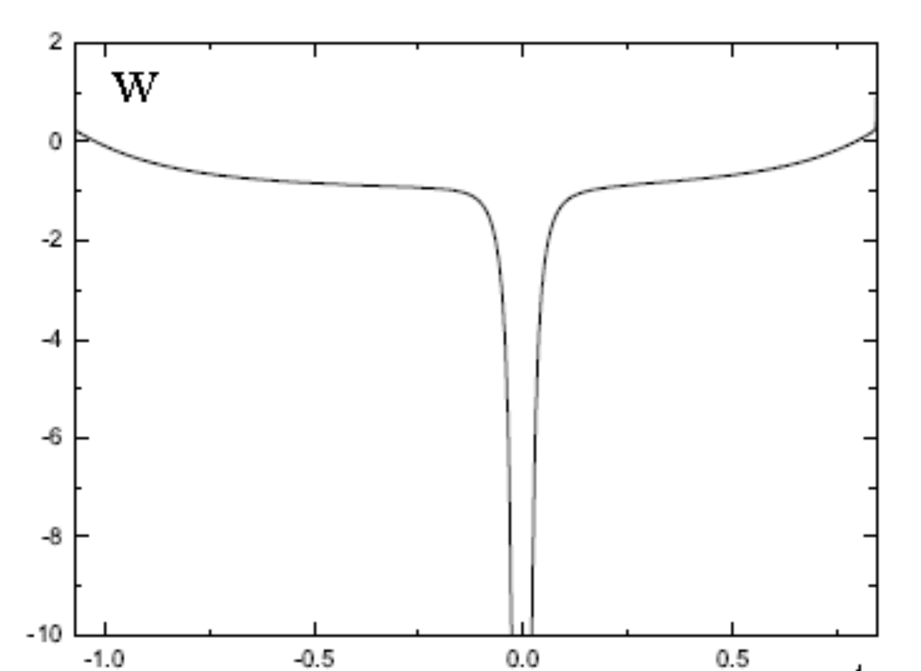


FIG.1 State parameter

a(t)

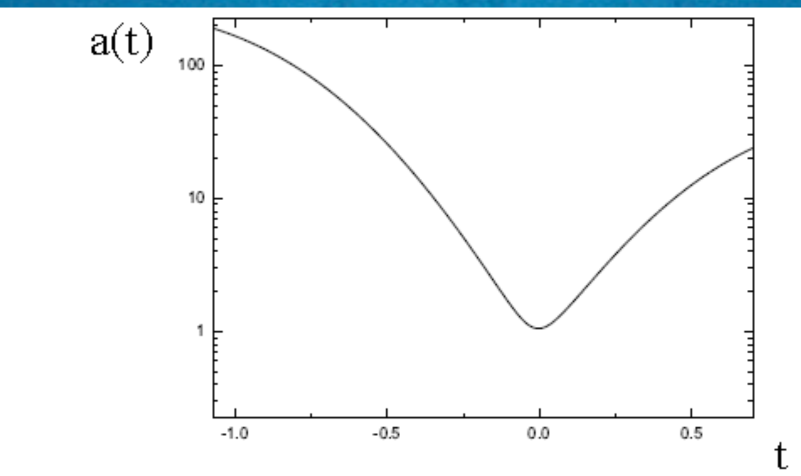
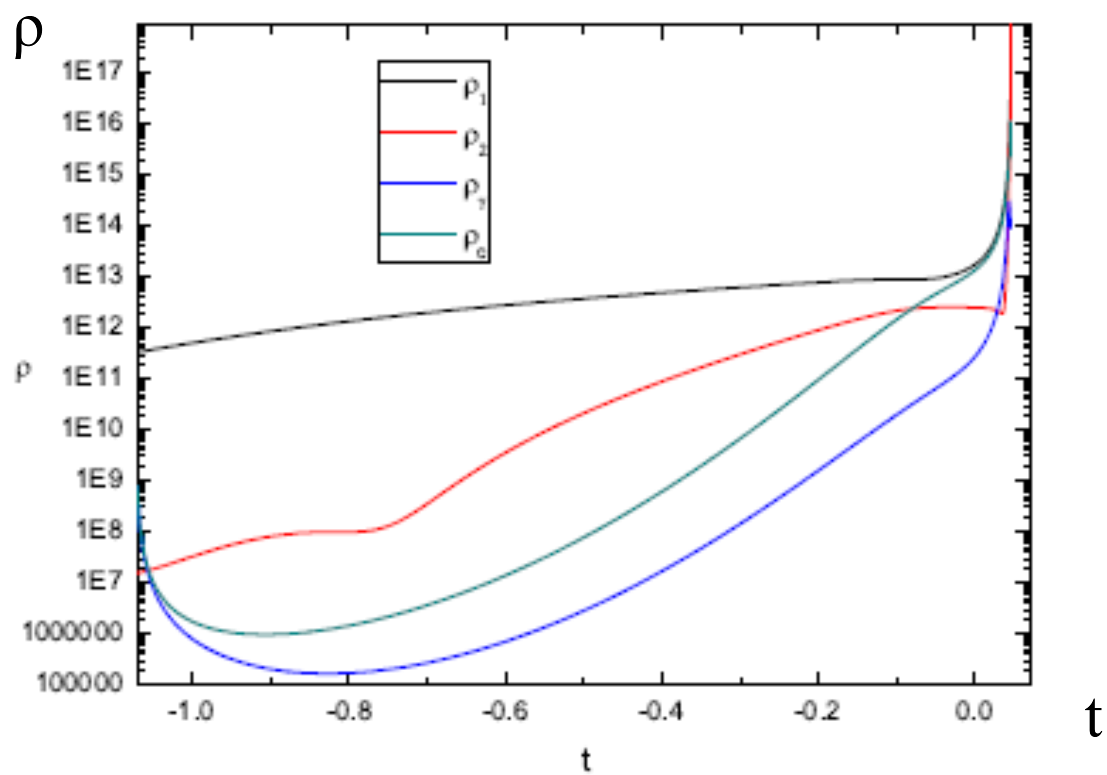


FIG. 2: The scale factor of the universe in the same simulation that leads to the evolution of the equation of state shown in Fig. 1. From the plot we see that the bounce happens at $t = 0$.

2 modes with opposite initial velocities for φ



Big Crunch !

FIG. 4: Energy densities of ϕ_1, ϕ_2 and γ in the system with different parameters . The curves from up to down are: ρ_{ϕ_1} (black), ρ_{ϕ_2} (red), ρ_c (dark cyan) and ρ_γ (blue), respectively. The variables are also normalized with the mass scale $m_{rec} = 10^{-6} m_{Pl}$.

5) Conclusion

No bounce unless fine tuning of initial conditions

Choice of the coupling may improve this result and help creating a bounce.

How to construct ghost free non singular bouncing models?

How to construct bouncing models inspired from the standard model?

Could we prove if this is possible or not?

Thanks!