Bose-Einstein Condensates and Primordial Magnetogenesis.

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Bose-Einstein Condensate (BEC)

The quantum distribution functions for fermions and bosons read:

$$f_F = \frac{1}{\exp[(E-\mu_F)/T]+1}\,, \qquad f_B = \frac{1}{\exp[(E-\mu_B)/T]-1}\,,$$

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Evidently $\mu_B > m_B$ would lead to a negative distribution!
Way out $\Longrightarrow$ Accumulation of identical bosons in the same (lowest energy) quantum state:

$$f_B = C\delta^{(3)}(\mathbf{p}) + \frac{1}{\exp[(E - m_B)/T] - 1},$$

as one can verify from kinetic theory (vanishing of the collision integral):

$$I_{coll} \sim \int d\tau \left[\Pi f_{in}(1 \pm f_{fin}) - \Pi f_{fin}(1 \pm f_{in})\right] .$$

W boson condensation

W bosons could condense in the primordial universe if the lepton asymmetry was large enough :

$$n_\nu^c = \frac{m_W^3}{6\pi^2} \qquad (T=0)$$

Linde, PRD 14 (1976), PLB 86 (1979)

as one can understand by considering the inverse decay:

$$W^+ \leftrightarrow e^+ + \nu \, .$$

Defining d as the inter-particle separation:

$$\lambda_{dB} \sim \frac{2\pi}{\sqrt{2mT}} > d \qquad \Longrightarrow \text{Condensation}$$

Analysis of temperature effects above and below the EW phase transition:

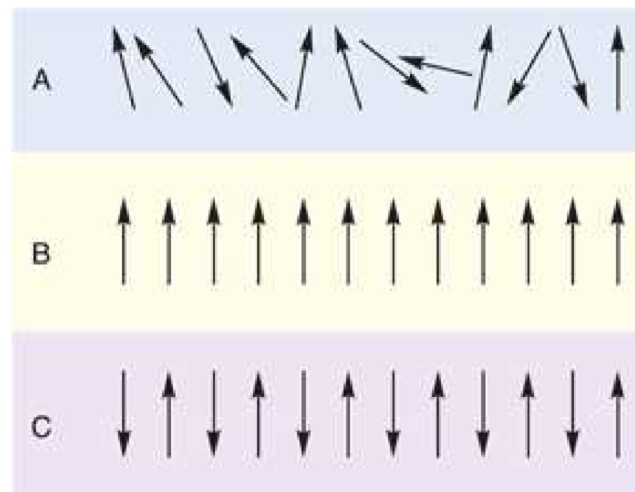
Ferrer et al. PLB 185 (1987), Kapusta PRD42 (1990)

Magnetic properties

New factor: interactions of the spins!

As in any multi-particle system, the spins of the condensed W bosons in the primordial universe can be:

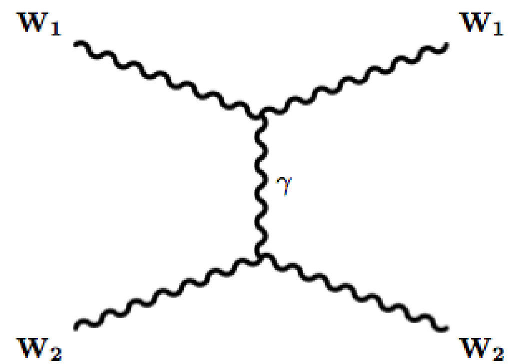
- Randomly oriented (paramagnetism)
- Aligned (ferromagnetism)
- Anti-aligned (anti-ferromagnetism)



For W bosons only direct spin-spin interactions are relevant!

Ferromagnetism of W bosons

Electromagnetic interaction



Contribution similar to the Breit's potential in electronic systems:

$$U_{em}^{spin}(r) = \frac{e^2\rho^2}{4\pi m_W^2}\left[\frac{(\mathbf{S}_1\cdot\mathbf{S}_2)}{r^3} - 3\,\frac{(\mathbf{S}_1\cdot\mathbf{r})\,(\mathbf{S}_2\cdot\mathbf{r})}{r^5} - \frac{8\pi}{3}\,(\mathbf{S}_1\cdot\mathbf{S}_2)\,\delta^{(3)}(\mathbf{r})\right].$$

ρ is the ratio of the real magnetic moment to the SM prediction.

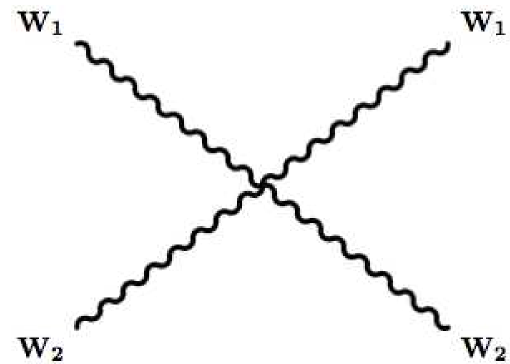
$$\delta E = \int \frac{d^3r}{V}\,U_{em}^{spin}(r) = -\frac{2\,e^2\rho^2}{3\,V m_W^2}\,(\mathbf{S}_1\cdot\mathbf{S}_2)\,,$$

Dolgov, AL & Piccinelli, JCAP 1008 (2010)

Ferromagnetism of W bosons

Quartic self-coupling of W

$$U_{4W}^{(spin)} = \frac{e^2}{8m_W^2 \sin^2\theta_W} (\mathbf{S}_1 \cdot \mathbf{S}_2)\, \delta^{(3)}(\mathbf{r})$$



By comparing the two contributions one can see that the energetically favored configuration of a multi-W state has a macroscopically large spin. In other words, the condensed primordial Ws constitute a ferromagnetic system. As a consequence, the primordial universe could be spontaneously magnetized on macroscopically large scales.

Dolgov, AL & Piccinelli, JCAP 1008 (2010)

Screening

In QED, once medium effects are taken into account, the photon EOM in medium becomes:

$$[k^\rho k_\rho g^{\mu\nu} - k^\mu k^\nu + \Pi^{\mu\nu}(k)]\, A_\nu(k) = \mathcal{J}^\mu(k)$$

In standard cases, e.g when BEC is absent:

- $G_{00}(k)$ has a pole in Π_{00}, $\qquad \Pi_{00}(k) = \Pi_{00} \equiv m_D^2$.
- $G_{ij} \sim G_{ij}^{(vacuum)}$ when $k \rightarrow 0$

As a consequence, only electrostatic interactions are screened:

$$U \sim \frac{Q_1 Q_2}{r} \qquad \text{(Coulomb potential in vacuum)}$$

$$U \sim \frac{Q_1 Q_2}{r} \exp^{-m_D r} \qquad \text{(Yukawa potential in medium)}$$

QED + BEC

The space-space component of the polarization operator does not vanish as k^2:

$$\Pi_{ij} = \left(e^2 m_C^2 + \frac{e^2 T}{16}\, k\right)\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) \qquad m_C \equiv \frac{C}{8\pi^3 m_B}$$

By comparing the screening length $\lambda = 1/e m_C$ with the typical inter-particle distance, $d \sim C^{-1/3}$, it follows that magnetic interactions are not screened as long as:

$$\frac{m_B}{C^{1/3}} \leq 10^{-4}\,.$$

If this condition was realized, there could be spontaneous magnetization at macroscopic scales in the early universe.

Dolgov, & AL arXiv:1105.2308 [hep-th]

QED + BEC

Without a BEC, the time-time component of the polarization operator, Π_{00}, tends to a constant of k. In the presence of the BEC, it becomes k dependent:

$$\Pi_{00} = -e^2 \left[\left(m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2} \right) \right]$$

$$m_0^2 = \left(\frac{2T^2}{3} \right) + \frac{C}{(2\pi)^3 m_B} \qquad m_1^3 = \frac{m_B^2 T}{2} \qquad m_2^4 = \frac{4 C m_B}{(2\pi)^3}$$

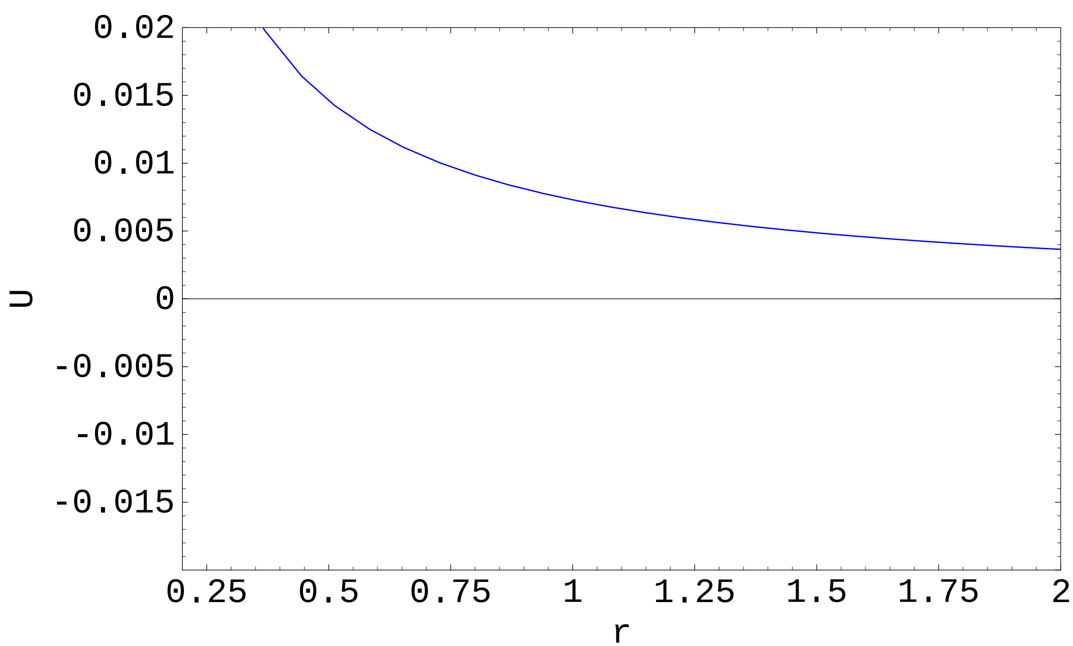
As a consequence, the potential is not only screened, but also oscillating with distance.

Dolgov, AL & Piccinelli, JCAP 0902 (2009); Phys.Rev.D80:125009 (2009).

Electrostatic interaction

What happens when an electric charge is surrounded by a BEC?

- Vacuum $\rightarrow$ standard Coulomb potential $U \sim \frac{Q_1 Q_2}{r}$



Electrostatic interaction

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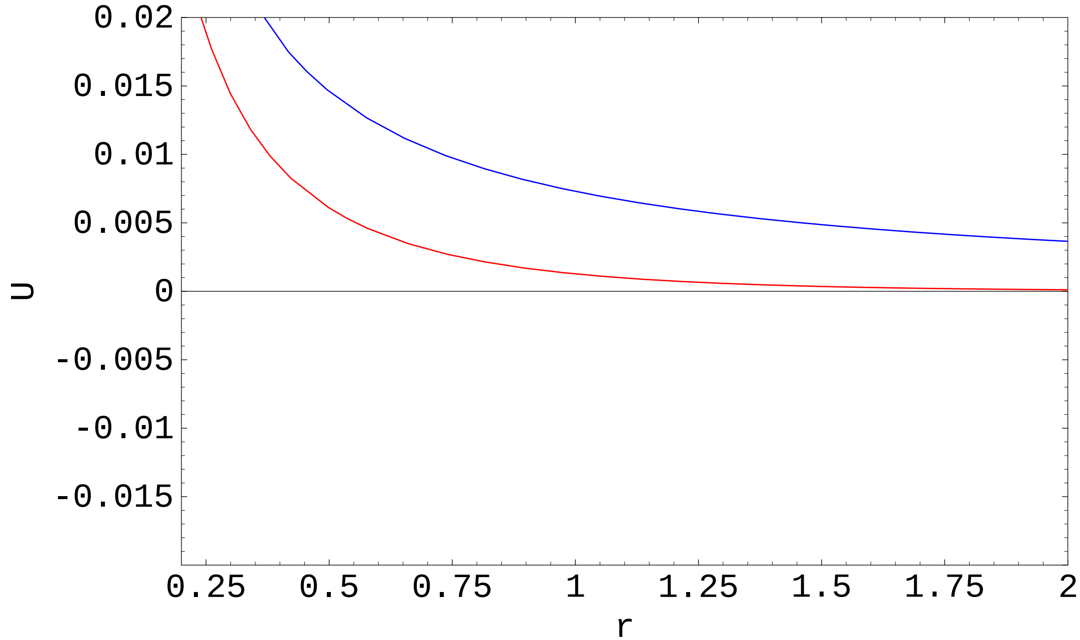
- Vacuum $\rightarrow$ standard Coulomb potential $U \sim \frac{Q_1 Q_2}{r}$
- Plasma $\rightarrow$ screening and Yukawa potential: $U \sim \frac{Q_1 Q_2}{r} \exp^{-m_D r}$



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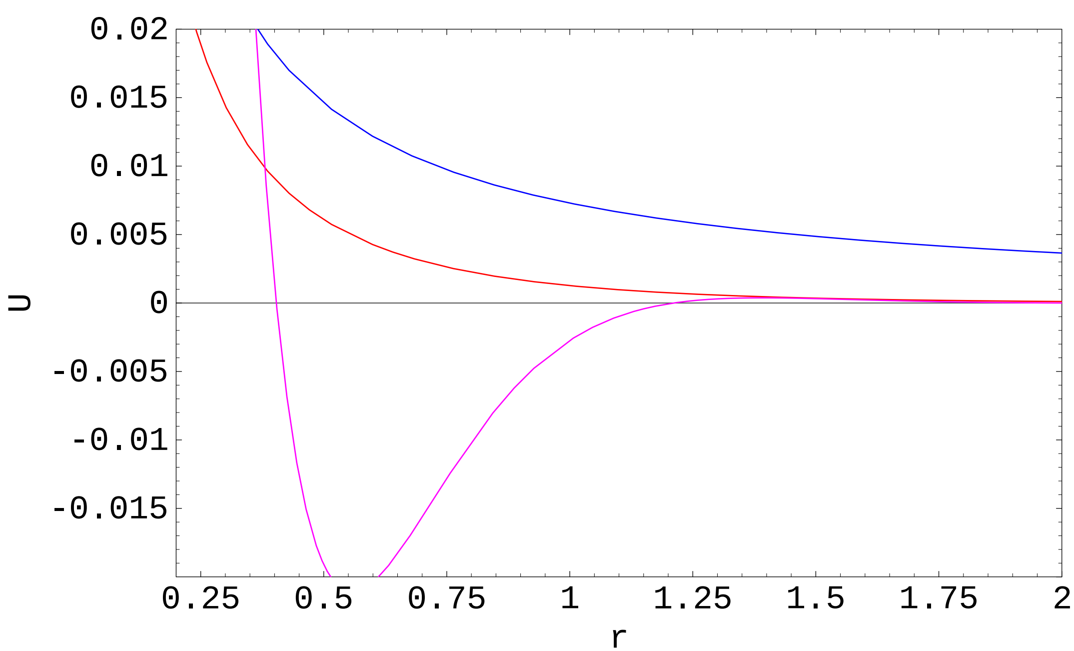
- Vacuum $\rightarrow$ standard Coulomb potential $U \sim \frac{Q_1 Q_2}{r}$
- Plasma $\rightarrow$ screening and Yukawa potential: $U \sim \frac{Q_1 Q_2}{r} \exp^{-m_D r}$
- Plasma including a Bose condensate:

$$U(r)_{pole} = \frac{Q_1 Q_2}{4\pi r} \exp\left(-\sqrt{e/2} m_2 r\right) \cos\left(\sqrt{e/2} m_2 r\right) .$$



Conclusions

- W bosons could condense in the early universe in the presence of a large lepton asymmetry.
- Spin-spin interactions are ferromagnetic in vacuum $\longrightarrow$ possible spontaneous magnetization of the early universe.
- Medium effects:
  - in some regions of the parameter space screening of magnetic interactions destroys ferromagnetism.
  - The electrostatic potential has a very peculiar (oscillating) shape.

The end

Thank you for your kind attention

The lagrangian

The system we studied is standard QED plus BEC.

$$\mathcal{L} = -\frac{\mathbf{1}}{\mathbf{4}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \mathbf{m}_\mathbf{B}^\mathbf{2}|\phi|^\mathbf{2} + |(\partial_\mu + \mathbf{i}\,\mathbf{e}\mathbf{A}_\mu)\phi|^\mathbf{2} + \bar{\psi}(\mathbf{i}\not{\partial} - \mathbf{e}\not{\mathbf{A}} - \mathbf{m_F})\psi$$

Electrostatic interactions

The time-time component of the polarization operator:

$$\Pi_{00} = -e^2 \left[\left(m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2} \right) \right]$$

$$m_0^2 = \left(\frac{2T^2}{3} \right) + \frac{C}{(2\pi)^3 m_B} \qquad m_1^3 = \frac{m_B^2 T}{2} \qquad m_2^4 = \frac{4Cm_B}{(2\pi)^3}$$

- Π_{00} is k-dependent and infrared singular.
- The dependence of the potential on the coupling constant e is not analytic anymore.
- Debye screening is stronger than in standard cases.

The potential

The potential is calculated from the photon EOM:

$$U(r) = Q_1 Q_2 \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{k}\mathbf{r})}{k^2 - \Pi_{00}(k)} = \frac{Q_1 Q_2}{2\pi^2} \int_0^\infty \frac{dk k^2}{k^2 - \Pi_{00}(k)} \frac{\sin kr}{kr}$$

The electrostatic potential in the presence of a Bose condensate strongly deviates from the standard Yukawa one:

$$U(r)^{(poles)} = -\left(\frac{Q_1 Q_2}{4\pi r}\right) \exp(-\sqrt{e/2} m_2 r) \cos(\sqrt{e/2} m_2 r)$$

$$m_2^4 = \frac{4 C m_B}{(2\pi)^3}$$

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The electrostatic potential in the presence of a Bose condensate strongly deviates from the standard Yukawa one:

$$U(r)^{(axis)} = -\frac{12 Q_1 Q_2 m_1^3}{\pi^2 m_2^8} \frac{1}{e^2} \frac{1}{r^6}$$

$$m_1^3 = \frac{m_B^2 T}{2}, \qquad m_2^4 = \frac{4 C m_B}{(2\pi)^3}$$

The potential

The potential is calculated from the photon EOM:

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The electrostatic potential in the presence of a Bose condensate strongly deviates from the standard Yukawa one:

$$U(r)^{(cuts)} = \frac{Q_1 Q_2 8(2\pi)^6 T^3 m_B^2}{e^2 C^2 r^2} \exp\left(-2\sqrt{2\pi m_B T}r\right) \cos\left(2\sqrt{2\pi m_B T}r\right)$$

A similar term arises in standard QED (Friedel oscillations).

Friedel oscillations

A similar behavior can be found in highly degenerate fermionic QED, where there are so called Friedel oscillations:

$$U(r, T=0) = \frac{Q_1 Q_2 e^2}{64\pi^3} \frac{\sin(2k_F r)}{k_F^3 r^4} \qquad (m_F = 0)$$

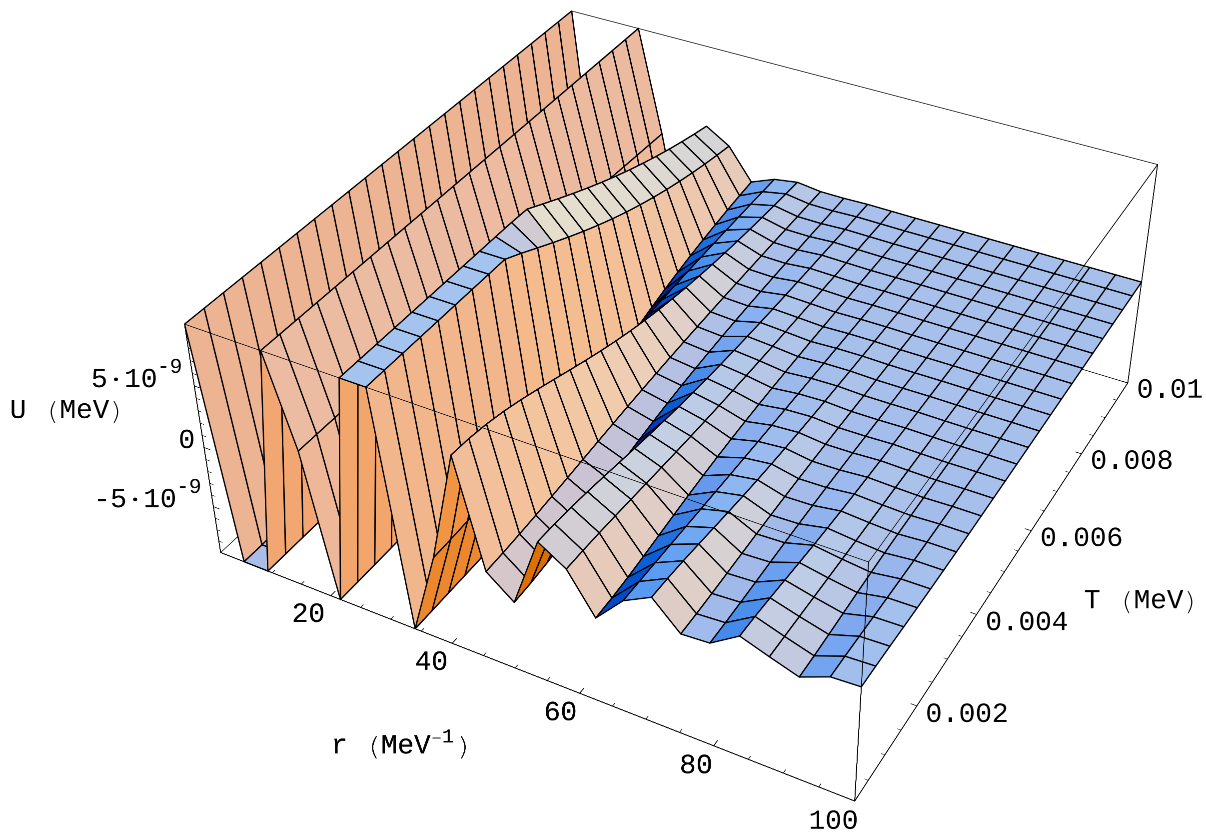
$$U(r, T=0) = \frac{Q_1 Q_2 e^2 m_F}{64\pi^3} \frac{\cos(2k_F r)}{k_F^3 r^3} \qquad (m_F \gg k_F)$$

which are analogous to the last term in the condensate potential.

These oscillations were experimentally observed. They exponentially vanish at finite temperature, while the bosonic ones don't.

Friedel oscillations

Plotting Friedel oscillations for $m_F = 0.5$ MeV, $\mu_F = 0.55$ MeV for $T \sim (10^6 - 10^8)K$ and $r \sim (2 \cdot 10^{-11} - 2 \cdot 10^{-9})cm$ we get:



Boson oscillations

Here it is an image of the bosonic oscillations due to branch cuts for $\mu_B = m_B = 100MeV$. Temperatures vary from $1.16 \cdot 10^4 K$ to $5.8 \cdot 10^8 K$ and distances from $2 \cdot 10^{-13}$ cm to $2 \cdot 10^{-10}$ cm.