

Holographic Models of the Quantum Hall Effect

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Motivational Slide

Many phenomena involve strongly-coupled fermions:

- Chiral Symmetry Breaking
- High T_c Superconductors
- Fractional quantum Hall effect (FQHE)
- ...

But, difficult  Use Gauge/Gravity Duality

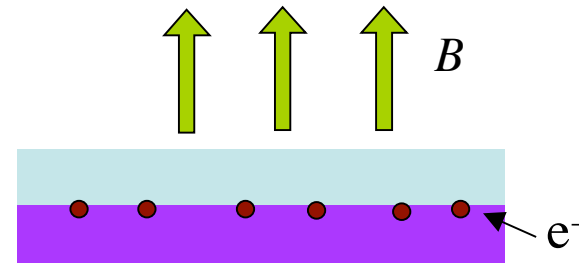
Top-Down Holographic Approach:

- Study concrete string solutions
- Known field theory duals
- Look for universal features

Quantum Hall Effect (QHE)

Experimental Setup:

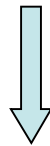
e^- in 2+1 dimensions
 high magnetic field B
 low temperature T



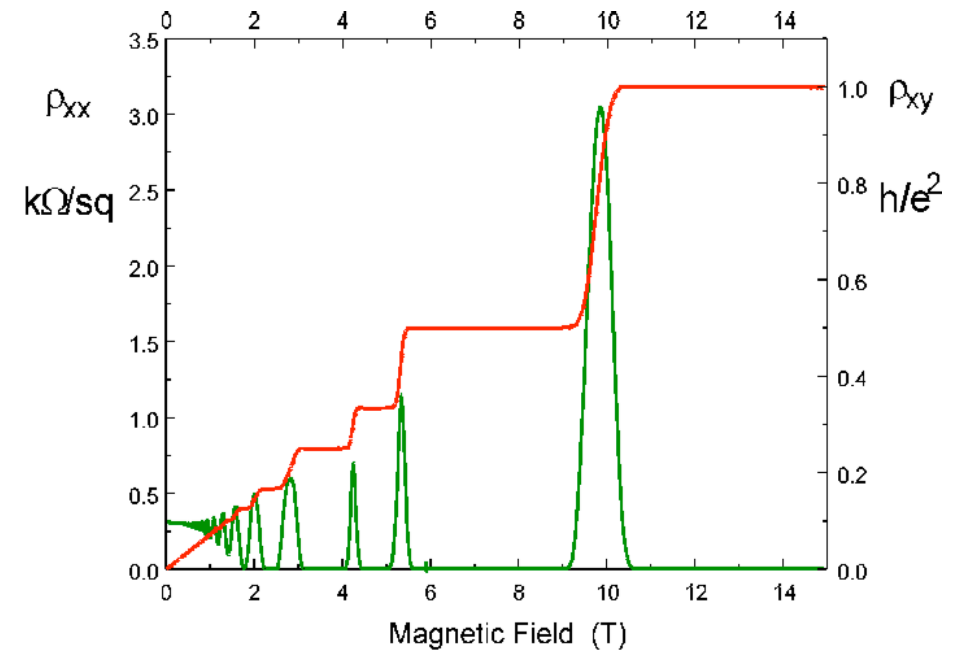
Conductivity

Longitudinal: $\sigma_{xx} = \frac{j_x}{E_x} = 0$

Hall: $\sigma_{xy} = \frac{j_y}{E_x} = \frac{e^2}{2\pi\hbar} \nu$



Filling fraction



Filling Fraction

$$\nu \equiv \frac{2\pi\hbar D}{e B} \sim \frac{\# \text{ electrons}}{\# \text{ flux quanta}}$$

QH states for particular values of ν

$\nu \in \mathbf{Z}$  Integer QHE

$\nu \notin \mathbf{Z}$  Fractional QHE

Open questions:

- microscopic description
- allowed ν 's
- Transitions between

Dp - Dq Models

Dp - Dq brane intersections with $\#ND=6$

- fundamental fermions at intersection
- Dq probe in Dp background
- SUSY \longrightarrow stability?

Example:

Sakai Sugimoto model $D4$ - $D8$ - $\overline{D8}$

Rey

Kraus & Keski-Vakkuri

Myers & Wapler, etc

Hong & Yee

Takayanagi et al.

...

Two other examples:

- $D3$ - $D7'$ Model

2+1-dim fermions, 3+1-dim gauge

FQHE, $\nu =$ irrational, set by internal flux

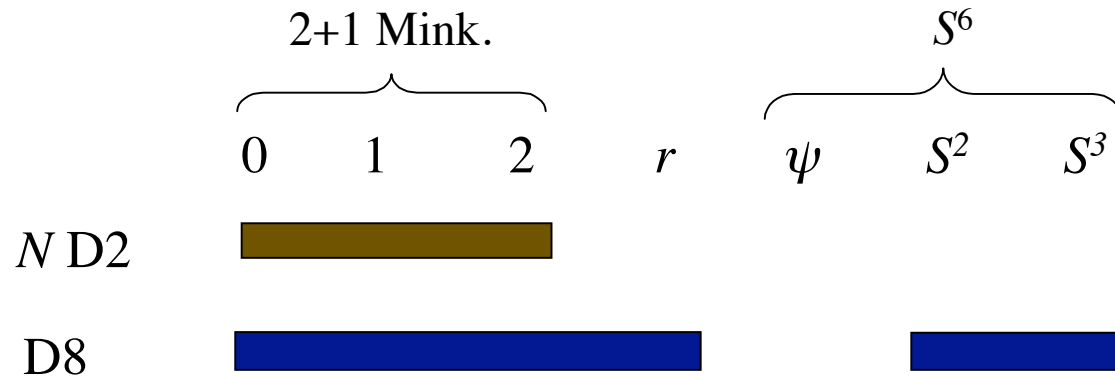
- $D2$ - $D8'$ Model

2+1-dim gauge + fermions

IQHE, $\nu = 1 \forall$ internal flux

Focus of today's talk

D2-D8' system



D8-brane embedding:
 wraps $S^2 \times S^3 \subset S^6$
 solve for $\psi(r)$

Stabilization

lowest mode for ψ tachyon (slipping mode)
 wrap magnetic flux on internal S^2

Add charges and magnetic field

Charge density

$$2\pi\alpha' F_{r0} = a'_0(r)$$

Magnetic field

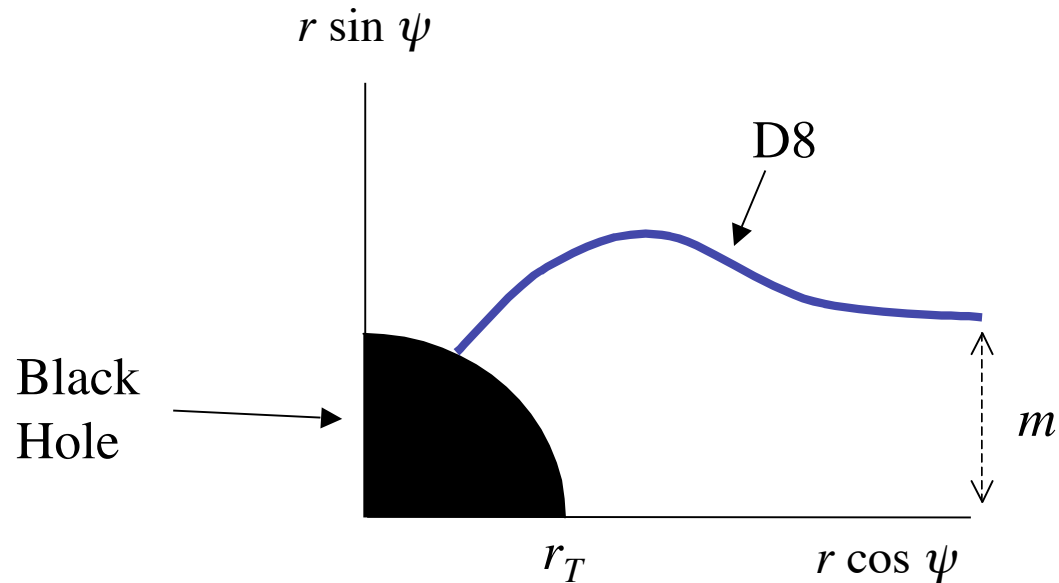
$$2\pi\alpha' F_{xy} = B$$

Chern-Simons

$$S_{CS} \sim \int C_5 \wedge F \wedge F$$

 C_5 flux and B induce charge

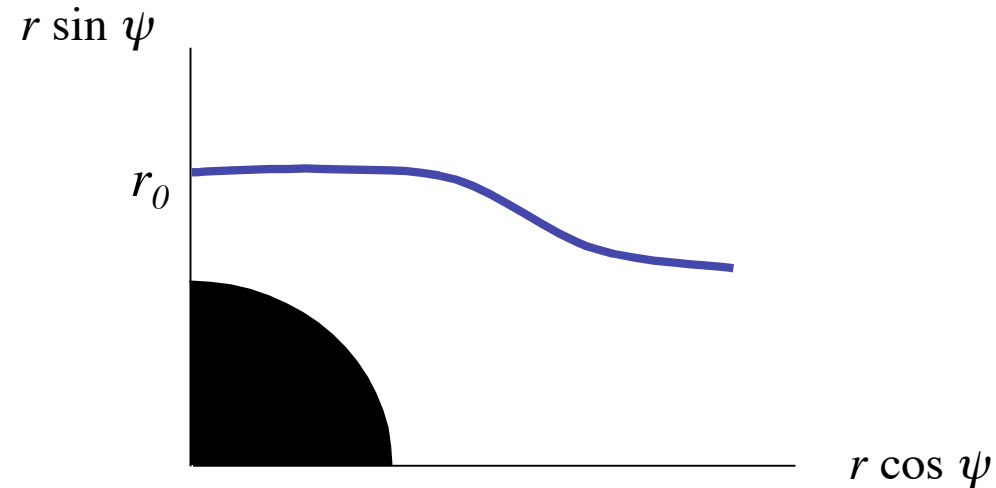
Black Hole Embeddings



Metallic state

- Gapless charged excitations
- $\sigma_{xx} \neq 0$ (via Karch-O'Bannon)
- Instability at high density \longrightarrow Spatially-modulated state

Minkowski Embeddings



Quantum Hall state

- D8 ends smoothly at r_0 where S^3 shrinks
- no sources at tip, all charge induced by CS term

$$\longrightarrow D = \frac{N}{2\pi} B$$

- $\nu = 1$ (filling fraction per fermion)

More Minkowski Embeddings

- Conductivity (via modified Karch-O'Bannon):

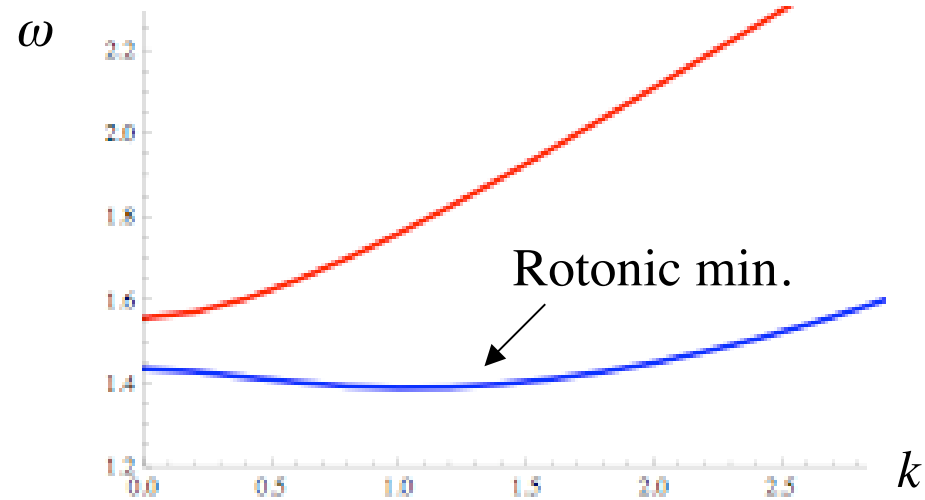
$$\sigma_{xx} = 0$$

$$\sigma_{xy} = \nu/2\pi$$

- Fluctuations:

mass gap $m_g \sim r_0$

lowest mode is (usually) a magneto-roton



Summary

Two top-down models of quantum Hall effect

Features:

- Quantized ν
- Mass gap
- Conductivities
- Fluctuations

To do list:

- multiple filling fractions & transitions
- impurities, plateaux
- boundaries, edge states
- quasiparticles, fractional charge
- connect to bottom-up models

e.g. [Burgess et al. arXiv/1008.1917](#)

References

D3-D7'

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D2-D8'

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A holographic quantum hall model at integer filling
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