Goldstone bosons in Higgs inflation

Sander Mooij
Nikhef, Amsterdam
Supervisor: Marieke Postma

NIKHEF

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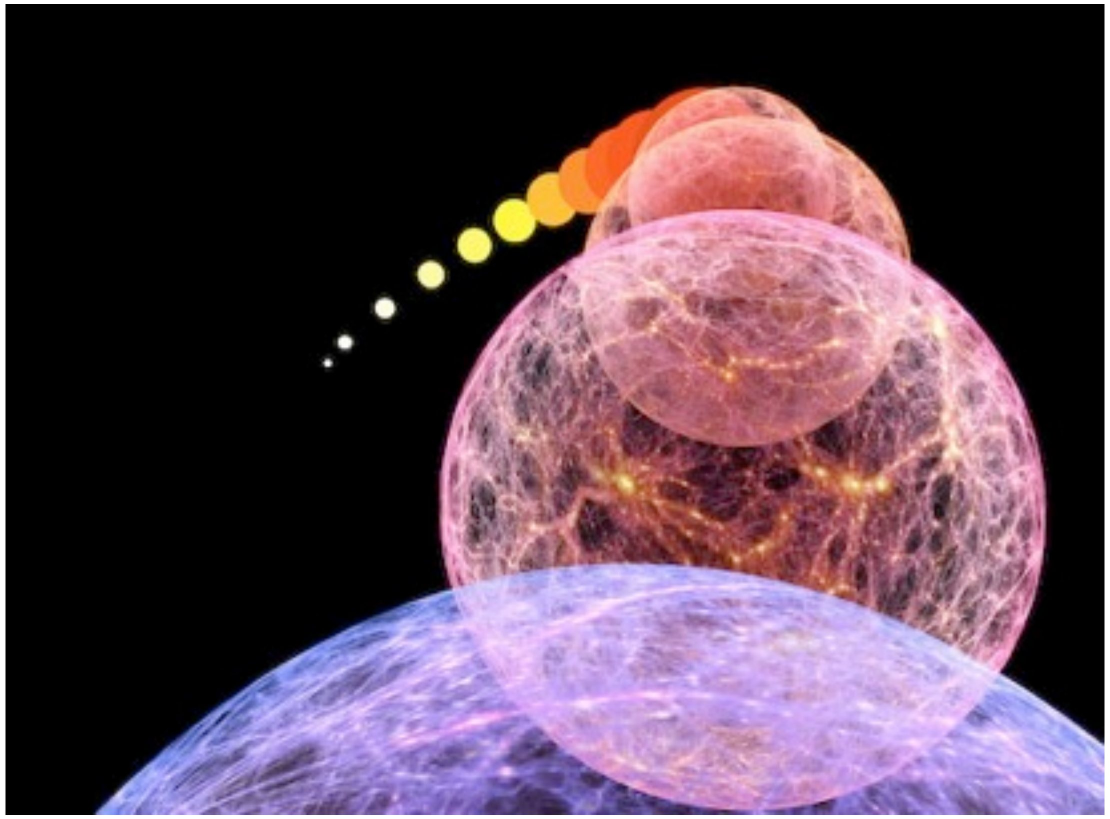
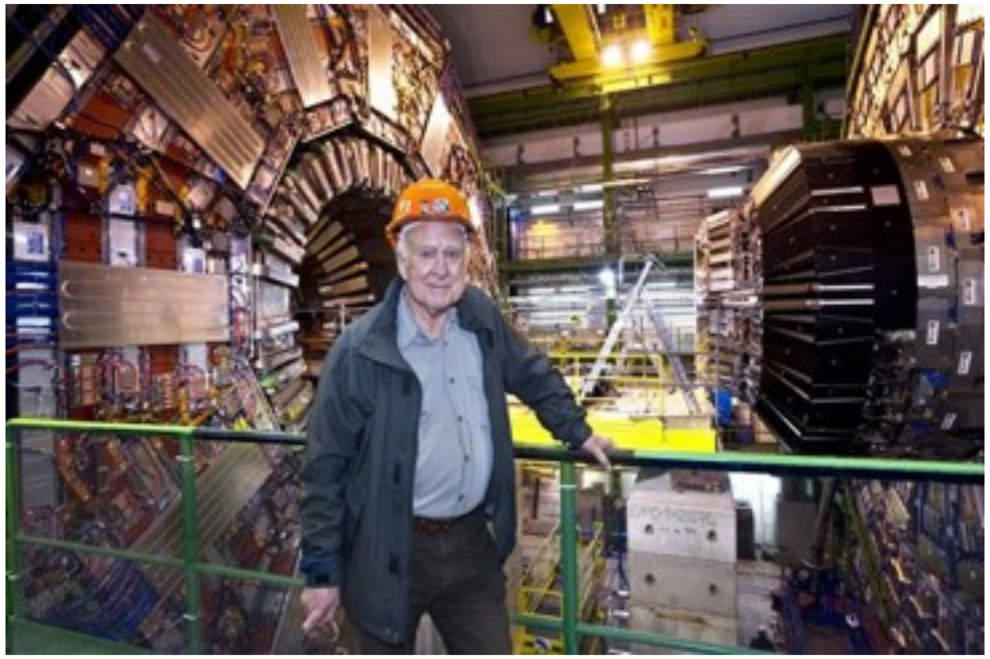
Inflation



- Need a scalar field slowly rolling down an almost flat potential
- “Elegant” approach: susy, sugra, strings
- “Economical” approach: Higgs

Economical approach

- Didn't the Standard Model contain a scalar field?



$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - g^{\mu\nu}(D_\mu\mathcal{H})^\dagger(D_\nu\mathcal{H}) - \lambda\left(\mathcal{H}^\dagger\mathcal{H} - v^2\right)^2\right]$$

PROBLEM: flat potential ≠ Mexican hat

Solution: non-minimal Higgs-graviton coupling

Bezrukov & Shaposhnikov, 0710.3755

$$S_J = \int d^4x\sqrt{-\hat{g}}\Bigg[\left(\frac{M_p^2}{2} - \xi\mathcal{H}^\dagger\mathcal{H}\right)R(\hat{g}_{\mu\nu}) - \hat{g}^{\mu\nu}(D_\mu\mathcal{H})^\dagger(D_\nu\mathcal{H}) - \lambda\big(\mathcal{H}^\dagger\mathcal{H} - v^2\big)^2\Bigg]$$

(Jordan)

Conformal transformation

$$\hat{g}_{\mu\nu} = \omega^2 g_{\mu\nu} \to g_{\mu\nu} \qquad \omega = \left(1 + \frac{2\xi\mathcal{H}^\dagger\mathcal{H}}{M_p^2}\right)^{-1/2}$$

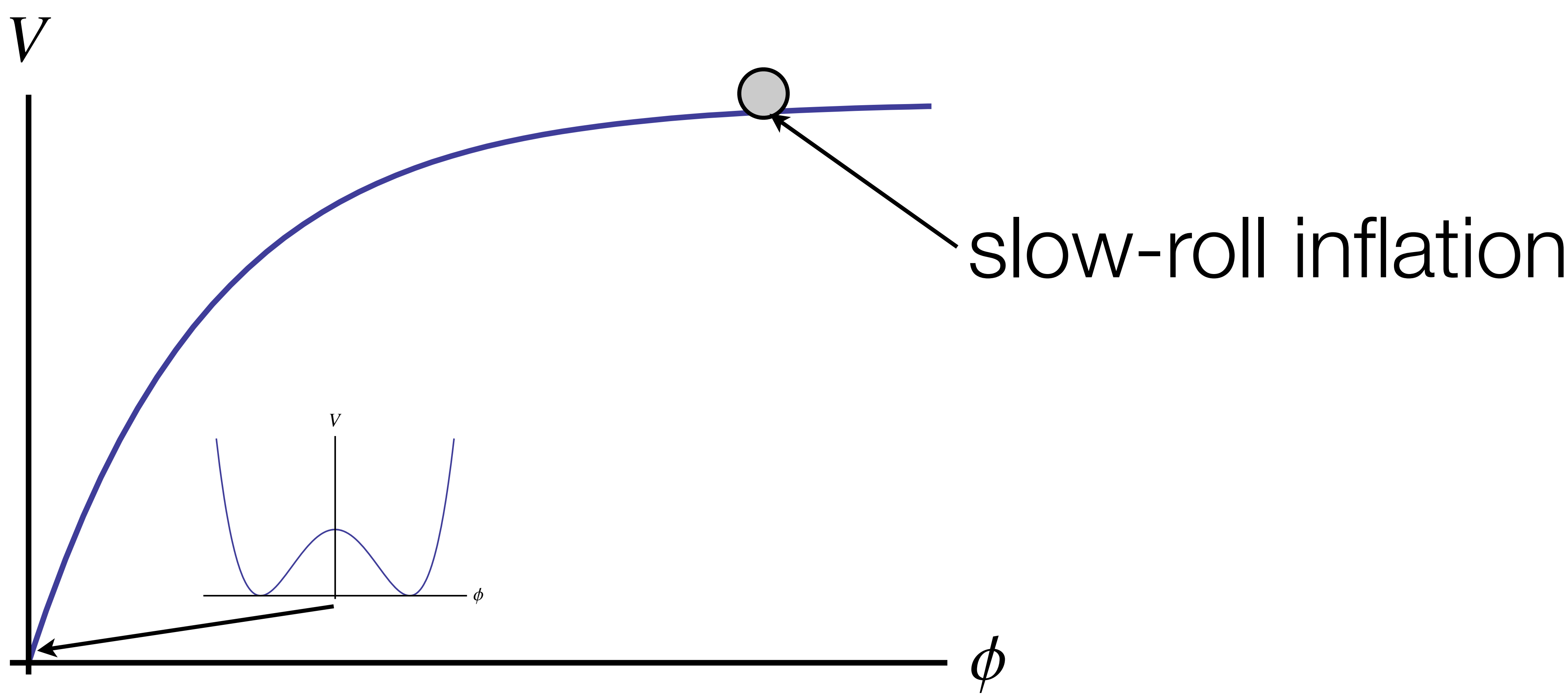
$$\begin{aligned} S_E \;=\; & \int d^4x\sqrt{-g}\Bigg[\frac{M_p^2}{2}R(g_{\mu\nu}) - g^{\mu\nu}\Bigg(3\frac{\omega^4\xi^2}{M_p^2}\partial_\mu\left(\mathcal{H}^\dagger\mathcal{H}\right)\partial_\nu\left(\mathcal{H}^\dagger\mathcal{H}\right) \\ & +\omega^2(D_\mu\mathcal{H})^\dagger(D_\nu\mathcal{H})\Bigg) - \omega^4\lambda\big(\mathcal{H}^\dagger\mathcal{H} - v^2\big)^2\Bigg]. \end{aligned}$$

(Einstein)

- non-minimal coupling removed (back to Einstein gravity)
- non-canonical kinetic terms, deformed potential

Different field regimes

- low field: Standard Model Higgs potential
- high field: slow-roll inflation potential $\lambda \to \lambda' \ll \lambda$
- Higgs inflation should ultimately connect these regimes



Higgs field: gauge variant inflaton

- During inflation, the Higgs field slowly rolls down (it is the inflaton)
- It is **NOT** in the minimum of the Mexican hat potential
- A gauge variant scalar field in a symmetry-breaking configuration has Goldstone bosons associated to it

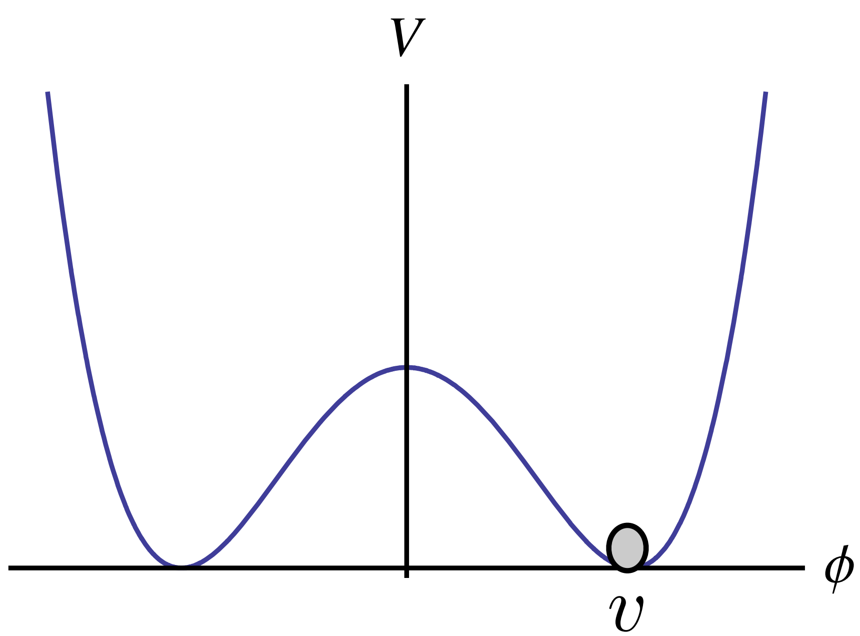
Goldstone Mechanism in Standard Model

[U(1) toy model]

$$\Phi(t,\vec{x}) = \frac{1}{\sqrt{2}}\left[v + h(t,\vec{x}) + i\theta(t,\vec{x})\right]$$

- v: time-independent VEV (246 GeV)
- h: quantum Higgs field
- θ: massless Goldstone boson

$$m_\theta^2 \equiv \left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\rm cl} = \left.\frac{1}{\phi}\frac{\partial V}{\partial\phi}\right|_{\rm cl} = 0$$



- Goldstone boson is unphysical: disappears from theory in unitary gauge. Its associated degree of freedom renders the U(1) gauge boson massive.

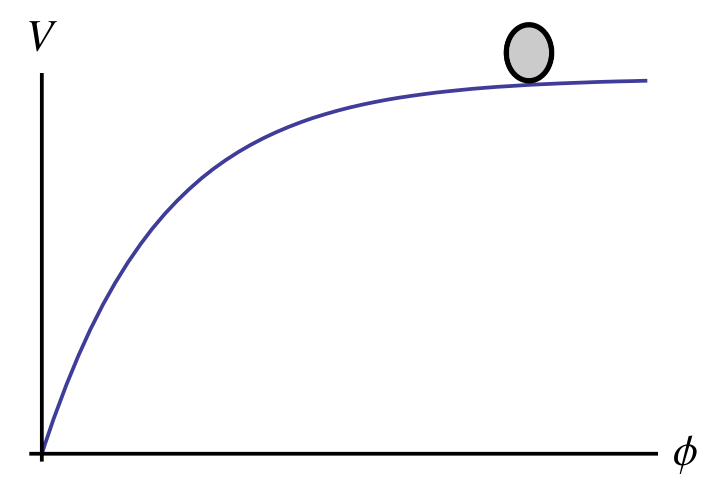
Goldstone Mechanism in Higgs Inflation

[U(1) toy model]

$$\Phi(t,\vec{x}) = \frac{1}{\sqrt{2}}\left[\phi(t) + h(t,\vec{x}) + i\theta(t,\vec{x})\right]$$

- φ: time-dependent classical background field
- h: quantum Higgs field
- θ: massive Goldstone boson

$$m_\theta^2 \equiv \left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\rm cl} = \left.\frac{1}{\phi}\frac{\partial V}{\partial\phi}\right|_{\rm cl} = -\left.\frac{\ddot{\phi}}{\phi}\right|_{\rm cl} \neq 0$$



- Goldstone boson still disappears from theory in unitary gauge. Its associated degree of freedom still renders the U(1) gauge boson massive. Is it still unphysical?

Coleman-Weinberg corrections

$$V_{\rm CW} = \frac{1}{32\pi^2} \sum_i (-1)^{2J_i} (2J_i + 1) m_i^2 \left(\Lambda^2 - m_i^2 \ln \Lambda \right)$$

Sum is over all fields... what about the Goldstone boson mass?
Cross it out?

- YES because it is unphysical

Linde et al, 1008.2942

- "YES" would have dramatic consequences for SUSY Higgs inflation

- NO because that produces a discontinuous CW potential

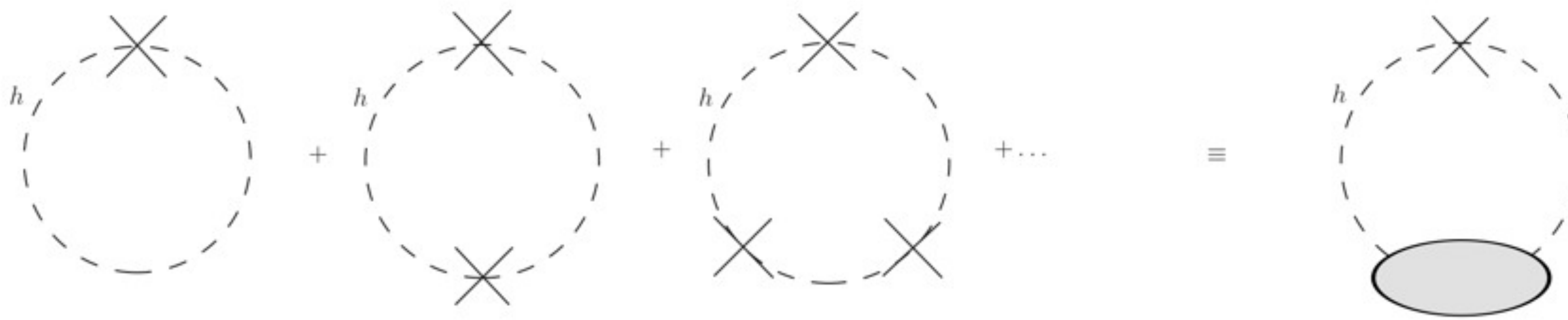
Solution: compute CW-corrections for time-dependent background

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{gaugekinetic}} + \mathcal{L}_{\text{higgskinetic}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{gaugefixing}} + \mathcal{L}_{\text{faddeev-popov}} \\ &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi (D^\mu \Phi)^\dagger - V(\Phi\Phi^\dagger) - \frac{1}{2\xi} G^2 + \bar{\eta} g \frac{\delta G}{\delta \alpha} \eta\end{aligned}$$

$$\Big(G = \partial_\mu A^\mu - \xi g (\phi + h)\theta\Big)$$

- Use Schwinger-Keldysh formalism (closed time path, in-in)
- Computation has been done before in other context (Boyanovsky, Heitmann & Baacke), we find other coefficients

Calculation...



- Perturbative and non-perturbative approach
- Massless propagators with one-loop time-dependent mass insertions
- Contributions from all fields in the problem (h, θ, A^{μ}, η)
- Complication from mixing A^0-θ terms

Result

$$V_{\rm CW} = \frac{1}{32\pi^2}\left[\Lambda^2\left(V_{hh}+V_{\theta\theta}+3(g\phi)^2\right) - \log\Lambda\left(V_{hh}^2+V_{\theta\theta}^2+3(g\phi)^4-6V_{\theta\theta}(g\phi)^2\right)\right]$$

- Continuous effective potential
- Reduces to ordinary Coleman-Weinberg in static limit
- **Goldstone bosons do contribute to effective potential** (induced by massive gauge boson?)
- Gauge invariant (ξ-independent)
- unitary gauge ill-defined (limit ξ→∞ does not commute with limit k→∞ in momentum integrals)
- all details in arXiv[1104.4897]!!

Conclusion & remark

- From a U(1) toy model it already follows that in Higgs inflation Goldstone bosons will contribute to the effective potential

- Effects will be small during inflation, but significant at its end

To do...

- Generalize from U(1) to SU(2) x U(1) (trivial)
- Generalize from Minkowski to FRW (complicated: gauge field components couple in their equations of motion)
- include non-canonical kinetic terms (difficult for fast field evolution after inflation)