Existence of relativistic stars in $f(T)$ gravity

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PASCOS 2011

Outline

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Why $f(T)$ gravity?

- $f(T)$ gravity preferred over $f(R)$ gravity.
- Provides an alternative to dark energy.
- Unification of early inflation and late expansion.
- Fits observational data and supports crossing of the phantom line divide.

Investigation of a recent paper.

- Static solutions not studied until recently.
- 'Absence of relativistic stars in $f(T)$ gravity' *(C. Deliduman and B. Yapiskan, arXiv: 1103.2225v1)*.
- Use conservation equation from GR to prove this.
- If true, could be problematic for $f(T)$ gravity.

Absence of relativistic stars in $f(T)$ gravity?

- The metric $ds^2 = e^{a(r)}dt^2 - e^{b(r)}dr^2 - r^2 d\Omega^2$ gives the conservation equation

$$p' = \frac{T'}{8\pi r^2} f_{TT} - \frac{a'}{2}(\rho + p)\,.$$

- Compare with conservation equation from GR

$$p' = -\frac{a'}{2}(\rho + p)\,.$$

- Conclusion: $f_{TT} = 0$, hence solutions only exist in teleparallelism.

Outline

Problem: GR not equivalent to $f(T)$ gravity.

- TEGR by definition is equivalent to GR.
- $f(T)$ gravity is based on modifications to TEGR.

$$S = \frac{1}{16\pi} \int e\, f(T)\, d^4x + S_{\text{matter}} \,.$$

- The presence of $f(T)$ changes the field equations **and** the conservation equation.

Outline

Impact of $f(T)$

- The $f(T)$ field equations can be written as follows

$$e\delta^{\sigma}_{\rho} S_{\sigma}{}^{\mu\nu}\partial_{\mu}(T) f_{TT} + e^{i}{}_{\rho}\partial_{\mu}(e e_{i}{}^{\sigma} S_{\sigma}{}^{\mu\nu}) f_{T}$$

$$-e\delta^{\sigma}_{\rho} T^{\gamma}{}_{\mu\sigma} S_{\gamma}{}^{\nu\mu} f_{T} + \frac{e}{4}\delta^{\nu}_{\rho} f = 4\pi e\delta^{\sigma}_{\rho} \mathcal{T}_{\sigma}{}^{\nu} .$$

- Step 1: We exploit the antisymmetry of $S_{\sigma}{}^{\mu\nu}$.

Gauge current.

- Step 2: We can define the gauge current

$$j_i^{\ \nu} = -\frac{1}{4\pi}\left(e_i^{\ \sigma} S_\sigma^{\ \mu\nu}\partial_\mu(T) f_{TT} - e_i^{\ \sigma} T^\gamma{}_{\mu\sigma} S_\gamma^{\ \nu\mu} f_T + \frac{1}{4} e_i^{\ \nu} f\right) .$$

- $j_i^{\ \nu}$ represents the energy and momentum of the gravitational field.

- When $f(T) = T$ reduces to gauge current of TEGR.

Conservation Equation.

- We now arrive at the conserved quantity

$$4\pi\partial_\nu \left(e(j_i{}^\nu + \mathcal{T}_i{}^\nu)\right) = \partial_\mu(eS_i{}^{\mu\nu})\partial_\nu f_T \,.$$

- Compare with conservation equation derived for particular metric

$$p' = -\frac{a'}{2}(\rho + p) + \frac{T'}{2r^2} f_{TT} \,.$$

- Hence comparison with correct equation does not impose $f_{TT} = 0$.

Outline

Searching for solutions.

- Field equations and conservation equation give three independent equations.
- But now have five variables to solve for, $f(T)$, $\rho(r)$, $p(r)$, $a(r)$ and $b(r)$.
- System is underdetermined, must fix two parameters:
    - Equation of state,
    - Fix $f(T)$,
    - Fix metric and solve.

Basic solutions with $T = 0$.

- Let $T = 0$ and solve for $a(r)$.
- The field equations then imply the metric

$$ds^2 = \frac{A}{r} dt^2 - \frac{r}{B - 4r} dr^2 - r^2 d\Omega^2 \, .$$

- The field equations now give a full solution

$$\rho_0 = -p_0 = \frac{f_0}{\pi}$$

Schwarzschild-type Solutions.

- Assume, $e^a = e^{-b}$ and a constant pressure p_0.
- Conservation equation leads to the metric

$$ds^2 = \ln(E_1 r)dt^2 - \frac{1}{\ln(E_1 r)}dr^2 - r^2 d\Omega^2 \,.$$

- Using the field equations we obtain

$$16\rho(r) = \frac{E_2}{r}\left(\frac{4\chi}{2\chi+1} + \frac{2(2\chi+1)(2\chi(r^2-3)-1)}{r^3(6\chi+1)}\right) - 16\pi p_0 \,,$$

$$f(r) = -16\pi p_0 + \frac{E_2}{r} \,.$$

Einstein Static Universe.

- Here, one would normally assume $\rho = \rho_0$, $p = p_0$, and $a' = 0$.
- Can consider alternatives.
- $\rho = \rho_0$, $a' = 0$ and p free.
- $\rho = \rho_0$, $p = p_0$ and a free.

Generalised Einstein Static Universe.

- Need a generalisation

$$ds^2 = \frac{1}{1-kr^2}dt^2 - \frac{1}{1-kr^2}dr^2 - r^2 d\Omega^2$$

- Now we can solve for $f(T)$, $\rho(r)$ and $p(r)$

$$\rho(r) = \beta + 2\alpha k e^{-kr^2}(-1 - 6kr^2 + 4k^2r^4)$$

$$p(r) = \beta + 2\alpha k e^{-kr^2}(3 + 2kr^2)$$

$$f(T) = \alpha e^{2k/(2k+T)}(2k + T) + \beta\,.$$

Summary

- Have shown that the conservation equation does allow for relativistic stars in $f(T)$ gravity.
- We explicitly constructed several counter examples to support this.