$\mathcal{N}=4$ Instanton effective action in Ω-background and D3/D(-1)-brane system in R-R background

Speaker : Takuya Saka (Tokyo Tech.)

Collaboration with
Katsushi Ito, Shin Sasaki (Tokyo Tech.)
And Hiroaki Nakajima (KIAS, Kyungpook National Univ.)

Granted from
Global Center of Excellence Program
"Nanoscience and Quantum Physics"

PASCOS 2011 @ Cambridge 2011/07/05

Our theme of work

Instanton calculus in super Yang-Mills theory

- About the instanton

Topologically non-trivial solutions of EOM for Wick rotated Yang-Mills theory

$$F_{mn} = \tilde{F}_{mn}$$

$$k = \frac{g^2}{32\pi^2} \mathrm{Tr} \int \mathrm{d}^4 x F_{mn} \tilde{F}^{mn} \in \mathbb{Z}$$

↓

$k \neq 0 \Rightarrow$ Non-perturbative vacua of YM

We want to know the instanton correction to the physical quantites.

● $\mathcal{N}$ = 2 SYM case

- Seiberg-Witten (1994)

Instanton contribution to the prepotential of the low-energy effective action of $\mathcal{N}$ = 2 SYM

$$\mathcal{F}(\phi) = \mathcal{F}_{\text{pert}}(\phi) + \mathcal{F}_{\text{inst}}(\phi), \qquad \mathcal{F}_{\text{inst}}^{\text{SU}(2)}(\phi) = \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{\phi}\right)^{4k} \phi^2$$

- Nekrasov (2002)

Instanton partition function of $\mathcal{N}$ = 2 SYM

$$Z_{\text{inst}}(\phi, \Lambda, \epsilon_1, \epsilon_2) = \sum_{k=1}^{\infty} \Lambda^{2Nk} \int_{\mathcal{M}_{N,k}} \mathrm{d}\mu \exp[-S_{\text{eff}}(\mu, \phi, \epsilon_1, \epsilon_2)]$$

$S_{\text{eff}}(\mu, \phi, \epsilon_1, \epsilon_2)$ Effective action for instanton moduli (Instanton effective action)

ϵ_1, ϵ_2 Deformation paramters (Ω-background)

$$\log Z_{\text{inst}}(\phi, \Lambda, \epsilon_1, \epsilon_2) = \frac{1}{\epsilon_1 \epsilon_2} \left(\mathcal{F}_{\text{inst}}(\phi) + (\epsilon_1 + \epsilon_2) \mathcal{F}_1(\phi) + \mathcal{O}(\epsilon^2) \right)$$

- About Ω-background

We start with 6 dim. $\mathcal{N} = 1$ super Yang-Mills with some non-trivial metric;

$$ds_{6\text{D}}^2 = 2\mathrm{d}z\mathrm{d}\bar{z} + (\mathrm{d}x^m + \bar{\Omega}^m\mathrm{d}z + \Omega^m\mathrm{d}\bar{z})^2$$

$$\Omega^m = \Omega^{mn}x_n, \quad \bar{\Omega}^m = \bar{\Omega}^{mn}x_n$$

$$\Omega^{mn} = \frac{1}{2\sqrt{2}}\begin{pmatrix} & i\epsilon_1 & & \\ -i\epsilon_1 & & & \\ & & & -i\epsilon_2 \\ & & i\epsilon_2 & \end{pmatrix}, \bar{\Omega}^{mn} = \frac{1}{2\sqrt{2}}\begin{pmatrix} & -i\bar{\epsilon}_1 & & \\ i\bar{\epsilon}_1 & & & \\ & & & i\bar{\epsilon}_2 \\ & & -i\bar{\epsilon}_2 & \end{pmatrix}$$

Dimensional reduction to 4 dim. → 4D $\mathcal{N} = 2$ SYM with Ω-background

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S_{eff} is an exact form w.r.t. its supersymmetry. $S_{\mathrm{eff}} = QV$

→ Localization formula

Integration of the instanton moduli localizes on the fixed points of *Q*.

Continuous due to the translational symmetry of SYM

Ω-background isolates the fixed points. (Explicit presence of coordinate *x*)

→ Integral becomes the finite sum of the fixed points.

Apply the idea of "deformation of the field theory" to many other theories.

→ Interpret in terms of String theory

↓

Instanton corrections to many other gauge theories

- High dimensional instantons
- Instantons in quiver type gauge theory
- Field theories with more or less SUSY

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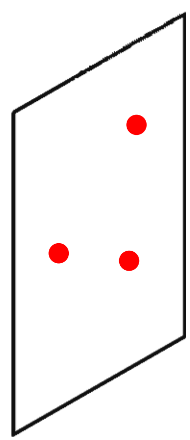
D3-branes on $\mathbb{Z}_2$ orbifolding point



● : D-instanton

Effective action on the D3-branes

=

4 dim. $\mathcal{N} = 2$ super Yang-Mills action

Effective action on the D-instantons

=

Instanton effective action of
4 dim. $\mathcal{N} = 2$ super Yang-Mills

- Billo-Frau-Fucito-Lerda (2006), Ito-Nakajima-TS-Sasaki (2010)

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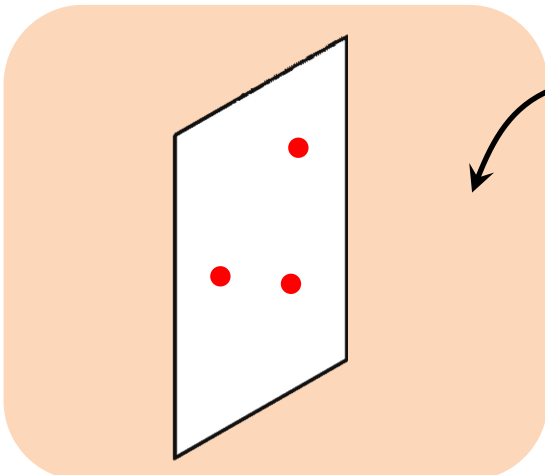

● : D-instanton

RR 3-form background

$C_{mna} \to C_{mn}, \bar{C}_{mn}$

Deformation to D-instanton eff. action

=

Ω-background deformation to the instanton effective action of 4D $\mathcal{N} = 2$ SYM

$\varepsilon_1 + \varepsilon_2$ part → BFFL

Generic ε_1 and ε_2 → INSS

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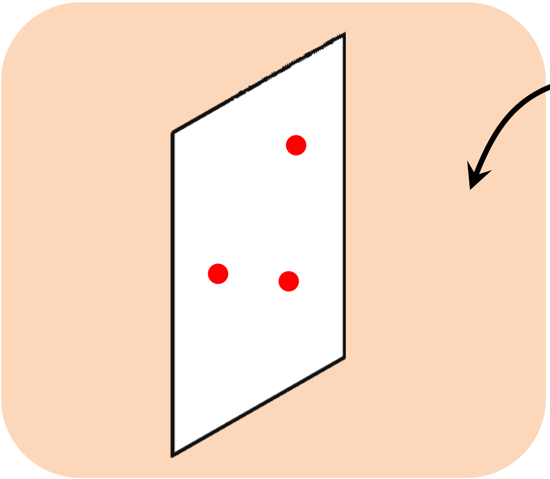

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- Our idea

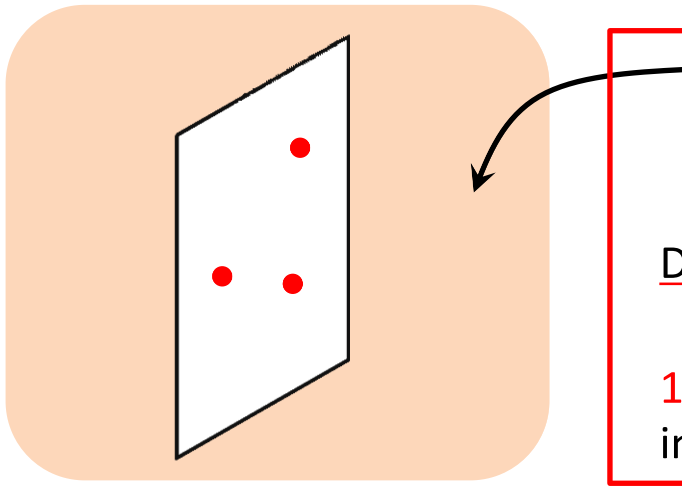
If we do not orbifold spacetime

→ • 4D $\mathcal{N} = 4$ SYM and its instanton eff. action on D3/D(-1)-brane

• "Extension" of the Ω-background ("10 dimensional Ω-background") corresponding to original C_{mna}

Our present work (WIP)

D3-branes on flat 10D spacetime



RR 3-form background

C_{mna}

Deformation to D-instanton eff. action

=

10D Ω-background deformation to the instanton effective action of 4D $\mathcal{N}$ = 4 SYM

● : D-instanton

- More deformation parameters $\epsilon_a \ (a = 1, \cdots, 6)$

  → We can consider many kinds of defromation

  - Recover $\mathcal{N}$ = 2 case
  - Mass deformation of $\mathcal{N}$ = 4 SYM → $\mathcal{N}$ = 2* theory

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Thank you for listening!!