Cosmological Simulations: 
Successes & Tensions of $\Lambda$CDM

Volker Springel

- Large scale predictions
- Small scale predictions
- Challenges for $\Lambda$CDM from non-linear structure formation
Want to bridge 13.6 billion years of cosmological evolution
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Mathematical Bridge in Cambridge

Rumored to be built by Isaac Newton... without nuts and bolts
Currently the fastest supercomputers carry out about ~1 Petaflop, which are one thousand billion floating point operations per second.

JUGENE IN JUELICH
Why are **cosmological simulations** of structure formation useful for studying the dark universe?

Simulations are the theoretical tool of choice for calculations in the non-linear regime.

They connect the (simple) cosmological initial conditions with the (complex) present-day universe.

**Predictions from N-body simulations:**
- Abundance of objects (as a function of mass and time)
- Their spatial distribution
- Internal structure of halos (e.g. density profiles, spin)
- Mean formation epochs
- Merger rates
- Detailed dark matter distribution on large *and* fairly small scales
- Galaxy formation models
- Gravitational lensing
- Baryonic acoustic oscillations in the matter distribution
- Integrated Sachs-Wolfe effect
- Dark matter annihilation rate
- Morphology of large-scale structure (“cosmic web”)
- ....
Simulations provide accurate measurements for halo abundance as a function of time

CONVERGENCE RESULTS FOR HALO ABUNDANCE

Simulated and observed large-scale structure in the galaxy distribution

MOCK PIE DIAGRAMS COMPARED TO SDSS, 2DFGRS, AND CFA-2

Springel et al. (2006)
The two-point correlation function of galaxies in the Millennium run is a very good power law.  

**GALAXY TWO-POINT FUNCTION COMPARED WITH 2dFGRS**
The galaxy distribution is **biased** with respect to the mass distribution.

**GALAXY AND MASS CLUSTERING AT DIFFERENT EPOCHS**

The graphs show the correlation function $\xi(r)$ for galaxies and dark matter at different redshifts ($z=0.00, 1.39, 5.72, 8.55$) as a function of comoving distance ($r$) in Mpc/$h$. The galaxy distribution evolves little in time, while the dark matter distribution evolves strongly in time.
The large-scale clustering pattern of halos and galaxies is already imprinted on the initial conditions.

TIME EVOLUTION OF THE MATTER AND GALAXY DISTRIBUTION
The baryonic wiggles remain visible in the galaxy distribution down to low redshift and may serve as a "standard ruler" to constrain dark energy.

DARK MATTER AND GALAXY POWER SPECTRA FROM THE MILLENNIUM SIMULATION IN THE REGION OF THE WIGGLES

Springel et al. (2005)
Large volume simulations are needed for ongoing and future surveys

GOALS FOR A “MILLENIUM-XXL” CALCULATION

**Science goals**
- Impact of galaxy physics on BAO/growth factor measurements
- Realistic mock catalogs for Pan–STARRS, SDSS–III/BOSS, BigBOSS, etc.
- Exploring galaxy physics in different cosmologies
- Integrated Sachs–Wolfe effect
- Clustering up to ~500 Mpc/h, universality of halo bias and mass function, rare events, environmental effects, first quasars, etc.

**Desired simulation characteristics**
- Box-Size: > 3000 Mpc/h
- Particle mass: < ~ 6 x 10^9 M☉/h
- Particle number: > 300 billion particles
Millennium-XXL

303 billion particles

Largest N-body simulation ever
Millennium-XXL was successfully executed on JUROPA in 2010

PARAMETERS OF FINAL RUN

- $6720^3 \sim 303$ billion particles
- 3000 Mpc/h box, Millennium cosmology
- 12288 cores: 3072 MPI-task / 4 threads (70% of Juropa)
- $9216^3$ FFT mesh
- 86 trillion force calculations
- Cost: 2.7 million CPU hours (~300 years), corresponding to 9.3 days wallclock time (including FOF+SUBFIND)
- Peak memory usage: 29 TB (105 bytes/particle)
- 700 million halos at z=0 (44% of particles)
- About 25 billion (sub)halos in merger trees
- Largest cluster has $9 \times 10^{15} \, M_\odot$
- Size of a full snapshot: ~10 TB
- More than 120 TB stored for science

JUROPA
Jülich
Forschungszentrum
Are the presently known high-mass clusters still consistent with $\Lambda$CDM?

**CLUSTER MASS FOR GIVEN ABUNDANCE AS A FUNCTION OF TIME**

Detection of one violating cluster would invalidate $\Lambda$CDM

Holz & Perlmutter (2010) argued XMMU J2235.3-2557 to be inconsistent with $\Lambda$CDM at 3$\sigma$

Boyle, Jimenez & Verde (2010) argue that $\sigma_8$ would have to be $\sim 4\sigma$ higher to accommodate massive clusters. Suggest non-Gaussian ICs as a solution.

**Abundance per Gpc$^3**

- $0.1$
- $1$
- $10$
- $100$
Massive clusters are not homogenous and are often fairly perturbed systems.

- **Snapshots z=0.32**
- **15 most massive clusters**
  - according to $M_{200}$
  - $M = [2.5 - 4] \times 10^{15}$ Msun/h
Diversity in the Extreme

THE MOST MASSIVE AND RAREST CLUSTERS FOLLOW THE SCALING RELATIONS EXPECTED FROM MORE ABUNDANT SMALLER SYSTEMS

So far reported massive clusters not in conflict with $\Lambda$CDM (yet)
NASA Press Release Aug 21, 2006:

1E 0657-56: NASA Finds Direct Proof of Dark Matter
Fitting the density jump in the X-ray surface brightness profile allows a measurement of the shock's Mach number.

**X-RAY SURFACE BRIGHTNESS PROFILE**

*Markevitch et al. (2006)*

**shock strength:**

\[ M = 3.0 \pm 0.4 \]

**shock velocity:**

\[ v_s = 4700 \text{ km/s} \]

Usually, shock velocity has been identified with velocity of the bullet.
How rare is the bullet cluster?

DISTRIBUTION OF VELOCITIES OF THE MOST MASSIVE SUBSTRUCTURE IN THE MILLENNIUM RUN

Adopted mass model from Clowe et al. (2004):

NFW-Halo with:

\[ M_{200} = 2.96 \times 10^{15} \, M_{\odot} \]
\[ R_{200} = 2.25 \, \text{Mpc} \]
\[ V_{200} = 2380 \, \text{km/sec} \]
\[ V_{\text{shock}} = 4500 \, \text{km/sec} \]
\[ V_{\text{sub}} / V_{\text{shock}} = 1.9 \quad \text{chance: } 10^{-2} \]

But, revised data from Clowe et al. (2006) and Markevitch et al. (2006):

\[ M_{200} = 1.5 \times 10^{15} \, M_{\odot} \]
\[ V_{200} = 1680 \, \text{km/sec} \]
\[ V_{\text{shock}} = 4740 \, \text{km/sec} \]
\[ V_{\text{sub}} / V_{\text{shock}} = 2.8 \quad \text{chance: } 10^{-7} \]
Observed X-ray map compared with simulation images of a standard zero-energy orbit merger

SIMULATED X-RAY MAP COMPARED TO OBSERVATION

Candra 500 ks image

bullet cluster simulation

Springel & Farrar (2007)
The model also matches the observed temperature and mass profiles.

**COMPARISON OF SIMULATED TEMPERATURE AND MASS PROFILE WITH OBSERVATIONS**

- Data from Markevitch et al. (2006)
- Data from Bradac et al. (2006)
Despite a shock speed of \(~4500\) km/s, the bullet moves considerably slower.

**VELOCITIES AND POSITIONS OF MAIN BULLET CLUSTER FEATURES AS A FUNCTION OF TIME**

- **Shock speed:** 4500 km/s
- **Pre-shock infall:** -1100 km/s
- **Shock speed relative to bullet:** -800 km/s
- **Speed of bullet:** 2600 km/s

[Graph showing velocities and positions as a function of time]
Models with a “fifth force” in the dark sector can speed up the bullet, but seem not required to match the bullet system

**SPEED OF THE BULLET IN FIFTH FORCE MERGERS** (proposed by Farrar & Rosen 2006)

\[
\phi_s(r) = -\beta \frac{G m}{r} \exp \left( -\frac{r}{r_s} \right)
\]

- \( \beta = 1.0, \ r_s = 4 \text{ Mpc} \)
- \( \beta = 0.3, \ r_s = 4 \text{ Mpc} \)
- \( \beta = 1.0 \)
- \( v_b = 3800 \text{ km/s} \)
- \( \beta = 0.3 \)
- \( v_b = 3010 \text{ km/s} \)
- \( \beta = 0 \)
- \( v_b = 2600 \text{ km/s} \)
Small-scale dark matter structure
Zooming in on dark matter halos reveals a huge abundance of dark matter substructure.

DARK MATTER DISTRIBUTION IN A MILKY WAY Sized HALO AT DIFFERENT RESOLUTION.
Spherically averaged density profiles of dark matter halos have a nearly universal shape.

**DENSITY PROFILE AS A FUNCTION OF RADIUS**

- Rotation curve of galaxies
- Internal structure of galaxy clusters
- Gravitational lensing
- DM annihilation
- Galaxy mergers
Our simulations allow us to study the convergence of subhalo density profiles. SPHERICALLY AVERAGED DENSITY PROFILES IN THE AQ-A HALO AT DIFFERENT RESOLUTION.
A long standing issue in galaxy formation theory: The shapes of the CDM halo mass function and the galaxy luminosity function are very different.

THE OBSERVED LF COMPARED TO THE SHAPE OF THE CDM HALO MASS FUNCTION

van den Bosch et al. (2004)
Taken at face value, the number of luminous satellites in the Milky Way is much smaller than the number of dark matter lumps

**THE SATELLITE PROBLEM**

Moore et al. (1999)
Hydrodynamical simulations aim to predict:

- Morphology of galaxies
- Fate of the diffuse gas, WHIM, metal enrichment
- X-ray atmospheres in halos
- Turbulence in halos and accretion shocks
- Large-scale regulation of star formation in galaxies through feedback processes from stars and black holes
- Transport processes (e.g. conduction)
- Radiative transfer
- Dynamical transformations (e.g. ram-pressure stripping)
- Magnetic fields
Current cosmological hydrodynamic simulations have severe trouble to explain the low galaxy formation efficiency required for $\Lambda$CDM.

**Galaxy Formation Efficiency as a Function of Halo Mass**

- Abadi et al. (2003)
- Okamoto et al. (2005)
- Governato et al. (2007)
- Scannapieco et al. (2009)
- Piontek & Steinmetz (2009)

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Sawala & White (2010)
Relatively dense massive subhalos are predicted that apparently cannot host any of the luminous satellites of the Milky Way.

**THE PUZZLING DARKNESS OF SOME OF THE LARGE DARK MATTER SATELLITES**

*Boylan-Kolchin, Bullock & Kaplinghat (2011)*
Warm dark matter models also reduce the number of Milky Way satellites substantially – for 1 keV there are still enough

**SATELLITE ABUNDANCE AS A FUNCTION OF DM PARTICLE MASS**

Maccio & Fontanot (2010)
The radial distribution of substructures is strongly antibiased relative to all dark matter, and independent of subhalo mass.

**RADIAL SUBSTRUCTURE DISTRIBUTION IN Aq-A-1**

Most subhalos are at large radii, subhalos are more effectively destroyed near the centre.

Subhalos are far from the Sun.

see also Diemand et al. (2007, 2008)
The subhalo abundance per unit halo mass is surprisingly uniform

**VELOCITY FUNCTION IN OUR DIFFERENT HALOS**

![Graph showing velocity function across different halos.](image)
The cumulative mass fraction in resolved substructures reaches about 12-13%, we expect up to ~18% down to the thermal limit.
Dark matter annihilation predictions
Dark matter could be self-annihilating, in which case the presence of subhalos should **boost** the expected flux

**THE ANNIHILATION SIGNAL DUE TO SUBSTRUCTURES**

Annihilation flux:

\[
F = \frac{N_\gamma \langle \sigma v \rangle}{2 \ m_{DM}^2} \int_V \frac{\rho_{DM}^2(x)}{4\pi \ d^2(x)} \ d^3 x
\]

Luminosity of a halo with maximum circular velocity \(V_c(r_{\text{max}})= V_{\text{max}}\):

\[
L = \int \rho_{DM}^2(x) \ d^3 x
\]

**NFW-Profile:** \(L = 1.23 \frac{V_{\text{max}}^4}{G^2 \ r_{\text{max}}}\)

**Einasto-Profile:** \(L = 1.87 \frac{V_{\text{max}}^4}{G^2 \ r_{\text{max}}}\)

**\(\alpha = -1.4\) Profile:** \(L = 3.97 \frac{V_{\text{max}}^4}{G^2 \ r_{\text{max}}}\)

**Moore-Profile:** \(L = \infty\)
The dark matter annihilation flux is boosted significantly by dark matter substructures

**EXTRAPOLATED ALL-SKY MAP OF THE DM ANNIHILATION FLUX FROM THE MILKY WAY**

\[ L \propto \rho^2 \, dV \]

*total emission*
Dark matter annihilation can be best discovered with an optimal filter against a bright background.

**The Signal-to-Noise for Detection With an Optimal Filter**

The optimal filter is proportional to the signal

\[
S/N = \sqrt{\tau A_{\text{eff}}} \left[ \int \frac{n_\gamma^2(\theta, \phi)}{n_\gamma(\theta, \phi) + b_\gamma(\theta, \phi)} \, d\Omega \right]^{1/2}
\]

The background dominates, then:

**Main Halo’s Smooth Component:**

\[
(S/N)_{\text{Main Sm}} = f_{\text{Main Sm}} \left[ \frac{\tau A_{\text{eff}}}{b_\gamma} \right]^{1/2} \frac{F}{\theta_h}
\]

**Subhalo’s Smooth Component:**

\[
(S/N)_{\text{Sub Sm}} = f_{\text{Sub Sm}} \left( \frac{\tau A_{\text{eff}}}{b_\gamma(\bar{\alpha})} \right)^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}}
\]

**Sub-substructure of a subhalo:**

\[
(S/N)_{\text{Sub Sub}} = f_{\text{Sub Sub}} \left( \frac{\tau A_{\text{eff}}}{b_\gamma(\bar{\alpha})} \right)^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}}
\]
Detectability of different annihilation emission components in the Milky Way

S/N for detecting subhalos in units of that for the main halo

30 highest S/N objects, assuming the use of optimal filters

\[ S/N \propto CV_{\text{max}}^4 / (r_{\text{half}}^2 d) \]

Highest S/N subhalos have 1% of S/N of main halo
Highest S/N subhalos have 10 times S/N of known satellites
Substructure of subhalos has no influence on detectability
But what about other nearby structures, like galaxy clusters?

High resolution “Phoenix” project

Gao et al. (2011)
The nearest massive galaxy clusters are attractive targets for annihilation detection

### Comparison of Signal-to-Noise for Detection of Nearby Sources

<table>
<thead>
<tr>
<th>Object Name</th>
<th>Half-light radius [arcmin]</th>
<th>Distance [Mpc]</th>
<th>$M_{200}$ [$M_\odot$]</th>
<th>$L$ [$L_{\text{MW}}$]</th>
<th>$F = L/(4\pi d^2)$ [$F_{\text{MW}}$]</th>
<th>S/N [$(S/N)_{\text{MW}}$]</th>
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<tbody>
<tr>
<td>AWM 7</td>
<td>35.5</td>
<td>67.0</td>
<td>$4.2 \times 10^{14}$</td>
<td>$7.1 \times 10^4$</td>
<td>$3.2 \times 10^{-4}$</td>
<td>$6.8 \times 10^{-3}$</td>
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<td>Fornax Cluster</td>
<td>84.1</td>
<td>17.5</td>
<td>$1.0 \times 10^{14}$</td>
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<td>$8.0 \times 10^{-4}$</td>
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<td>M49</td>
<td>59.6</td>
<td>18.2</td>
<td>$0.4 \times 10^{14}$</td>
<td>$3.9 \times 10^3$</td>
<td>$2.4 \times 10^{-4}$</td>
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<td>NGC 4636</td>
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<td>17.4</td>
<td>$0.24 \times 10^{14}$</td>
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<td>$1.4 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-3}$</td>
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<td>Centaurus (A3526)</td>
<td>40.1</td>
<td>50.5</td>
<td>$2.6 \times 10^{14}$</td>
<td>$3.9 \times 10^4$</td>
<td>$3.1 \times 10^{-4}$</td>
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<td><strong>Coma</strong></td>
<td><strong>36.1</strong></td>
<td><strong>95.8</strong></td>
<td><strong>$1.3 \times 10^{15}$</strong></td>
<td><strong>$2.9 \times 10^5$</strong></td>
<td><strong>$6.4 \times 10^{-4}$</strong></td>
<td><strong>$1.3 \times 10^{-2}$</strong></td>
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<td>Draco</td>
<td>16.4</td>
<td>0.082</td>
<td>N/A</td>
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<td>$1.6 \times 10^{-5}$</td>
<td>$6.3 \times 10^{-4}$</td>
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<td>UMaI</td>
<td>18.4</td>
<td>0.066</td>
<td>N/A</td>
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<td>$2.0 \times 10^{-5}$</td>
<td>$7.5 \times 10^{-4}$</td>
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<td>Leol</td>
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<td>N/A</td>
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<td>$1.2 \times 10^{-6}$</td>
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<tr>
<td>Fornax dwarf</td>
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<td>0.138</td>
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<td>Carina</td>
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<td>UMaII</td>
<td>28.8</td>
<td>0.032</td>
<td>N/A</td>
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<td>Comber</td>
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<td>$1.7 \times 10^{-5}$</td>
<td>$6.8 \times 10^{-4}$</td>
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<tr>
<td>Will</td>
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<td>0.066</td>
<td>N/A</td>
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<td>N/A</td>
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<tr>
<td><strong>M31</strong></td>
<td><strong>351.5</strong></td>
<td><strong>0.807</strong></td>
<td><strong>$1.8 \times 10^{12}$</strong></td>
<td><strong>$1.3 \times 10^2$</strong></td>
<td><strong>$4.2 \times 10^{-3}$</strong></td>
<td><strong>$9.3 \times 10^{-3}$</strong></td>
</tr>
</tbody>
</table>
The Millennium-II simulation can be used to construct full backwards lightcones of the expected gamma-ray annihilation background.

Thin redshift slices of the gamma-ray background reveal cosmic large-scale structure

A PARTIAL MAP AT ENERGY 10 GEV NEAR Z~0
In the complete coadded map, individual structures vanish in the high background level except for very near halos

THE FULL BACKGROUND MAP OUT TO REDSHIFT $Z = 10$
The full-sky maps can be used to extract direct predictions for the properties of the background

ENERGY SPECTRUM OF THE BACKGROUND
Coupled dark energy models get around some of the fine-tuning problems associated with dark energy, but introduce non-trivial modifications of long-range forces

**COUPLED DARK ENERGY IN THE NEWTONIAN LIMIT**

**Basic equations:**

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{dV}{d\phi} = \kappa \beta(\phi) \rho_{DM}
\]

\[
\dot{\rho}_{DM} + 3H \rho_{DM} = -\kappa \beta(\phi) \rho_{DM}
\]

Wetterich (1995)

Amendola (2000)

**Expanding to linear order in Newtonian gauge:**

\[\dot{v} = -\left[ H - \beta \frac{\dot{\phi}}{M} \right] v - G \left[ (1 + 2\beta^2) \frac{\nabla\Phi_c}{a} + \frac{\nabla\Phi_b}{a} \right]\]

Varying dark matter particle mass term

extra friction term

Modified gravitational interaction

Implemented by **Baldi et al. (2009)** in the GADGET-3 code
The numerical implementation accurately matches the linear theory on large-scales and predicts shallower halo profiles on small scales

SOME RESULTS FOR COUPLED DARK ENERGY SIMULATIONS

Interestingly, the above results is opposite to earlier numerical findings:
Summary points

Direct numerical simulations have become indispensable for studying the non-linear growth of structures in $\Lambda$CDM cosmologies.

Current numerical techniques allow high-resolution simulations with an unprecedented dynamic range. One presently reaches $N>10^{11}$, with a dynamic range of $10^5 – 10^7$ in 3D.

The future observation of a sufficiently massive cluster may easily rule out the $\Lambda$CDM model. The predicted satellite population may still be in tension with the observations.

Understanding galaxy formation physics remains a serious challenge in $\Lambda$CDM, both at the faint and the bright end.

Dark matter annihilation radiation should be more easily detectable from Coma or Fornax compared with nearby satellites.

Simulations of structure formation in modified gravity still in their infancy, but they promise to put powerful constraints on the viable model space.