The holographic fluid dual to vacuum Einstein gravity

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Holography

Any gravitational theory is expected to be holographic, *i.e.* it should have a description in terms of a non-gravitational theory in one dimension less.

- If gravity is indeed holographic, one should be able to recover generic features of quantum field theories through gravitational computations.

Holography and long wavelength behavior

- A generic feature of QFTs is the existence of a *hydrodynamic description* capturing the long-wavelength behavior near to thermal equilibrium.
- One then expects to find the same feature on the gravitational side, *i.e.*, there should exist a bulk solution corresponding to the thermal state, and nearby solutions corresponding to the hydrodynamic regime.

Hydrodynamics and AdS/CFT

This picture is indeed beautifully realized in AdS/CFT:

Thermal state	$\Leftrightarrow$	AdS black hole
Relativistic hydrodynamics	$\Leftrightarrow$	Relativistic gradient expansion solution of bulk

- Solutions describing non-equilibrium configurations are well approximated by hydrodynamics at late times.

[Witten (1998)] ... [Policastro, Son, Starinets (2001)] ... [Janik, Peschanski (2005)] ... [Bhattacharyya, Hubeny, Minwalla, Rangamani (2007)] ... [Chestler, Yaffe (2010)] ...

Hydrodynamics and vacuum Einstein gravity

We will see that a similar picture can be developed for vacuum Einstein gravity:

Thermal state	$\Leftrightarrow$	Rindler space
Incompressible Navier-Stokes expansion $+$ corrections	$\Leftrightarrow$	Non-relativistic gradient solution of bulk

One may then use the properties of these solutions in order to obtain clues about the nature of the dual theory.

References

- The talk is based on

  Geoffrey Compère, Paul McFadden, Kostas Skenderis, Marika Taylor,
  The holographic fluid dual to vacuum Einstein gravity, [arXiv:1104.3894].

- Related work

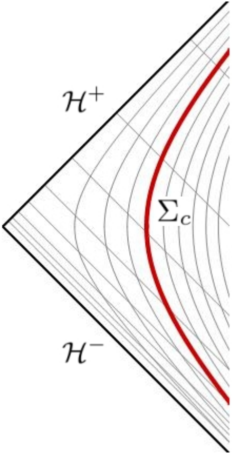
  I. Bredberg, C. Keeler, V. Lysov, A. Strominger, [arXiv:1101.2451]; V. Lysov, A. Strominger [arXiv:1104.5502].

Rindler spacetime

- Flat spacetime in ingoing Rindler coordinates is given by:

$$ds^2 = -r d\tau^2 + 2 d\tau dr + dx_i dx^i$$

  We consider the portion of spacetime between $r = r_c$ and the future horizon, $\mathcal{H}^+$.
- The induced metric γ_{ab} on Σ_c $(r = r_c)$ is flat.
- We can obtain a more general equilibrium configuration via diffeomorphisms respecting flatness of the induced metric..



Equilibrium configurations

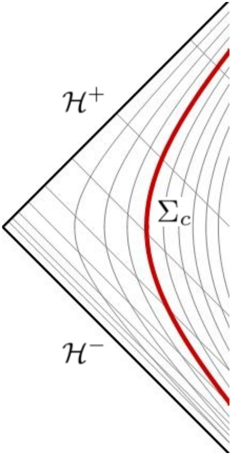
$$ds^2 = -p^2(r-r_c)u_a u_b dx^a dx^b - 2p u_a dx^a dr + \gamma_{ab} dx^a dx^b.$$

- The Rindler horizon has constant Unruh temperature $T = \frac{1}{4\pi\sqrt{r-r_c}}$.
- The Brown-York stress energy tensor takes the perfect fluid form:

$$T_{ab} = \rho u_a u_b + p h_{ab}$$

with

$$\rho = 0, \quad p = \frac{1}{\sqrt{r_c - r_h}}, \quad u^a = \frac{1}{\sqrt{r_c - v^2}}(1, \vec{v_i}).$$



From equilibrium to hydrodynamics

We now wish to consider near-equilibrium configurations.

- We consider the pressure field p and velocities v_i as slowing varying functions of the coordinates.
- We will further consider the limit

$$v_i^{(\epsilon)}(\tau, \vec{x}) = \epsilon v_i(\epsilon^2 \tau, \epsilon \vec{x}), \qquad P^{(\epsilon)}(\tau, \vec{x}) = \epsilon^2 P(\epsilon^2 \tau, \epsilon \vec{x}), \qquad \epsilon \to 0$$

  where P is the pressure fluctuation around the background value p.
- Keeping terms through order ϵ^2, one finds that the resulting metric satisfies Einstein's equations to $O(\epsilon^3)$, provided one imposes

$$\partial_i v^i = O(\epsilon^3).$$

Solution to order ϵ^3

- At next order, one can add a new term, $g_{\mu\nu}^{(n)}$, of order ϵ^3 such that the resulting metric solves Einstein equations through order ϵ^3.
- In order for the metric to be Ricci flat one must impose

$$\partial_\tau v_i + v^j \partial_j v_i - \eta \partial^2 v_i + \partial_i P = O(\epsilon^4),$$

which is precisely the incompressible Navier-Stokes equation!

- Higher-derivative correction terms are then naturally organized according to their scaling with ϵ.

Solution to all orders

A vacuum Einstein solution can be iteratively constructed to arbitrarily high order in ϵ.

- Given a solution at order ϵ^{n-1} we add a new term $g^{(n)}_{\mu\nu}$ at order ϵ^{n}.
- We then solve for $g^{(n)}_{\mu\nu}$ in terms of the metric at lower orders.
- For this to be possible, integrability conditions must be satisfied, which are equivalent to the conservation of the Brown-York stress tensor on Σ_c:

$$\nabla^b T_{ab}\big|_{\Sigma_c} = [2\nabla^b(K\gamma_{ab} - K_{ab})] = 0.$$

Features of solution

- Conservation of the Brown-York stress tensor is equivalent to incompressibility Navier-Stokes plus specific corrections from order ϵ^4:
  - At even orders in ϵ we recover the incompressibility equation plus corrections,

$$\partial_i v_i = \frac{1}{r_c} v_i \partial_i P - v_i \partial^2 v_i + 2\partial_{(i} v_{j)} \partial_i v_j + O(\epsilon^6),$$

  - At odd orders we recover Navier-Stokes plus corrections,

$$\partial_\tau v_i + v_j \partial_j v_i - r_c \partial^2 v_i + \partial_i P = (-\frac{3r_c^2}{2} \partial^4 v_i + 2 r_c v_k \partial^2 \partial_k v_i + \ldots) + O(\epsilon^7).$$

- We impose boundary conditions such that:
  - the metric on Σ_c is preserved;
  - the solution is regular on the future horizon $\mathcal{H}^+$.

The underlying *relativistic* fluid

As the ϵ-expansion is non-relativistic, T_{ab} appears to be non-relativistic. In fact, however, there is an underlying *relativistic* stress tensor which, when expanded out in ϵ, reproduces our above results.

- This is in agreement with the expectation that the dual holographic theory should be relativistic.
- The relativistic stress tensor is much simpler: all information is encoded in only a few transport coefficients. In general,

$$T_{ab} = \rho u_a u_b + p h_{ab} + \Pi^{\perp}_{ab}, \qquad u^a \Pi^{\perp}_{ab} = 0,$$

where $\Pi^{\perp}_{ab}$ represents dissipative corrections and may be expanded in fluid gradients.

Characterizing the dual fluid

- The equilibrium energy density vanishes:

$$\rho = T_{ab} u^a u^b = -\frac{1}{2\sqrt{r_c}} \sigma_{ij}\sigma_{ij} + O(\epsilon^6), \qquad \sigma_{ij} = 2\partial_{(i} v_{j)},$$

  which is zero when v_i is constant, and is otherwise negative!
- The Hamiltonian constraint on Σ_c imposes

$$d T_{ab} T^{ab} = T^2.$$

  determining ρ in terms of p and $\Pi^{\perp}_{ab}$ and giving the analogue to an equation of state.
- Leading order transport coefficients are universal $(\eta/s = 1/4\pi)$ but at higher orders depend on the bulk field equations (e.g. Einstein versus Gauss-Bonnet), as expected.

A model for the dual fluid

- A simple Lagrangian model for the dual fluid is the action:

$$S = \int d^{d+1}x \sqrt{-\gamma}\sqrt{-(\partial\phi)^2},$$

which reproduces the non-dissipative part of the stress tensor,

- The equilibrium configuration with $p = 1/\sqrt{r_c}$ in the rest frame corresponds to taking

$$\phi = \tau,$$

so $v_i \sim \partial_i \phi = 0$. This breaks Lorentz invariance, as does any choice of u_a.

- To model small fluctuations about this background we then set

$$\phi = \tau + \delta\phi(\tau, \vec{x}).$$

and solve for the 3-velocity and pressure fluctuations in terms of $\delta\phi$.

Conclusions

➢ Direct relation between $(d+1)$-dimensional Ricci flat metrics and d-dimensional fluids satisfying the incompressible Navier-Stokes equations, corrected by specific higher-derivative terms.

➢ The dual fluid has vanishing equilibrium energy density but nonzero pressure. There is an underlying relativistic hydrodynamic description.

➢ A simple sqrt Lagrangian captures the non-dissipative properties of the fluid.

Open questions

- Is there a manifestly relativistic construction of the bulk metric? Does the solution exist globally? What if we add matter to the bulk?
- Does the correspondence extend beyond the hydrodynamic regime on the field theory side, and/or the classical gravitational description on the bulk side? Is there a string embedding? Can we get the sqrt action from branes?
- How far can flat space holography be developed? Is there a holographic dictionary relating bulk computations to quantities in the dual field theory on Σ_c?
- By the equivalence principle, our construction should hold locally in any small neighbourhood. Can one patch together such a 'local' holographic description of neighbourhoods to obtain a global holographic description of general spacetimes?