A simple harmonic universe

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The current Standard Model of cosmology beautifully explains known data.

However, the central question of the singularity in the far past is not addressed.

From cosmological singularity theorems [Penrose, Hawking, ...] **we expect that the universe started from a singularity.**

- Is it possible to evade these singularity theorems?
- Can we construct stable nonsingular cosmologies with infinite cycles of expansion and contraction? Quantum fate?

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Our goal: present stable cosmological solutions where

1) the universe is oscillatory with $a_{max} \gg a_{min}$ (bangs and “crunches”)

2) classical GR is always valid (distances $\gg$ Planck length)

3) use ‘ordinary’ matter, satisfying the null energy condition.

I. Evading the singularity theorems

[As far as we know, our analysis and solution are novel. But basic ideas go back to Lemaitre and Tolman. We were also inspired by more recent work by Molina-Paris, Visser, Ellis, Maartens...]

• Main point that we exploit: While for $K = 0, -1$ the singularity theorems require the NEC, for $K = +1$ they require much stronger assumptions (like the SEC), which are violated by many physical systems.

Basic tools: FRW equations assuming matter with $p = w\rho$

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega_2^2\right] \\ \frac{\ddot{a}}{a} &= -\frac{4\pi}{3}G(1+3w)\rho_m + \frac{\Lambda}{3} \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi}{3}G\rho_m - \frac{K}{a^2} + \frac{\Lambda}{3} \\ \rho_m &= \frac{c}{a^{3+3w}} \end{aligned}$$

We want to find min&max values $a_\pm$ from 3-term structure

$$\dot{a}^2 = -K + \frac{8\pi}{3}\frac{Gc}{a^{3w+1}} + \frac{\Lambda}{3}a^2$$

Solution: $K = +1 \ , \ \Lambda < 0 \ , \ -1 < w < -\frac{1}{3} \ .$

Thus it seems we can evade the inflationary singularity theorems using ordinary and well-understood sources!

- a_- produced by K and ρ, while a_+ produced by Λ and ρ
- sols w/oscillatory behavior and nonsingular bounces and crunches

$$M_{Pl}^{-1} \ll a_- \ll a_+$$

Let's study this in more detail for the simplest solution $w = -2/3$.

II. A simple harmonic universe (SHU)

When $w = -2/3$ the FRW eqs. describe a harmonic oscillator:

$$\ddot{a} + \frac{|\Lambda|}{3} a = \frac{4\pi}{3} Gc$$

$$\begin{aligned} \Rightarrow \quad a(t) &= \frac{1}{\sqrt{\gamma}\omega} \left(1 + \sqrt{1-\gamma}\cos(\omega t)\right) \\ \omega &\equiv \sqrt{\frac{|\Lambda|}{3}}, \; \gamma \equiv \frac{3|\Lambda|}{(4\pi Gc)^2} \end{aligned}$$

$\surd$ Pheno interesting limit: $\gamma \ll 1 \quad \Rightarrow \quad \frac{a_-}{a_+} \approx \gamma$

- **Stability of the universe?**

The SHU is stable under homogeneous perturbations, unlike the Einstein static universe.

We also have to study inhomogeneous perturbations, changing the density and metric.

If matter were really a perfect fluid, since we needed negative pressure ($w < -1/3$), sound wave perturbations would have an imaginary speed! $\rightsquigarrow$ Disastrous high momentum instabilities!

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$\star$ **Solution:** consider a matter source that behaves like a fluid with $w < -1/3$ at long distances, but under inhomogeneous perturbations it behaves like a solid.

[Bucher, Spergel: Solid dark matter]

This adds extra elastic resistance to deformations, producing a positive speed of sound and stabilizing the inhomogeneous modes.

Concrete example: frustrated network of domain walls, with $w = -2/3$, $c_s^2 > 0$.

III. Classical and quantum dynamics

Dynamics of massless scalar (e.g. graviton) in the SHU:

$$S = \int d^4x\, a^3(t) \left((\partial_t \phi)^2 - \frac{1}{a^2(t)} (\nabla_{S^3} \phi)^2 \right)$$

Normalizing $\phi = a^{-3/2}\chi$ and expanding in spherical harmonics,

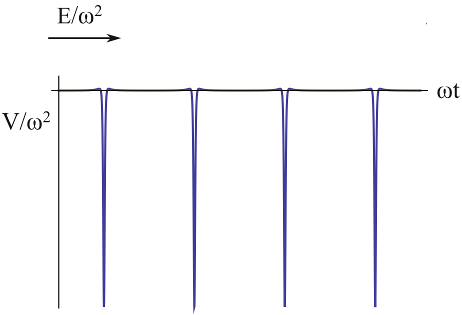
$$-\ddot{\chi}_k + \left(\frac{3}{2}\frac{\ddot{a}}{a} + \frac{3}{4}\frac{\dot{a}^2}{a^2} - \gamma\omega^2 \frac{k(k+2)}{a^2} \right) \chi_k = 0$$

- at high momenta $k \gg 1/\sqrt{\gamma}$ $\rightsquigarrow$ flat space modes
- we then focus on $k \ll 1/\sqrt{\gamma}$

⇒ Schrödinger problem for a particle in a periodic δ-fc potential

$$V \approx -\omega^2 \frac{k(k+2)}{\sqrt{\gamma}} \sum_n \delta(\omega t - 2\pi n)$$

$$E \approx \frac{\omega^2}{4}\left[3 + k(k+2)\gamma\right]$$



Dynamics of particles in the harmonic universe with $\gamma \ll 1$
⇔ electrons in a Bloch potential

$\surd$ Instead of periodic b.c., we want to give initial conditions for χ

$\surd$ Set of decoupled harmonic oscillators in each cycle

$\surd$ Full sol: impose continuity and correct jump in first derivative

$$\text{e.g.} \qquad \chi^{(N)}(t) = \chi_0 \, e^{i\sqrt{E}\,t} \left[1 + const \times k(k+2) \frac{N}{\sqrt{\gamma}} \right]$$

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- Classically, perturbations after N cycles are small if

$$\frac{|\chi_0|}{M_{Pl}} \ll \frac{\gamma^{5/4}}{N}$$

- Quantum-mechanically, $\chi_0 \sim \gamma^{3/4}\omega$. Particle production important when

$$N_* \sim \gamma^{1/2}\,\frac{M_{Pl}}{\omega}$$

Large number of bounces for small $|\Lambda|/M_{Pl}^2$.

IV. Conclusions

- We presented new GR solutions of cosmologies that cycle through an infinite set of bounces
- The solutions have $K = +1$, negative c.c. and matter with $-1 < w < -1/3$
- The bounces are nonsingular, and the curvature is always small
- QM: particle production eventually becomes important, and a more detailed analysis is needed (singular crunch?)
- Next step, combine our solution with other components. Inflaton, radiation/matter domination, ...