Tunneling in The Landscape : an Experimental Test

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Cosmic Landscape

- String theory suggests a very rich complicated landscape for the wavefunction of our universe
- Tunneling among multiple local minima should be studied.
- Eternal Inflation and Anthropic Principle ?

Motivation

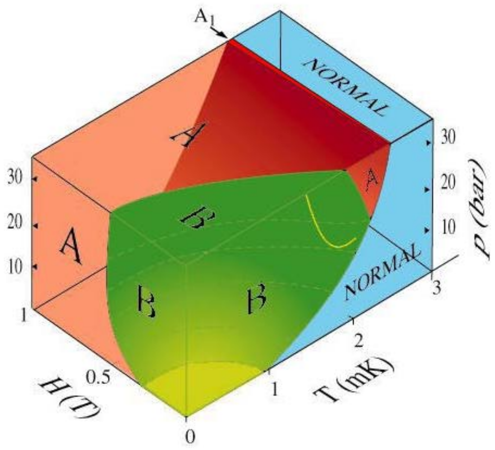
- It will be very useful to find a laboratory system that has a complicated effective potential to mimic the cosmic landscape and study its properties.
- In particular, tunneling among the multiple minima in such a system should be studied.
- Even better if there are parameters that we can tune to vary the properties of the system.
- He-3 superfluid is a good toy model.

He-3 Superfluid

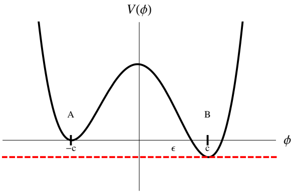
He-3 superfluid is one of the best studied condensed matter systems, both in theory and in experiments.

- Pairing of 2 He-3 atoms is in the *p*-wave and Spin 1 state, so the order parameter (Higgs field) in the Ginsburg-Landau theory is a 3×3 complex matrix.
- Many possible phases :
  Normal, A_1, planar, polar, α, β, A, B, ...
- Observed : normal, A_1, A and B phases.
- defects, like boojums ..., observed.
- A and B are degenerate. Their degeneracies are largely lifted by container surface effect, magnetic field and other higher order corrections.

Phase Diagram



Tunneling from $A \to B$



- ϕ is the interpolating field between the A phase and the B phase of He-3 superfluid.
- Because of its superfluidity properties, no impurities in the sample.
- Tunneling probability (in thin wall approx.) $\Gamma \sim e^{-S_E}$ where $S_E = \frac{27\pi^2}{2}\frac{\tau^4}{\epsilon^3}$, where τ is the domain wall tension.

Tunneling Rate

- $\Gamma \sim e^{-S_E}$ where $S_E = \frac{27\pi^2}{2}\frac{\tau^4}{\epsilon^3}$
- One can calculate as well as measure the domain wall tension τ and the free energy density difference ϵ between the A phase and the B phase. Theory and measurements are in excellent agreement. The thin wall approximation is very good. [1]

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[2] Bailin and Love

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- Optimistic $A \to B$ transition time due to quantum tunneling is $\sim 10^{20,000}$ years (at $T = 0^o$K) [2].
- At $T \sim 0.7T_{AB}$ in experiments, decay time $> 10^{720,000}$ years.

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- Because of the superfluidity property, there are no impurities in the sample to provide seeds of bubble nucleation; so, according to calculation, the $A \to B$ transition should never have happened.

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- But the A to B transition typically occurs in seconds, minutes or at most hours.
- Container wall effect actually stabilizes the A phase so transition happens inside the sample.

Outline

Why ?

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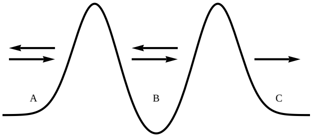
- We'll suggest some simple experiments to be carried out in the He-3 superfluid to test our proposal.

Double Barrier



- Tunneling rate for single-barrier tunneling is $\Gamma_{A\to B} = Ce^{-S}$
- Tunneling probability for single-barrier tunneling is $P_{A\to B} = Ke^{-S} \approx e^{-S}$.
- Suppose $P_{B\to C} \approx e^{-S}$ also. What is $P_{A\to C}$?

A Simple Question

Probability $P_{A\to B} \approx \Gamma_{A\to B} \approx e^{-S}$ and time $t_{A\to B} \approx e^{S}$

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Which is correct ? $P_{A\to C} \approx e^{-2S}$ or $\approx e^{-S}$?

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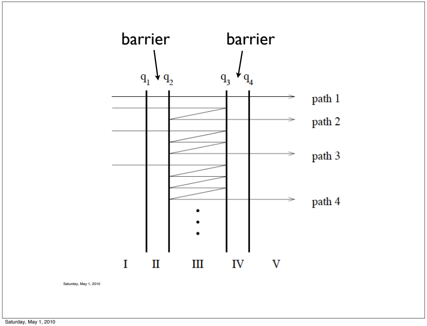
Answer :

$$P_{A\to C} \approx e^{-S}$$

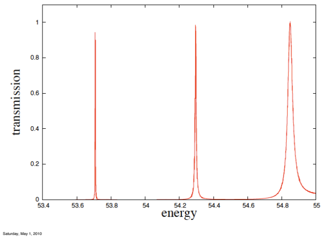
Coherent Sum of Paths

Barriers at $q_1 \to q_2$ and $q_3 \to q_4$



- Comercialized : resonant tunneling diodes

Tunneling Probability versus Energy

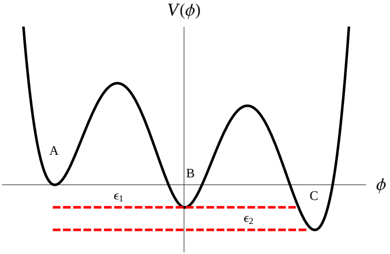


- $P_{A \to C} \sim 1$ at resonance energies.
- Away from resonance energies, $P_{A \to C} \sim P_{A \to B} P_{B \to C} \sim e^{-2S}$.
- On average, $P_{A \to C} = P_{A \to B} P_{B \to C} / (P_{A \to B} + P_{B \to C}) \sim e^{-S}$

Double-Barrier Tunneling

- Same method of analysis
- $P_{A\to C} = 4\left(\left(\Theta\Phi + \frac{1}{\Theta\Phi}\right)^2 \cos^2 W + \left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)^2 \sin^2 W\right)^{-1}$
- $\Theta \simeq 2\exp\left(\frac{1}{\hbar}\int_{x_1}^{x_2} dx \sqrt{2m(V(x)-E)}\right)$
- $\Phi \simeq 2\exp\left(\frac{1}{\hbar}\int_{x_3}^{x_4} dx \sqrt{2m\,(V(x)-E)}\right)$
- $W = \frac{1}{\hbar}\int_{x_2}^{x_3} dx \sqrt{2m(E-V(x))}$
- If B has zero width, $P_{A\to C} \simeq 4\Theta^{-2}\Phi^{-2} = P_{A\to B}P_{B\to C}/4$
- If $W = (n_B + 1/2)\pi$, then $P_{A\to C} = \frac{4}{(\Theta/\Phi+\Phi/\Theta)^2}$. If $\Theta = \Phi$, $P_{A\to C} = 1$

QFT ?



- $t_{A \to C} = t_{A \to B} + t_{B \to C}$ implies that resonant tunneling have to take place in QFT as well.

Functional Schrödinger Method

Basic Idea

Scalar QFT $\rightarrow$ one-dimensional QM problem

- In semiclassical limit, the vacuum tunneling rate is dominated by a discrete set of classical paths [3]
- Equivalent to Euclidean instanton method for single-barrier tunneling
- Easily generalizes to multiple-barrier tunneling

[3] Bender, Banks, Wu ; Gervais and Sakita; Bitar and Chang

Functional Schrödinger Equation

- $H = \int d^3x \left(\frac{\dot{\phi}^2}{2} + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right)$
- Quantize using $[\dot{\phi}(x), \phi(x')] = i\hbar\delta^3(x - x')$
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- Make ansatz $\Psi(\phi) = A \exp(-\frac{i}{\hbar} S(\phi))$
- $H\Psi(\phi(x)) = E\Psi(\phi(x))$

Semiclassical Expansion

- $S(\phi) = S_{(0)}(\phi) + \hbar S_{(1)}(\phi) + \ldots$
- $\int d^3x \left[\frac{1}{2} \left(\frac{\delta S_{(0)}(\phi)}{\delta \phi} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] = E$
- $\int d^3x \left[-i \frac{\delta^2 S_{(0)}(\phi)}{\delta \phi^2} + 2 \frac{\delta S_{(0)}(\phi)}{\delta \phi} \frac{\delta S_{(1)}(\phi)}{\delta \phi} \right] = 0$
- Solve $S_{(0)}$, then $S_{(1)}$, and so on.
- Here, we ignore higher-order terms.

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Goal

Determine value of $S_{(0)}$ that gives dominant contribution to the tunneling probability.

Approach

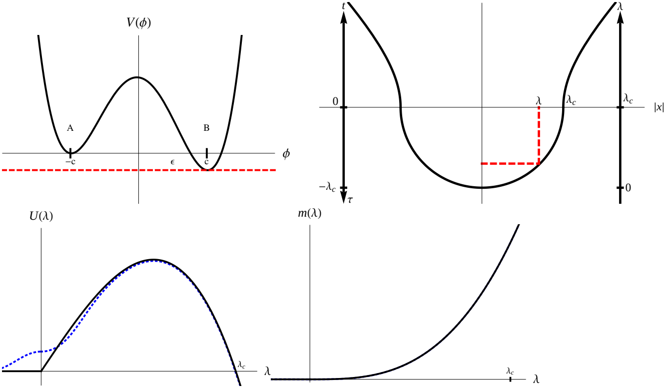
Reduction

Choose MPEP or ansatz : $\phi_0(x,\lambda)$

- Position-dependent mass $m(\lambda) \equiv \int d^3x \left(\frac{\partial\phi_0(x,\lambda)}{\partial\lambda}\right)^2$
- Effective tunneling potential $U(\lambda)$:
  $U(\lambda) = \int d^3x \left(\frac{1}{2}(\nabla\phi_0(x,\lambda))^2 + V(\phi_0(x,\lambda))\right)$
- Now have a one-dimensional time-independent QM problem, with potential $V(\lambda) = m(\lambda)U(\lambda)$ and $E = 0$:

$$\left(-\frac{\hbar^2}{2}\frac{d^2}{d\lambda^2} + m(\lambda)U(\lambda)\right)\Psi_0(\lambda) = 0$$

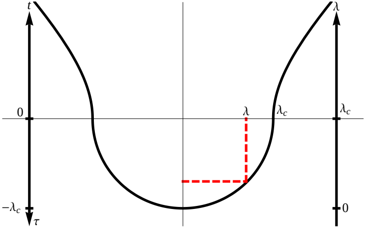
Single Barrier Case

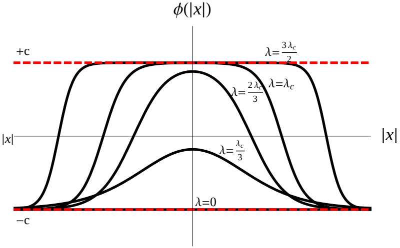


Advantages of Functional Schrödinger Method

- Same arguments lead to $S_{(0)}(\phi(x,\lambda)) = \int d\lambda \sqrt{2m((\phi(x,\lambda))[-U((\phi(x,\lambda))]}$ and Lorentzian EOM in classically allowed regions
- If we choose $\lambda = \sqrt{\lambda_c^2 + t^2}$ as parameter, MPEP takes form $\phi_0(x,\lambda) = -c\tanh\left(\frac{\mu}{2}(|x| - \lambda)\frac{\lambda}{\lambda_c}\right) = -c\tanh\left(\frac{\mu}{2}\frac{(|x|-\lambda)}{\sqrt{1-\dot{\lambda}^2}}\right)$
- Single real parameter λ describes entire system in both the classically allowed and the classically forbidden regions.

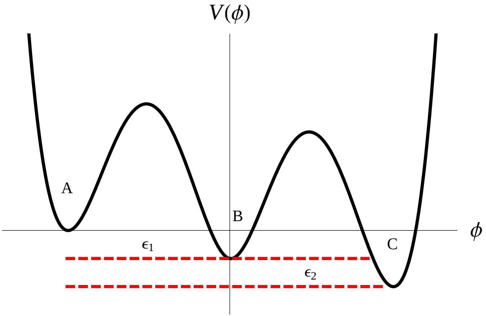
Advantages of Functional Schrödinger Method II





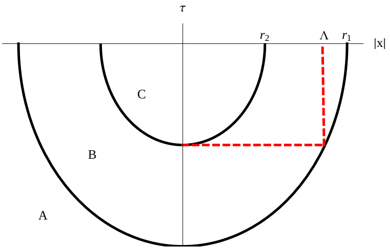
- $\lambda = \sqrt{\lambda_c^2 - \tau^2} \leq \lambda_c$ for $\tau \leq 0$ in classically forbidden region.
- $\lambda = \sqrt{\lambda_c^2 + t^2} \geq \lambda_c$ for $t \geq 0$ in classically allowed region.
- Single real parameter λ describes both.

Effective Potential with Double Barriers



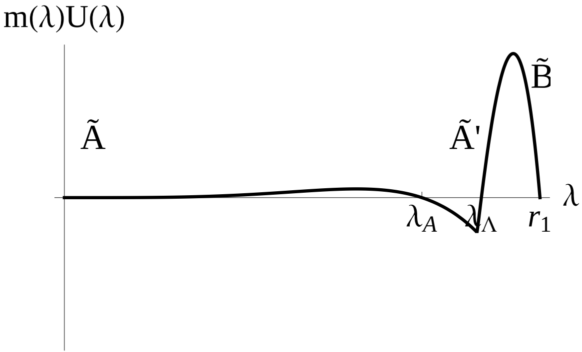
- $$V(\phi) = \begin{cases} \frac{1}{4}g_1((\phi + c_1)^2 - c_1^2)^2 - B_1\phi - 2B_1c_1 & \phi < 0 \\ \frac{1}{4}g_2((\phi - c_2)^2 - c_2^2)^2 - B_2\phi - 2B_1c_1 & \phi > 0 \end{cases}$$

MPEP



- $\phi_0(|x|,\lambda) = -c_1 \tanh\left(\frac{\mu_1}{2}\frac{\lambda}{r_1}(|x|-\lambda)\right) - \Theta\left(\frac{\lambda}{\Lambda} - 1\right) c_2 \tanh\left(\frac{\mu_2}{2}\frac{\lambda'}{r_2}(|x|-\lambda')\right) + c_2 - c_1$ solves both Euclidean and Lorentzian equation of motion

Effective Tunneling Potential



Consistency Conditions

- We require zero total energy at nucleation
- $\mathcal{E}_{(2)} = 4\pi(S_1^{(1)} - \frac{1}{3}r_1\epsilon_1)r_1^2 + 4\pi(S_1^{(2)} - \frac{1}{3}r_2\epsilon_2)r_2^2 = 0.$
- We do not demand that the energy of each bubble vanishes individually
- Equivalent to condition that action is stationary.
- Must also ensure existence of a classically allowed region $U(\lambda) < 0$ for $\Lambda > \lambda > \lambda_B$.
- Also require the existence of a second classically forbidden region.

Resonant Tunneling or Catalyzed Tunneling

Resonant Tunneling

If the inside bubble is large enough $\lambda_{2c} > r_2 > 2\lambda_{2c}/3$ tunneling from A to C will complete.

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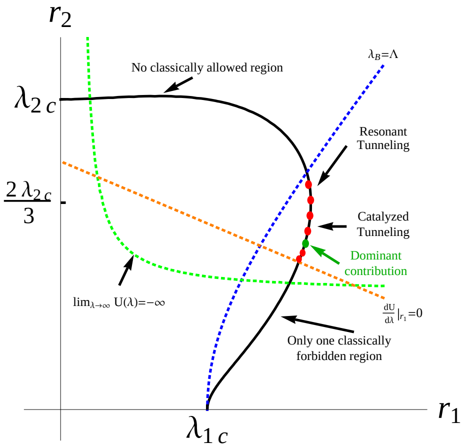
Catalyzed Tunneling

- If the inside bubble is too small $0 < r_2 < 2\lambda_{2c}/3$, inside bubble will collapse after nucleation
- Effect will be tunneling from A to B
- Tunneling rate is exponentially enhanced by presence of C

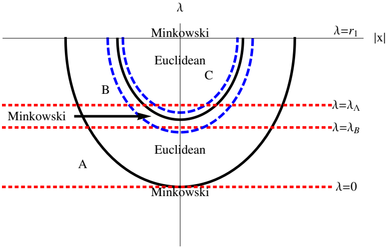
Consistency Conditions



Resonant Tunneling in QFT

- $P_{A\to C} = 4\left(\left(\Theta\Phi + \frac{1}{\Theta\Phi}\right)^2 \cos^2 W + \left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)^2 \sin^2 W\right)^{-1}$
- $W = \int_{\lambda_2}^{\lambda_3} d\lambda \sqrt{2m(\lambda)(-U(\lambda))}$
- $W = \frac{S_1^{(1)}\lambda_\Lambda}{\lambda_B}\sqrt{\lambda_\Lambda^2 - \lambda_B^2} - S_1^{(1)}\lambda_B \log\left[\frac{\lambda_\Lambda + \sqrt{\lambda_\Lambda^2 - \lambda_B^2}}{\lambda_B}\right]$
- Resonance condition is $W = (n + \frac{1}{2})\pi$.
- $P_{A\to C}$ is smaller of $P_{A\to B}/P_{B\to C}$, $P_{B\to C}/P_{A\to B}$. So preferred choice is when $\Gamma_{A\to B}$ is closest to $\Gamma_{B\to C}$.
- The figure uses $S_1^{(1)} = 1$ and $S_1^{(1)} = 5$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.3$, $P_{A\to B} = e^{-10^5} \to e^{-10^4}$

Euclidean vs Minkowski Description



This is why it is difficult to extend Coleman's approach to resonant tunneling case.

Probability of Hitting Resonance

- Treat tunneling probability as function of λ_Λ
- Expand around resonance at $\lambda_\Lambda = \lambda_R$ of width Γ_{λ_Λ}
- $\Gamma_{\lambda_\Lambda} = \frac{2}{\Theta\Phi(\frac{\partial W}{\partial \lambda_\Lambda})}\left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)$

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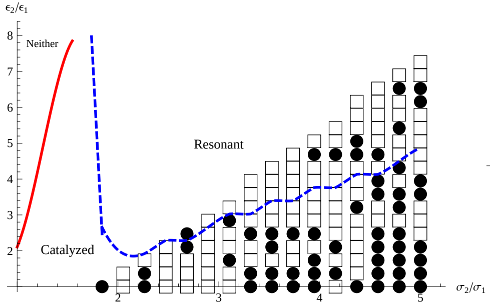
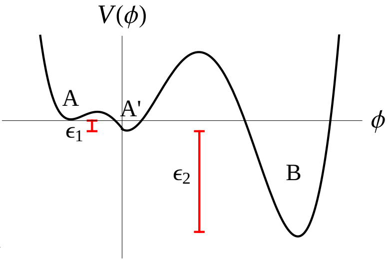
- $\Gamma_{\lambda_\Lambda} = \frac{2}{\Theta\Phi(\frac{\partial W}{\partial \lambda_\Lambda})}\left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)$
- Separation between resonances $\Delta\lambda \simeq \frac{\pi}{(\frac{\partial W}{\partial \lambda_\Lambda})}$
- Probability of hitting resonance $p(A \to C) = \frac{\Gamma_\Lambda}{\Delta\Lambda} \simeq \frac{2}{\pi\Theta\Phi} = \frac{1}{2\pi}\left(P_{A\to B} + P_{B\to C}\right)$ is the larger of two decay probabilities
- Average tunneling probability $< P_{A\to C} >= p(A \to C) P_{A\to C} \sim \frac{P_{A\to B}P_{B\to C}}{P_{A\to B}+P_{B\to C}}$ given by smaller of two tunneling probabilities

Enhancement of Tunneling

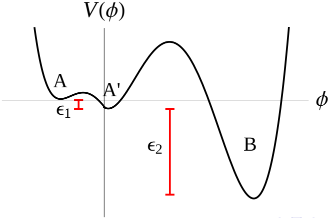




Resonant Tunneling in He-3 ?

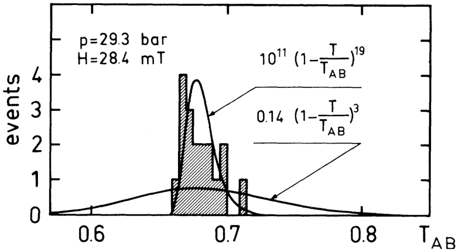
Starting with the initial A^i sub-phase, the tunneling goes via $A^i \to A^j \to B^k$, where A^j is a lower sub-phase of the A phase andB^k is a B sub-phase.

- A^j and B^k are chosen by nature to offer the best chance for the resonant tunneling phenomenon.
- Free energy difference between the A sub-phases are small and may be spatially varying.



$A \to B$ Transition

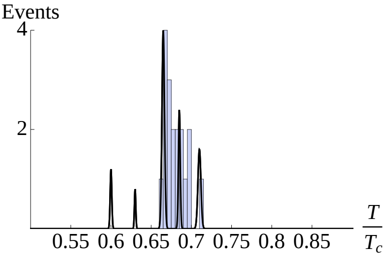
Hakonen etc. PRL 45, 245, 1985



- Data for He-3 as the temperature is slowly lowered.
- No transition until $T \sim 0.7 T_{AB}$.
- No transition when sit at the gap.

Prediction

- Resonant tunneling phenomenon happens only under some fine-tuned conditions, say, at certain values of the temperature, pressure and magnetic field. Away from these resonant peaks, the transition simply will not happen.
- Cooling may provide the tuning needed to satisfy the resonant conditions



Summary

- One uses the functional Schrödinger method to show how resonant tunneling takes place in scalar QFT.
- This phenomenon can be tested by simple experiments in He-3 superfluid. If true, this solves an outstanding puzzle in condensed matter physics since the early 1970s.
- The rich complicated potential for the He-3 order parameter mimics the cosmic landscape. If confirmed in He-3, tunneling in the cosmic landscape should be much more efficient than naively expected.