Warped de Sitter compactifications

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[1] Introduction

・De Sitter compactification of higher-dimensional theory has been explored from several points of view because this provides a fairly direct explanation of the accelerating expansion of four-dimensional universe.

・In the gravity theory, an initial clue of the de Sitter compactification was that the hyperbolic space associated to a higher-dimensional theory can be regarded as internal space.

・The warped de Sitter compactification shed light on whether there was the exact solution of field equations because the solution such as D-branes or M-branes in the supergravity can be embedded in warped compactification.

・We will discuss a new solution of warped compactification including the bulk matter. In these solutions, the spacetime is given by the warped product of the infinite volume extra dimension, the internal spherical direction and the four-dimensional de Sitter spacetime.

・As the bulk volume diverges due to the presence of the infinite volume direction, we construct a codimension-one braneworld, by cutting the spacetime at a certain place of it, and gluing to its copy at the same position imposing the Z_2-symmetry.

・We will find that the junction condition requires the positive brane tension. Hence, this solution is not a counterexample of the NO-GO theorem.

[2] Compactifications with field strength

◇ D-dimensional action ：

$$S = \frac{1}{2\kappa^2}\int\left[\left\{R - 2e^{-\alpha\phi/(p-1)}\Lambda\right\} * \mathbf{1} - \frac{1}{2}d\phi \wedge * d\phi - \frac{1}{2\cdot p!}e^{\alpha\phi}F \wedge * F\right]$$

R : Ricci scalar, ϕ : scalar field, F : p-form, α : constant
κ : gravitational constant, Λ : cosmological constant,

◆ Ansatz for fields :

n-dim universe

$$ds^2 = e^{2A(y)}\left[q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + dy^2 + \gamma_{ab}(\mathrm{Z})dz^a dz^b\right]$$

Internal space

$$\phi = \frac{2}{\alpha}(p-1)A(y)\,, \qquad F = f\left(\sqrt{\gamma}\,dz^1 \wedge \cdots \wedge dz^p\right)$$

constant

★ Solution :

dS$_n$

$$ds^2 = e^{2\ell(y-y_0)}\left[-dt^2 + e^{2Ht}\delta_{\rho\sigma}dx^\rho dx^\sigma + dy^2 + \gamma_{ab}(Z)dz^a dz^b\right]$$

$$\ell = \pm\alpha\sqrt{\frac{1}{(p-1)(D-2)}\left(-\frac{\Lambda}{p-1}+\frac{f^2}{4}\right)}, \qquad H^2 = \frac{1}{n-1}\left(\frac{\alpha^2}{p-1}-\frac{2(p-1)}{D-2}\right)\left(-\frac{\Lambda}{p-1}+\frac{f^2}{4}\right)$$

$$R_{ab}(Z) - \left[\beta\left(-\frac{\Lambda}{p-1}+\frac{f^2}{4}\right)+\frac{f^2}{2}\right]\gamma_{ab}(Z) = 0\,, \qquad \beta = \frac{\alpha^2}{p-1}-\frac{2(p-1)}{D-2}$$

・As the internal space is essentially supported by the field strength, we can keep the geometrical property of the internal space in the limit H → 0.

・If the constant field strength F has the components along our four-dimensional spacetime, the Ricci tensor on 4d is proportional to the four-dimensional metric with negative sign that is no longer de Sitter spacetime.

[3]. Lower-dimensional effective theory

Ansatz for fields

(n+1) dimension

compactification of (D-n-1)-dim

$$ds^2 = \mathrm{e}^{2A(v)}\left[q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + dy^2 + \mathrm{e}^{2\psi(v)}\gamma_{ab}(\mathrm{Z})dz^a dz^b\right],$$

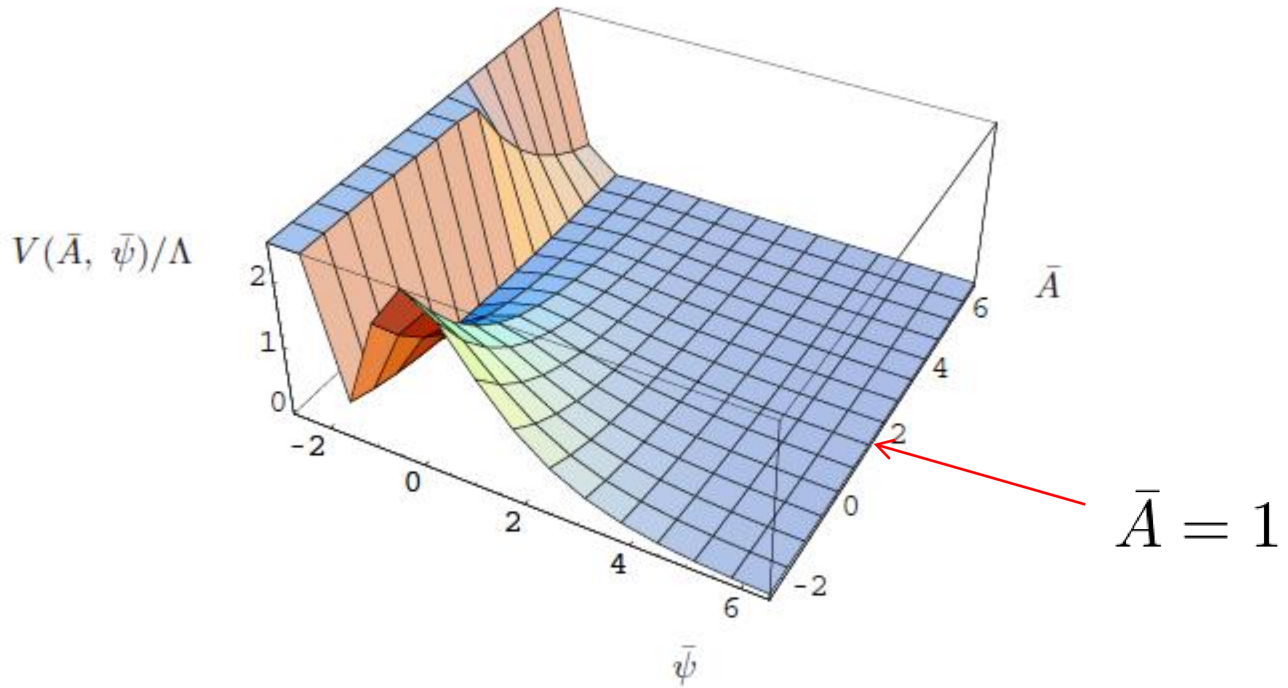
$$q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + dy^2 \equiv \mathrm{e}^{-[(D-2)A+p\psi]/(n-1)}w_{ij}(\bar{\mathrm{M}})dv^i dv^j,$$

$$\phi = \frac{2}{\alpha}(p-1)A(y), \qquad F = f\left(\sqrt{\gamma}\,dz^1 \wedge \cdots \wedge dz^p\right)$$

(n+1)-dimensional effective action

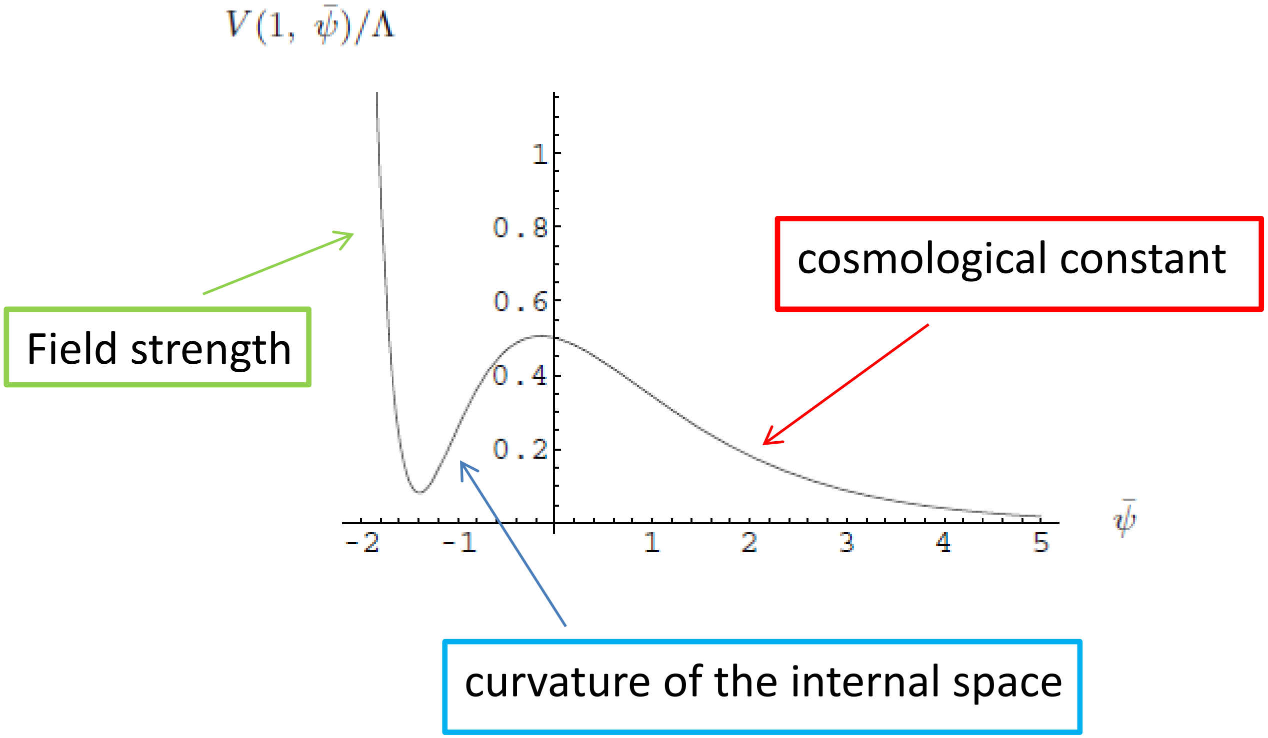
$$S = \frac{1}{2\tilde{\kappa}^2}\int_{\bar{\mathrm{M}}}\left[\left\{R(\bar{\mathrm{M}}) - V\left(\bar{A},\ \bar{\psi}\right)\right\} *_{\bar{\mathrm{M}}} \mathbf{1}_{\bar{\mathrm{M}}} - \frac{1}{2}d\bar{A}\wedge *_{\bar{\mathrm{M}}}d\bar{A} - \frac{1}{2}\frac{c_2}{\sqrt{c_1c_3}}d\bar{A}\wedge *_{\bar{\mathrm{M}}}d\bar{\psi} - \frac{1}{2}d\bar{\psi}\wedge *_{\bar{\mathrm{M}}}d\bar{\psi}\right],$$

$$
\begin{aligned}
V\left(\bar{A},\bar{\psi}\right) &= \exp\left[-\frac{2(D-2)\bar{A}}{(n-1)\sqrt{c_1}}\right]\left[2\Lambda\exp\left\{-\frac{2p\bar{\psi}}{(n-1)\sqrt{c_3}}\right\}\right. \\
&\left. +\frac{f^2}{2}\exp\left\{-\frac{2np\bar{\psi}}{(n-1)\sqrt{c_3}}\right\}-p\lambda\exp\left\{-\frac{2(D-2)\bar{\psi}}{(n-1)\sqrt{c_3}}\right\}\right]
\end{aligned}
$$

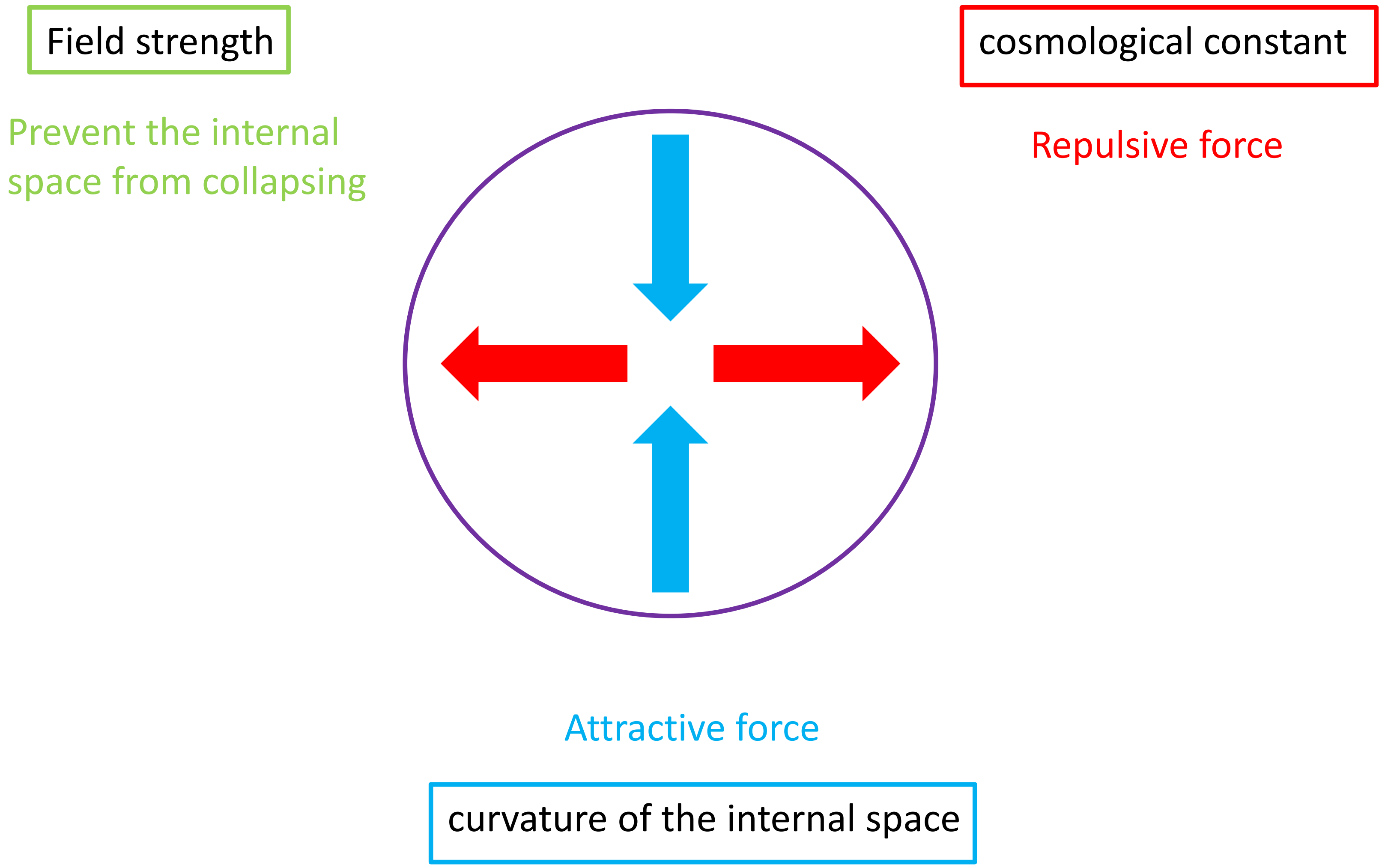


- Set n=4, p=5, f=0.17, and α=0.5 in the unit of Λ=1.

▪Set $n = 4,\ p = 5,\ \bar{A} = 1,\ \Lambda = 1,\ f = 0.17,\ \alpha = 0.5$



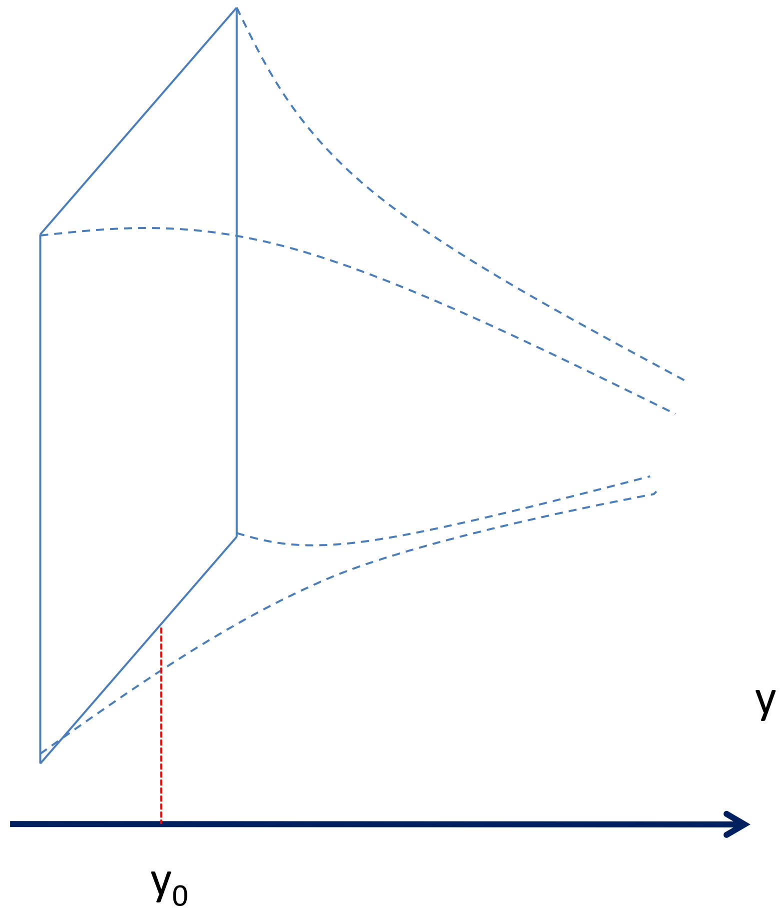
Stabilization of moduli: Balancing between field strength, curvature of the internal space and cosmological constant.



Field strength
Prevent the internal space from collapsing
cosmological constant
Repulsive force
Attractive force
curvature of the internal space

[4]. Braneworld

- We construct a codimension-one braneworld, by cutting the spacetime at a certain place of it, and gluing to its copy at the same position imposing the Z_2-symmetry.



- The extrinsic curvature is discontinuous across the boundary.

  ↓

  Israel junction condition.

・The difference from the five-dimensional case is that the braneworld involves the spherical dimensions as well as the de Sitter spacetime. We compactify the spherical dimensions to obtain the four-dimensional cosmology.

・The extrinsic curvature is discontinuous across the boundary and its jump is determined by the Israel junction condition.

・We will find that the junction condition requires the positive brane tension. The insertion of the braneworld is hence equivalent to add a positive potential energy to the effective theory.

▪Induced metric on the brane

$$ds^2_{\rm ind} = \underbrace{-dt^2 + e^{2Ht}\delta_{ij}dx^i dx^j}_{\rm dS_4} + \gamma_{ab}(Z)dz^a dz^b$$

▪The junction conditions at the brane boundary:

$$\kappa_D^2 \sigma_f = 2H\sqrt{3\left(D-2+\frac{D-6}{\zeta}\right)}, \qquad \zeta = \frac{\alpha^2}{2(D-6)} - \frac{D-6}{D-2}$$

κ_D^2: gravitational constant σ_f: brane tension

▪The junction condition gives the positive brane tension. Therefore, these solutions are not the counterexample of the NO-GO theorem.

★NO-GO theorem of de Sitter compactification :

(J.M.Maldacena & C.Nunez,Int.J.Mod.Phys.A16:822-855,2001.[arXiv:hep-th/0007018])

D-dimensional spacetime

$$ds^2 = A^2(y)\left[\underbrace{q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu}_{d_1\text{-dimensional spacetime}} + \underbrace{u_{ij}(\mathrm{Y})dy^i dy^j}_{(D-d_1)\text{-dimensional internal space}}\right]$$

- The potential is non-positive.
- massless fields with positive kinetic terms
- No higher curvature correction in the background
- d_1-dimensional effective Newton constant is finite.

⇒ There is no non-singular warped compactification to dS.

[5] Summary :

・ For the compactification with field strength, we have simply required that non-vanishing components of the field strength should be along the internal space.

・To make the volume of extra dimensions finite, we have inserted the braneworld boundary. The junction condition gives the positive brane tension. These solutions are not the counterexample of the NO-GO theorem.

・The scale of internal space is stabilized by balancing the gauge field strength wrapped around the internal space, the curvature term of the internal space and the cosmological constant.

・As the moduli potential without the cosmological constant leads to the AdS_{n+1} at the local minimum even if we have a field strength, we need the contribution of the cosmological constant to obtain de Sitter compactifications.