Precise Form Factor Measurement of $K_{\mu 3}^\pm$ decays at NA48/2

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On behalf of the NA48/2 Collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien
• Physics motivations

• The NA48/2 experiment and detector

• $K_{\mu 3}^\pm$ analysis
  • Event selection
  • Fitting procedure
  • Preliminary results

• $K_{e 3}^\pm$ Analysis

• Summary
Form Factors: Physics Motivations

- Kaon semi–leptonic decays are due to $s \rightarrow u$ transitions
  \[ K^\pm \rightarrow \pi^0 \ell^\pm \nu_\ell \quad K^0 \rightarrow \pi^{\pm} \ell^{\mp} \nu_\ell \quad (\ell = \mu, \ e) \]

- These decays are known to be the “golden” modes to extract $|V_{us}|$ and one of the main playgrounds for low–energy theories such as ChPT

  \[ \Rightarrow \text{Precision frontier in CKM studies} \]

- $|V_{us}|$ and $|V_{ud}|$ are by far the most precisely known CKM matrix elements
  \[ \sigma_{|V_{ud}|}/|V_{ud}| \simeq 0.02\% \quad \sigma_{|V_{us}|}/|V_{us}| \simeq 0.5\% \]

- Unitarity test in the first CKM row
  \[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006 \]
  Error budget: $0.0004V_{ud} \oplus 0.0004V_{us}$

- Outstanding agreement with unitarity

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The form factors appear because the $\pi$ and the $K$ are not pointlike particles.
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Include the effects of the strong force using two form factors $f_+$ and $f_-$

$$\mathcal{M} = G_F/\sqrt{2} \, V_{us} \left[ f_+(t) (P_K + P_\pi)^\mu \bar{u}_\ell \gamma^\mu (1 + \gamma_5) u_\nu + f_-(t) m_\ell \bar{u}_\ell (1 + \gamma_5) u_\nu \right]$$

t is the square of the four–momentum transfer to the lepton system

$$f_-(t) \Rightarrow m_\ell^2 / m_K^2$$

Can be measured only in $K_{\mu3}$ decays

Instead of $f_-$ usually a combination of the two, $f_0(t)$, is used:

$$f_0(t) = f_+(t) + \frac{t}{(m_K^2 - m_\pi^2)} f_-(t) \quad f_0(0) = f_+(0) \text{ by construction}$$

$f_+(0)$ is not directly measurable $\Rightarrow$ Theory input

$\Rightarrow$ factor out $f_+^{K^0 \pi^-}(0)$ and normalize the ff of all channels: $\bar{f}_{+0}(t) = \frac{f_{+0}(t)}{f_+(0)}$
**$K_{\ell 3}$ Form Factors and $|V_{us}|$ Determination**

$K_{\ell 3}$ decays $\Rightarrow$ most accurate and theoretically clean way to access $|V_{us}|$

The master formula for $K_{\ell 3}$ decay rates:

$$
\Gamma_{K\ell 3(\gamma)} = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} \ S_{EW} \ |V_{us}|^2 \ |f_+(0)|^2 \ I_K^\ell (\lambda_+0) \ (1 + \delta_{SU(2)}^\ell + \delta_{EM}^\ell)^2
$$

$k = k^0, k^\pm; \ c_K^2 = 1 \ c_K^2 = 1/2$

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**Experimental Inputs**

- $\Gamma(K_{\ell 3(\gamma)})$ Branching Ratios
- Kaon lifetimes
- $I_K^\ell [f^+_{+,0}(t)]$ Phase space integral
- Depends on ff

**Theory Inputs**

- $S_{EW}$ Universal short distance
- EW correction (1.0232±0.0003)
- $f_+(0)$ Calculated ff at t=0
- $2^{nd}$ order SU(3)
- $\delta_{SU(2)}^K$ Form factor correction for
- isospin breaking ($K^\pm$ only)
  $$f^K_{+,\pi^0}(0)/f^K_{+,\pi^-}(0) - 1 = 0.029 \pm 0.0004$$
- $\delta_{EM}^K$ Long distance EM effects
  $$\delta_{EM}^K \approx 0 \text{ for } K^\pm$$

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Class I: Make use of physical constraints - Only one parameter required

\[ \bar{f}_{+,0}(t) = \frac{m_{V,S}^2}{m_{V,S}^2 - t} \] \quad \text{POLE}

Exchange of \( K^* \) resonances with spin–parity \( 1^-/0^+ \) and mass \( m_V/m_S \)

For \( f_+ \) dominance of \( K^*(892) \), no obvious dominance for \( f_0 \)

\[ \bar{f}_+(t) = \exp \left[ \frac{t}{m_\pi^2} (\Lambda_+ + H(t)) \right] \] \quad \text{DISPERSEIVE}

\[ \bar{f}_0(t) = \exp \left[ \frac{t}{\Delta_{K\pi}} (\ln C - G(t)) \right] \]

This parametrization is based on a dispersive approach with a relation subtracted twice \( (t = 0, \ t = \Delta_{K\pi}) \) [PLB 638(2006) 480, PRD 80(2009) 034034]

Accurate polynomial approximations for the dispersive integrals \( G(t) \) and \( H(t) \)
Class II: No Physics input - Power series expansion

Well known and widely used are the linear and quadratic:

\[
\tilde{f}_{+,0}(t) = \left( 1 + \lambda_{+,0} \frac{t}{m_{\pi}^2} \right)
\]

\text{LINEAR}

\[
\tilde{f}_{+,0}(t) = \left[ 1 + \lambda_{+,0}’ \frac{t}{m_{\pi}^2} + \frac{1}{2} \lambda_{+,0}'' \left( \frac{t}{m_{\pi}^2} \right)^2 \right]
\]

\text{QUADRATIC}

- More parameters to be determined by fit \(\Rightarrow\) Correlations
- Not possible to determine \(\lambda''\) experimentally
  \(\Rightarrow\) Experiments usually provide: \(\tilde{f}_+\) quadratic / \(\tilde{f}_0\) linear
NA48: A fixed target experiment at the CERN SPS dedicated to the study of CP violation and rare decays in the Kaon sector.
$K^\pm$ Form Factors @ NA48/2 Experiment

- $K^\pm$ collected during 2004 data taking: NA48/2 experiment
  (Main purpose search for direct CP violation in $K^\pm \to 3\pi$ decays)
- Simultaneous $K^+$ and $K^-$ beams
- $K^+$ flux $\simeq 3.2 \times 10^6$; $K^+/K^- \simeq 1.78$ (production rate @target)
- Dedicated run with minimum bias trigger and low intensity ($\times 1/4 \ l_0$)
- Reduced momentum spread: (60 $\pm$ 1.8) GeV/c
- $K^+$ and $K^-$ beams coincide within 1 mm all along 114 m decay volume
The NA48 Detector

Min Bias trigger: \( Q1 \times E_{LKR} > 10 \)

**Magnetic Spectrometer**
4 drift chambers
\[
\frac{\sigma_p}{p} (\%) = 1 \oplus 0.044 \ p \ (GeV/c)
\]

**Hodoscope**
Two orthogonal planes of scintillator
Fast trigger
Precise track time measurement
\( \sigma_t \simeq 150 \ \text{ps} \)

**Liquid Krypton EM Calorimeter**
Quasi-homogeneous - High granularity
13248 cells of 2×2 cm
\[
\frac{\sigma_E}{E} (\%) = 3.2 \sqrt{\frac{E}{E}} \oplus 9.0 \frac{E}{E} \oplus 0.42 \ (GeV)
\]

**Muon Counter**
3 planes of scintillator
Each shielded by 80 cm of iron
25 × 25 cm² cells
\( \sigma_t \simeq 350 \ \text{ps} \)

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**Event Selection**

- 1 "good" track and 1 $\pi^0$
  - Geometrical detector acceptance
  - Vertex - dCA track and K nominal axis
  - Track $P > 10$ GeV/c for MUC efficiency
  - Timing
    - $|m_{\gamma\gamma} - m_{\pi^0}^{PDG}| < 10$ MeV
- $E/p < 0.2$
- 1 MUC Hit matched to the track
- $|t_{MUC} - t_{HOD}| < 3$ ns
- $|MM(\mu)|^2 < 10$ MeV$^2$

Missing Mass: $MM^2 = (P_K - P_\mu - P_{\pi^0})^2$

- $P_{\pi^0} > 15$ GeV
  (Trigger efficiency)
- Cut to remove $\pi^+\pi^-\pi^0$ BKG

3.4 $\times 10^6 K_{\mu3}^\pm$ events selected
Fitting Procedure

Dalitz Plot analysis: to extract the form factors perform a fit to the DP density

$$\rho(E^*_\mu, E^*_\pi) = \frac{d^2 N(E^*_\mu, E^*_\pi)}{dE^*_\mu dE^*_\pi} \propto A f_+^2 + B f_+ (f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t} + C \left[ (f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t} \right]^2$$

$E^*_\mu, E^*_\pi$ are the energies of $\mu$ and $\pi$ in the kaon CMS

$A, B$ and $C$ are kinematical terms

5x5 MeV$^2$ cells - Cells crossed by the Dalitz border are not used in the fit

Need to correct for:

- **Acceptance**
  
  $\epsilon = \frac{\rho(E^*_\mu, E^*_\pi_0)_{MC\ Rec}}{\rho(E^*_\mu, E^*_\pi_0)_{MC\ Gen}}$

- **Background subtraction**

<table>
<thead>
<tr>
<th>$\pi^\pm \pi^0$</th>
<th>0.6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm \pi^0\pi^0$</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

- **Radiative corrections**
  
  - Tree level parametrization
  - Need to cancel the distortion induced by radiative effects
  - Small effect: $\delta_{EM}^{K_{\mu3}} = 0.008 \pm 0.125(\%)$
### QUADRATIC ($\times 10^3$)

<table>
<thead>
<tr>
<th>$\lambda_+$</th>
<th>$\lambda_+''$</th>
<th>$\lambda_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30.3\pm2.7\pm1.4$</td>
<td>$1.0\pm1.0\pm0.7$</td>
<td>$15.6\pm1.2\pm0.9$</td>
</tr>
</tbody>
</table>

First error is stat second is syst

### POLE (MeV/c$^2$)

<table>
<thead>
<tr>
<th>$m_V$</th>
<th>$m_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$836\pm7\pm9$</td>
<td>$1210\pm25\pm10$</td>
</tr>
</tbody>
</table>

To the dispersive results theo uncertainty has been added

V. Bernard et al., PRD80 (2009) 034034

### DISPERSIVE ($\times 10^3$)

<table>
<thead>
<tr>
<th>$\Lambda_+$</th>
<th>$\ln C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$28.5\pm0.6\pm0.7\pm0.5$</td>
<td>$188.8\pm7.1\pm3.7\pm5.0$</td>
</tr>
</tbody>
</table>

Biggest sources of systematics:

- Vertexing
- K energy
- Background ($\pi \to \mu$) scale
Form Factors - Quadratic Fits

- Experimental situation on quadratic fit for $K_{\mu 3}$ decay
- $K^0_{\mu 3}$ results from KLOE, KTeV and NA48, ISTRA measures $K^-_{\mu 3}$
- First measurement which uses also $K^+_{\mu 3}$
- High precision - Very competitive with other results
- Small quadratic term - Larger $\lambda_0$ with respect to NA48 case
- Size and dispersion of ellipses indicate the difficulty of this measurement
Event selection is similar to $K_{\mu3}$

- Require 1 "good" track and 1 $\pi^0$
- Electron ID with $0.95 < E/p < 1.05$
- Event pT cut to remove $\pi^\pm \pi^0$ BKG
- "Easier" measurement
  - Only one form factor
    $\rightarrow$ Reduced correlations
  - BKG issues less critical
    $\rightarrow$ Need a $\pi$ with $E/p > 0.95$
- Expected more precise results
- Analysis is in progress

$4.2 \times 10^6 \, K_{e3}^\pm$ events selected
• NA48/2 has provided a new contribution to $|V_{us}|$ quest
• Preliminary results on $K_{\mu3}^{\pm}$ form factors
• For the first time both $K^+$ and $K^-$ decays have been studied
• High precision measurement very competitive with other results
• Soon also new results on $K_{e3}^{\pm}$ will appear
• NA62 will continue the CKM precision tests in the $|V_{us}|$ sector
• High statistics data samples of $K_{\ell3}^{\pm}$ and $K_{\ell3}^{0}$ have been collected by NA62 during the 2007 run → Ready to be analyzed!
Precise Form Factor Measurement of $K^{\pm} \to \mu^+\mu^-$ decays at NA48/2
**\( \pi^\pm \pi^0 \) Background**

<table>
<thead>
<tr>
<th>Decay</th>
<th>BR(%)</th>
<th>( \mathcal{P}(\pi^\pm \pi^0 \rightarrow K_{\mu3}) )(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^\pm \pi^0 )</td>
<td>20.66 ± 0.08</td>
<td>19.8</td>
</tr>
</tbody>
</table>

- \( \pi^\pm \pi^0 \) events with \( \pi \rightarrow \mu \) can fake a \( K_{\mu3}^\pm \) decay
- This BKG is well localized on the Dalitz plot !!
- Apply cut on \( m_{\pi\pi^0} \) vs \( \pi^0 pT \) plane
- The loss of \( K_{\mu3} \) signal is about 24%
- \( \pi^\pm \pi^0 \) contamination reduced to 0.6%
Background

<table>
<thead>
<tr>
<th>Decay</th>
<th>BR(%)</th>
<th>$\mathcal{P}(\pi^\pm\pi^0\pi^0 \rightarrow K_{\mu3})(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm\pi^0\pi^0$</td>
<td>1.761 ± 0.022</td>
<td>0.14</td>
</tr>
</tbody>
</table>

- Need to correct also for $\pi^\pm\pi^0\pi^0$ BKG
- Small contamination but localized on the Dalitz plot
- Shift of $\simeq 0.5 \sigma_{stat}$ if the correction is not applied
Radiative Corrections

Including first order radiative corrections the $K_{\ell 3}$ decay rate is:

$$\Gamma_{K_{\ell 3}} = \Gamma_{K_{\ell 3}}^0 + \Gamma_{K_{\ell 3}}^1 = \Gamma_{K_{\ell 3}}^0 (1 + 2\delta_{EM}^{K_{\ell}})$$

- For the normalization use: JHEP 11 (2008) 006

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\delta_{EM}^{K_{\ell}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{e 3}^0$</td>
<td>0.495±0.110</td>
</tr>
<tr>
<td>$K_{e 3}^{\pm}$</td>
<td>0.050±0.125</td>
</tr>
<tr>
<td>$K_{\mu 3}^0$</td>
<td>0.700±0.110</td>
</tr>
<tr>
<td>$K_{\mu 3}^{\pm}$</td>
<td>0.008±0.125</td>
</tr>
</tbody>
</table>

- Small effect on the acceptance
- Sign changes - Integral can be 0 even in presence of large corrections
- Smaller distortion w.r.t. the $K_{\mu 3}^0$ case
  $\rightarrow$ Only one charged particle in the f. s.
### Preliminary Survey of Systematics

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \lambda_+^\prime$</th>
<th>$\Delta \lambda_+^\prime\prime$</th>
<th>$\Delta \lambda_0$</th>
<th>$\Delta m_V$ (MeV/$c^2$)</th>
<th>$\Delta m_S$</th>
<th>$\Delta \Lambda_+$</th>
<th>$\Delta \ln C$</th>
<th>$\times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^\pm$ Energy</td>
<td>±0.7</td>
<td>±0.5</td>
<td>±0.6</td>
<td>±7</td>
<td>±2</td>
<td>±0.5</td>
<td>±2.6</td>
<td></td>
</tr>
<tr>
<td>Vertexing</td>
<td>±1.0</td>
<td>±0.4</td>
<td>±0.6</td>
<td>±2</td>
<td>±4</td>
<td>±0.1</td>
<td>±1.1</td>
<td></td>
</tr>
<tr>
<td>Acceptance</td>
<td>±0.3</td>
<td>±0.1</td>
<td>±0.2</td>
<td>±2</td>
<td>±7</td>
<td>±0.1</td>
<td>±1.8</td>
<td></td>
</tr>
<tr>
<td>$\pi \to \mu$ scale</td>
<td>±0.4</td>
<td>±0.2</td>
<td>±0.2</td>
<td>±1</td>
<td>±1</td>
<td>±0.0</td>
<td>±0.0</td>
<td></td>
</tr>
<tr>
<td>2$^{nd}$ analysis</td>
<td>±0.4</td>
<td>±0.1</td>
<td>±0.2</td>
<td>±6</td>
<td>±6</td>
<td>±0.5</td>
<td>±1.5</td>
<td></td>
</tr>
<tr>
<td>Total Systematic</td>
<td>±1.4</td>
<td>±0.7</td>
<td>±0.9</td>
<td>±10</td>
<td>±10</td>
<td>±0.7</td>
<td>±3.7</td>
<td></td>
</tr>
<tr>
<td>Statistical</td>
<td>±2.7</td>
<td>±1.0</td>
<td>±1.3</td>
<td>±7</td>
<td>±26</td>
<td>±0.6</td>
<td>±7.1</td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±0.5</td>
<td>±5.0</td>
<td>±0.5</td>
</tr>
<tr>
<td>Total Error</td>
<td>±3.0</td>
<td>±1.2</td>
<td>±1.6</td>
<td>±12</td>
<td>±28</td>
<td>±1.0</td>
<td>±10.1</td>
<td>±0.5</td>
</tr>
</tbody>
</table>

- $K^\pm$ Energy - Calculate two energy solutions (as in $K_L$ case) instead of using the nominal beam energy of 60 GeV
  $\Rightarrow$ Better resolution on CMS variables
- KType2 (2 sol) - KType1 (1 sol) - KType0 (sol outside allowed range)
- $\pi \to \mu$ scale - Accounts for not perfect modeling of $\pi$ decay in the region from LKR onwards
$K_{\ell 3}$ Form Factors at NA62

NA62, during the 2007 run, collected data for a dedicated measurement of $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$ and tests for the future $K^+ \to \pi^+\nu\bar{\nu}$ experiment

- 4 months data taking with minimum bias trigger
  - 1 track + 10 GeV deposition in EM calorimeter
- Simultaneous $K^+$ and $K^-$ beams of $P=(74 \pm 1.6)$ GeV/c
- Better track momentum resolution ($\to p_T$ kick doubled)
- Collected $\sim 150000$ $K_{e2}$ events
- First results for 40% of stat presented at BEACH2010 and ICHEP2010
- Expected precision on the full data sample: $\sigma(R_K)/R_K \approx \pm 0.4\%$

$K_{\ell 3}$ from NA62 2007 data

- Huge $K^+_{e3}/K^+_{\mu 3}$ statistics of $\approx 40/20 \times 10^6$
- Special $K_L$ run (15 h) to measure electron ID efficiency
  - $K^0_{e3}$ and $K^0_{\mu 3}$ statistics $\approx 4 \times 10^6$

NA48 analyses of $K^0_{\ell 3}$ and $K^\pm_{\ell 3}$ can be repeated with different/larger data sets