Potential-driven G-inflation

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arXiv:1012.4238, PRD 83, 083515 (2011), K. Kamada, T. Kobayashi, MY, J. Yokoyama

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$c = \hbar = M_G = 1$

Contents

- **Introduction**

  What is G-inflation ?

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Nicolis et al 2009

Deffayet et al. 2009, 2011

Burrage et al. 2011

G = Galileon field

Field equations have Galilean shift symmetry in flat space : $\partial_\mu \phi \longrightarrow \partial_\mu \phi + b_\mu$

Covariantization:

$$
\begin{cases}
\mathcal{L}_2 = K(\phi, X), \\
\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
\qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{cases}
$$

$$X = -\frac{1}{2}(\nabla\phi)^2, \qquad G_{iX} \equiv \partial G_i/\partial X.$$

This is the most general non-canonical and non-minimally coupled single-field model which yields second-order equations.

NB :
- **$G_4 = M_G^2 / 2$ yields the Einstein-Hilbert action**
- **$G_4 = f(\phi)$ yields a non-minimal coupling of the form $f(\phi)R$**
- **The new Higgs inflation with $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ comes from $G_5 \propto \phi$ after integration by parts.**

Horndeski's theorem

Horndeski 1974

(Tony & Tsutomu's talks)

In 1974, Horndeski presented the most general action (in four dimensions) constructed from the metric g, the scalar field ϕ, and their derivatives, $\partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \cdots, \partial\phi, \partial^2\phi, \partial^3\phi, \cdots$ still having second-order equations.

$$\begin{aligned}\mathcal{L}_H &= \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\left[\kappa_1\nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi + \kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\right] \\ &+\delta^{\alpha\beta}_{\mu\nu}\left[(F+2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\right] - 6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9.\end{aligned}$$

- κ1, κ3, κ8, κ9, F : functions of ϕ & X with $F_X = 2(\kappa_3 + 2X\kappa_{3X} - \kappa_{1\phi})$.
- W = W(ϕ)
- $\delta^{\alpha_1\alpha_2\ldots\alpha_n}_{\mu_1\mu_2\ldots\mu_n} = n!\delta^{[\alpha_1}_{\mu_1}\delta^{\alpha_2}_{\mu_2}\ldots\delta^{\alpha_n]}_{\mu_n}$.

What is the relation between Galileon model and Horndeski's model ?

⇒ Both models are completely equivalent :

$$\begin{cases} K = \kappa_9 + 4X\int^X dX'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right), \\ G_3 = 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X dX'(\kappa_8 - 2\kappa_{3\phi}), \\ G_4 = 2F - 4X\kappa_3, \\ G_5 = -4\kappa_1, \end{cases}$$

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

Models of G-inflation

Gravitational EOM under the Friedmann background

Under the homogeneous and isotropic background:

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2, \quad T^\mu_\nu = \mathrm{diag}(-\mathcal{E}, \mathcal{P}, \mathcal{P}, \mathcal{P}).$$

$$\mathcal{E} \equiv \sum_{i=2}^{5} \mathcal{E}_i = 0, \qquad \mathcal{P} \equiv \sum_{i=2}^{5} \mathcal{P}_i = 0,$$

$$\begin{cases} \mathcal{E}_2 = 2XK_X - K, \\ \mathcal{E}_3 = 6X\dot{\phi}HG_{3X} - 2XG_{3\phi}, \\ \mathcal{E}_4 = -6H^2G_4 + 24H^2X(G_{4X} + XG_{4XX}) \\ \qquad -12HX\dot{\phi}G_{4\phi X} - 6H\dot{\phi}G_{4\phi}, \\ \mathcal{E}_5 = 2H^3X\dot{\phi}(5G_{5X} + 2XG_{5XX}) \\ \qquad -6H^2X\left(3G_{5\phi} + 2XG_{5\phi X}\right). \end{cases}$$

$$\begin{cases} \mathcal{P}_2 = K, \\ \mathcal{P}_3 = -2X\left(G_{3\phi} + \ddot{\phi}G_{3X}\right), \\ \mathcal{P}_4 = 2\left(3H^2 + 2\dot{H}\right)G_4 - 12H^2XG_{4X} - 4H\dot{X}G_{4X} \\ \qquad -8\dot{H}XG_{4X} - 8HX\dot{X}G_{4XX} + 2\left(\ddot{\phi} + 2H\dot{\phi}\right)G_{4\phi} \\ \qquad +4XG_{4\phi\phi} + 4X\left(\ddot{\phi} - 2H\dot{\phi}\right)G_{4\phi X}, \\ \mathcal{P}_5 = -2X\left(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi}\right)G_{5X} \\ \qquad -4H^2X^2\ddot{\phi}G_{5XX} + 4HX\left(\dot{X} - HX\right)G_{5\phi X} \\ \qquad +2\left[2(HX)^{\cdot} + 3H^2X\right]G_{5\phi} + 4HX\dot{\phi}G_{5\phi\phi}. \end{cases}$$

$$G_4 = \frac{M_G^2}{2} \Rightarrow \begin{cases} \mathcal{E}_4 = -3M_G^2H^2, \\ \mathcal{P}_4 = M_G^2(3H^2 + 2\dot{H}). \end{cases} \quad (= -G^\mu_\nu)$$

Scalar field EOM under the Friedmann background

Under the homogeneous and isotropic background:

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2, \quad \phi = \phi(t).$$

$$\frac{1}{a^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3 J\right) = P_\phi,$$

$$\begin{cases} J = \dot{\phi}K_X + 6HXG_{3X} - 2\dot{\phi}G_{3\phi} \\ \quad +6H^2\dot{\phi}\left(G_{4X} + 2XG_{4XX}\right) - 12HXG_{4\phi X} \\ \quad +2H^3X\left(3G_{5X} + 2XG_{5XX}\right) \\ \quad -6H^2\dot{\phi}\left(G_{5\phi} + XG_{5\phi X}\right), \\ P_\phi = K_\phi - 2X\left(G_{3\phi\phi} + \ddot{\phi}G_{3\phi X}\right) \\ \quad +6\left(2H^2 + \dot{H}\right)G_{4\phi} + 6H\left(\dot{X} + 2HX\right)G_{4\phi X} \\ \quad -6H^2XG_{5\phi\phi} + 2H^3X\dot{\phi}G_{5\phi X}. \end{cases}$$

NB : P_ϕ vanishes if all of K & Gi depend only on X.

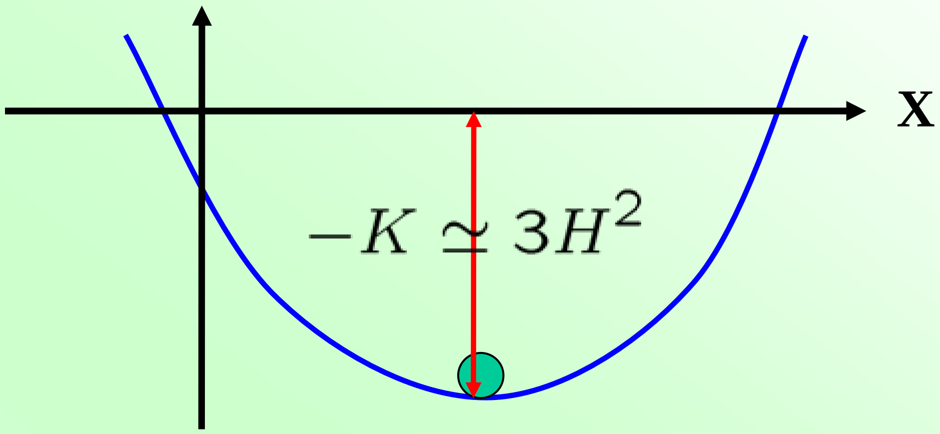
Two types of inflation model

Kinetically driven inflation :

k-inflation: Armendariz-Picon et al. 1999
Ghost inflation: Arkani-Hamed et al. 2004

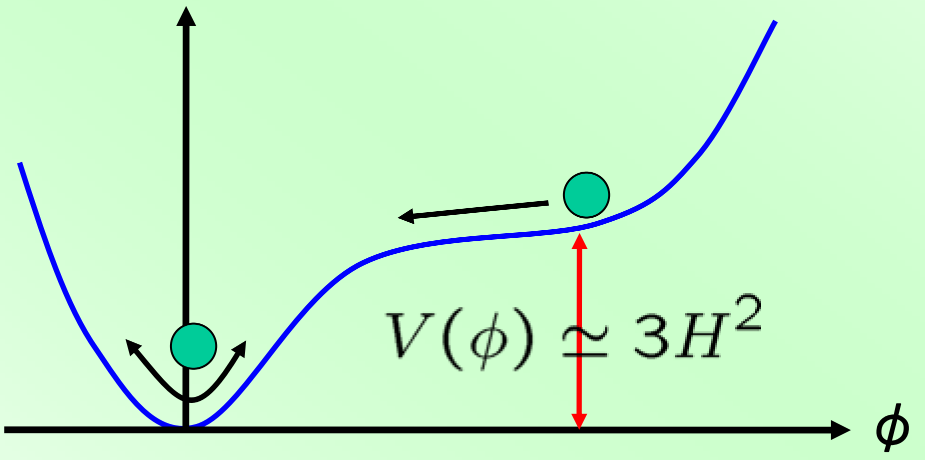
$$\mathcal{L} = K(X).$$

$$\Rightarrow \frac{d}{dt}\left(a^3 K_X \dot{\phi}\right) = 0.$$



Potential driven inflation :

$$\mathcal{L} = X - V(\phi).$$



Potential driven G-inflation

- **Expand the functions in terms of X :**

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots,$$
$$G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots,$$

- **Neglecting all the terms multiplied by dot{phi} in the gravitational field equations and assuming** $|\dot{H}| \ll H^2$ and $|\ddot{\phi}| \ll |H\dot{\phi}|$.

$$\Rightarrow \sum_{i=2}^{5} \mathcal{P}_i \simeq -\sum_{i=2}^{5} \mathcal{E}_i \simeq -V(\phi) + 6g_4(\phi)H^2 = 0.$$

Potential-driven slow-roll inflation is realized : $H^2 \simeq \frac{V}{6g_4}$.

- **During slow-roll, we can approximate** $|\dot{J}| \ll |HJ|,\ |\dot{g}_i| \ll |Hg_i|,\ |\dot{h}_i| \ll |Hh_i|,$

$$\Rightarrow 3HJ \simeq -V_\phi + 12H^2 g_{4\phi},$$

$$J \simeq \mathcal{K}\dot{\phi} - 2g_{3\phi}\dot{\phi} + 6\left(Hh_3X + H^2h_4\dot{\phi} - H^2g_{5\phi}\dot{\phi} + H^3h_5X\right).$$

We can set g3=0 & g5=0. $\Leftarrow$ $\mathcal{K} - 2g_{3\phi} \to \mathcal{K},\ h_4 - g_{5\phi} \to h_4.$

Powerspectrum of primordial fluctuations

(Tsutomu's talk)

Primordial tensor perturbations

Perturbed metric :

$$\begin{cases} ds^2 = -dt^2 + a^2(t)\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\right) \\ h_{ii} = 0 = h_{ij,j} \end{cases}$$

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\mathcal{G}_T\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right].$$

$$\begin{cases} \mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right], \\ \mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] \quad \left(= \frac{1}{2}\sum_{i=2}^{5}\frac{\partial \mathcal{P}_i}{\partial \dot{H}}\right), \\ c_T^2 := \frac{\mathcal{F}_T}{\mathcal{G}_T}. \end{cases}$$

For $G_{4X} \neq 0$ or $G_{5\phi} \neq 0$ or $G_{5X} \neq 0$,
the sound velocity squared c_T^2 can deviate from unity.

No ghost & gradient instabilities ⇔ $\mathcal{F}_T > 0, \quad \mathcal{G}_T > 0.$

Powerspectrum of tensor perturbations

Mode functions : $v_{ij} = \frac{\sqrt{\pi}}{2}\sqrt{-y_T}H^{(1)}_{\nu_T}(-ky_T)e_{ij}$ ← **polarization tensor**

⇒ $h_{ij}(\mathbf{k}, y_T) = \frac{v_{ij}(y_T)}{z_T}\hat{a}_{\mathbf{k}} + \frac{v^*_{ij}(y_T)}{z_T}\hat{a}^\dagger_{-\mathbf{k}}$

Commutation relations : $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$

⇒ $$\mathcal{P}_T = \frac{k^3}{2\pi^2}\left|\frac{v_{ij}}{z_T}\right|^2 = 2^{2\nu_T}\left|\frac{\Gamma(\nu_T)}{\Gamma(3/2)}\right|^2 \frac{(1-\epsilon-s_T)^2}{4\pi^2}\frac{H^2}{\mathcal{F}_T c_T}\Bigg|_{ky_T=-1}.$$

⇒ $$n_T = 3 - 2\nu_T = -\frac{2\epsilon + 3s_T + g_T}{1-\epsilon-s_T} = -\frac{4\epsilon + 3f_T - g_T}{2(1-\epsilon-s_T)}.$$

$$\epsilon := -\frac{\dot{H}}{H^2}, \quad f_T := \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T}, \quad g_T := \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}, \quad \left(s_T := \frac{\dot{c}_T}{Hc_T} = \frac{1}{2}(f_T - g_T)\right)$$

Note that the blue spectrum $n_T > 0$ can be easily obtained as long as $4\epsilon + 3f_T - g_T < 0$.

Primordial scalar perturbations

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2(1+2\zeta)d\boldsymbol{x}^2$$

Unitary gauge : $\phi = \phi(t), \quad \delta\phi = 0.$

$$S_S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\mathcal{G}_S\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla\zeta)^2\right]$$

Hamiltonian & Momentum constraints

$$\begin{cases} \mathcal{F}_S := \dfrac{1}{a}\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T, \\ \mathcal{G}_S := \dfrac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T, \\ c_s^2 := \dfrac{\mathcal{F}_S}{\mathcal{G}_S}. \end{cases}$$

$$\begin{aligned} \Sigma := {} & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\ & +6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\ & +6[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\ & -H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)] \\ & +30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\ & +4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X(6G_{5\phi} \\ & +9XG_{5\phi X} + 2X^2G_{5\phi XX}) \\ = {} & X\sum_{i=2}^{5}\frac{\partial\mathcal{E}_i}{\partial X} + \frac{1}{2}H\sum_{i=2}^{5}\frac{\partial\mathcal{E}_i}{\partial H}. \end{aligned}$$

$$\begin{aligned} \Theta := {} & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\ & -8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\ & -H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\ & +2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right) \\ = {} & -\frac{1}{6}\sum_{i=2}^{5}\frac{\partial\mathcal{E}_i}{\partial H}. \end{aligned}$$

No ghost & gradient instabilities ⇔ $\mathcal{F}_S > 0, \quad \mathcal{G}_S > 0.$

NB : In case of k-inflation with G3 = G5 = 0 and G4 = MG^2 / 2,
FS = MG^2 ε = - MG^2 dot{H} / H^2, which means that
dot{H} > 0 is prohibitted by the stability condition.

Powerspectrum of scalar perturbations

Mode functions : $u_k = \frac{\sqrt{\pi}}{2}\sqrt{-y_S}H_q^{(1)}(-ky_S)$

$$\zeta(\mathbf{k}, y_S) = \frac{u_k(y_S)}{z_S}\hat{a}_{\mathbf{k}} + \frac{u^*_{-k}(y_S)}{z_S}\hat{a}^\dagger_{-\mathbf{k}}$$

Commutation relations : $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$

$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z_S}\right|^2 = 2^{2q-3}\left|\frac{\Gamma(\nu_S)}{\Gamma(3/2)}\right|^2 \frac{(1-\epsilon-s_S)^2}{4\pi^2}\frac{H^2}{2\mathcal{F}_S c_s}\bigg|_{ky_S=-1}.$$

$$n_S = 3 - 2\nu_S = -\frac{2\epsilon + 3s_S + g_S}{1-\epsilon-s_S} = -\frac{4\epsilon + 3f_S - g_S}{2(1-\epsilon-s_S)}.$$

$$\epsilon := -\frac{\dot{H}}{H^2}, \quad f_S := \frac{\dot{\mathcal{F}}_S}{H\mathcal{F}_S}, \quad g_S := \frac{\dot{\mathcal{G}}_S}{H\mathcal{G}_S}, \quad \left(s_S := \frac{\dot{c}_S}{Hc_S} = \frac{1}{2}(f_S - g_S)\right)$$

Note that almost scale invariance requires 2 ε + 3s$_S$ + g$_S$ << 1, while each slow-roll parameter can be large.

Tensor-to-scalar ratio : $r := \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = 16\left(\frac{\mathcal{F}_S}{\mathcal{F}_T}\right)^{3/2}\left(\frac{\mathcal{G}_S}{\mathcal{G}_T}\right)^{-1/2} = 16\frac{\mathcal{F}_S}{\mathcal{F}_T}\frac{c_S}{c_T}.$

Powerspectrum of perturbations in potential-driven G-inflation

Powerspectrum of primordial tensor fluctuations

Slow-roll EOM :

$$\begin{cases} H^2 \simeq \dfrac{V}{6g_4}, \\ 3HJ \simeq -V_\phi + 12H^2 g_{4\phi}. \end{cases}$$

$$\begin{bmatrix} K(\phi, X) &=& -V(\phi) + \mathcal{K}(\phi)X + \cdots, \\ G_i(\phi, X) &=& g_i(\phi) + h_i(\phi)X + \cdots, \end{bmatrix}$$

⇒ $$\mathcal{F}_T \simeq \mathcal{G}_T \simeq 2g_4.$$

⇒ $$c_T^2 \simeq 1.$$

$$\begin{cases} f_T := \dfrac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T} \simeq \dfrac{\dot{g}_4}{Hg_4}, \quad g_T := \dfrac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T} \simeq \dfrac{\dot{g}_4}{Hg_4}, \\ \left(s_T := \dfrac{\dot{c}_T}{Hc_T} = \dfrac{1}{2}(f_T - g_T) \simeq 0 \right). \end{cases}$$

⇒ $$\begin{cases} \mathcal{P}_T \simeq \dfrac{4}{g_4}\left(\dfrac{H}{2\pi}\right)^2 \quad \left(\simeq \dfrac{8}{M_G^2}\left(\dfrac{H}{2\pi}\right)^2 \text{ for } g_4 = \dfrac{M_G^2}{2} \right), \\ n_T = -\dfrac{2\epsilon + g_T}{1-\epsilon} \simeq -(2\epsilon + g_T) \quad (\epsilon \ll 1) \end{cases}$$

Powerspectrum of primordial scalar fluctuations

$$\begin{cases} \mathcal{F}_S \simeq \dfrac{X}{H^2}\left(\mathcal{K}+6H^2h_4\right)+\dfrac{4\dot{\phi}X}{H}\left(h_3+H^2h_5\right), \\ \mathcal{G}_S \simeq \dfrac{X}{H^2}\left(\mathcal{K}+6H^2h_4\right)+\dfrac{6\dot{\phi}X}{H}\left(h_3+H^2h_5\right). \end{cases}$$

- **K or h4 term domination**

$$\mathcal{F}_S \simeq \mathcal{G}_S \simeq g_4(2\epsilon+g_T).$$

$$\begin{cases} c_S^2 \simeq 1. \\ \mathcal{P}_S \simeq \dfrac{1}{g_4(2\epsilon+g_T)}\dfrac{H^2}{8\pi^2} \\ \quad\simeq \dfrac{H^2}{8\pi^2 M_G^2\epsilon} \quad \text{for} \quad g_4=\dfrac{M_G^2}{2}. \end{cases}$$

$$r = 8(2\epsilon+g_T) = -8n_T.$$

- **h3 or h5 term domination**

$$\mathcal{F}_S \simeq \frac{2}{3}\mathcal{G}_S \simeq \frac{4}{3}g_4(2\epsilon+g_T).$$

$$\begin{cases} c_S^2 \simeq \dfrac{2}{3}. \\ \mathcal{P}_S \simeq \dfrac{1}{g_4(2\epsilon+g_T)}\dfrac{3\sqrt{6}H^2}{64\pi^2} \\ \quad\simeq \dfrac{3\sqrt{6}H^2}{64\pi^2 M_G^2\epsilon} \quad \text{for} \quad g_4=\dfrac{M_G^2}{2}. \end{cases}$$

$$r = \frac{32\sqrt{6}}{9}(2\epsilon+g_T) = -\frac{32\sqrt{6}}{9}n_T.$$

Thus, consistency relations are useful to discriminate which term dominates in the dynamics for the potential driven inflation.

Summary

- We have proposed a new inflation model named G-inflation, which is driven by a Galileon field. This model is shown to be equivalent to the Horndeski's theory and is the most general non-canonical and non-minimally coupled single-field model which yields second-order equations.
- The general formula for tensor and scalar perturbations are derived.
- Potential driven G-inflation predicts new consistency relations between r and n_T, which tells us what kind of term dominates the dynamics.