Superconformal Index for large N quiver Chern-Simons theories

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Motivations

- **We are interested in a variety of dualities, especially AdS/CFT , however, the duality are not verified and are needed to check.**
- **Superconformal Index (SI) tells us that whether the duality is correct or not.**
  - **If a pair of theories are dual each other, then the Superconformal Indices calculated from both theories match.**
- **We need an appropriate(large N) Conformal Field Theory for AdS/CFT and its Superconformal Index (Large N Index)**

AdS/CFT and Outline

Roughly speaking, AdS/CFT is a duality between a gravity theory with Anti de Sitter backgroud space and Conformal Field Theory with large N gauge group (assuming $U(N)$ gauge group) which lives in a one lower dimension.

From now on, we concentrate on $\mathrm{AdS}_4/\mathrm{CFT}_3$, which is appropriate for M-theory.

$$\mathrm{AdS}_4 \times \mathrm{M}_7 \quad \longleftrightarrow \quad \mathrm{CFT}_3 \text{ with global symmetry}$$

where M_7 is a 7-dimensional manifold.

Definition of the 3d Superconformal Index

$$I(x, z_i) = \mathrm{tr}\left[(-1)^F x'^{\{Q,Q^\dagger\}} x^{\Delta + j_3} \prod_i z_i^{F_i}\right]$$

where

- tr is a summantion of all gauge invariant operators
- F is a fermion number
- Q is a certain supercharge
- Δ is an energy (conformal dimension)
- j_3 is the 3rd component of angular momentum
- F_i are flavor symmetry generators
- $\{Q, Q^\dagger\} = \Delta - j_3 - R \geq 0$

Short History of Superconformal Index

	General formula	Large N extension
No anomalous dimension ($\mathcal{N} \geq 3$)	Bhattacharya, Bhattacharyya, Minwalla, Raju[1]	Bhattacharya et al.[2] (ABJM without monopole) Kim[3] (ABJM with mon) Imamura, Yokoyama[4] ($\mathcal{N} = 4$)
Anomalous dimension	Imamura, Yokoyama[5]	Imamura, D.Y, Yokoyama

We extended the formula for the SI to the case of $\mathcal{N} = 2$ superconformal theory with large N $U(N)$ gauge group.

[1] arXiv:0801.1435 JHEP 0802 (2008) 064
[2] arXiv:0806.3251 JHEP 0901 (2009) 014
[3] arXiv:0903.4172 Nucl.Phys. B821 (2009) 241-284
[4] arXiv:0908.0988 Nucl.Phys. B827 (2010) 183-216
[5] arXiv:1101.0577 JHEP 1104 (2011) 007

$V^{5,2}$ manifold and its dual CFT

$V^{5,2}$ manifold

$V^{5,2}$ manifold is the homogeneous space defined as the coset

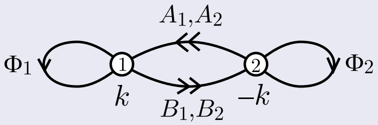
$$V^{5,2} = SO(5)/SO(3)$$

The isometry

$$SO(5) \times U(1)$$

$\mathbb{Z}_k$-orbifolding $(V^{5,2}/\mathbb{Z}_k)$

The dual CFT



The manifest global symmetry of this theory is

$$SU(2) \times U(1)_B \times U(1)_R$$

The Chern-Simons level k

The Calculation of The Superconformal Index

	Φ_1	Φ_2	A_1	A_2	B_1	B_2
Δ	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
F_1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
F_2	0	0	0	0	0	0

Fundamental matter contribution is

$$I^{(0)} = 1 + 2x^{2/3} + (\chi_1(z_1) + 4)x^{4/3} + (6 + 2\chi_1(z_1))x^2 + \cdots$$

where χ_s is the $SU(2)$ character,

$$\chi_s(z) = \frac{z^{s+1} - z^{-s}}{z-1} = z^s + z^{s-1} + \cdots + z^{-s}$$

Monopole contribution is

$$I^{(+)} = 1 + x^{2/3}\chi_{\frac{1}{2}}(z_1)z_2^{1/2} + x^{4/3}\left(\chi_{\frac{1}{2}}(z_1)z_2^{1/2} + 2\chi_1(z_1)z_2\right) + x^2\left(\chi_{\frac{3}{2}}(z_1)z_2^{1/2} + 2\chi_1(z_1)z_2 + \left(3\chi_{\frac{3}{2}}(z_1) + \chi_{\frac{1}{2}}(z_1)\right)z_2^{3/2}\right) + \cdots$$

$$I^{(-)}(x, z_1, z_2) = I^{(+)}(x, z_1, z_2^{-1})$$

The Superconformal Index is

$$I = 1 + x^{2/3}\left(\chi_{\frac{1}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + 2\right) + x^{4/3}\left(2\chi_1(z_1)\chi_1(z_2) + 3\chi_{\frac{1}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + 5\right) + x^2\Big(3\chi_{\frac{3}{2}}(z_1)\chi_{\frac{3}{2}}(z_2) + \chi_{\frac{3}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + \chi_{\frac{1}{2}}(z_1)\chi_{\frac{3}{2}}(z_2) + 6\chi_1(z_1)\chi_1(z_2) + 8\chi_{\frac{1}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + 10\Big) + \cdots$$

$$I = 1 + x^{2/3}\left(\chi^{SO(5)}_{(0,1)}(z_1, z_2) + 1\right) + x^{4/3}\left(2\chi^{SO(5)}_{(0,2)}(z_1, z_2) + \chi^{SO(5)}_{(0,1)}(z_1, z_2) + 2\right) + x^2\Big(3\chi^{SO(5)}_{(0,3)}(z_1, z_2) + \chi^{SO(5)}_{(2,1)}(z_1, z_2) + 2\chi^{SO(5)}_{(0,2)}(z_1, z_2) - \chi^{SO(5)}_{(2,0)}(z_1, z_2) + 3\chi^{SO(5)}_{(0,1)}(z_1, z_2) + 2\Big) + \cdots$$

Monopole contribution is

$$I^{(+)} = 1 + x^{2/3}\chi_{\frac{1}{2}}(z_1)z_2^{1/2} + x^{4/3}\left(\chi_{\frac{1}{2}}(z_1)z_2^{1/2} + 2\chi_1(z_1)z_2\right) + x^2\left(\chi_{\frac{3}{2}}(z_1)z_2^{1/2} + 2\chi_1(z_1)z_2 + \left(3\chi_{\frac{3}{2}}(z_1) + \chi_{\frac{1}{2}}(z_1)\right)z_2^{3/2}\right) + \cdots$$

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Monopole contribution is

$$I^{(+)} = 1 + x^{2/3}\chi_{\frac{1}{2}}(z_1)z_2^{1/2} + x^{4/3}\left(\chi_{\frac{1}{2}}(z_1)z_2^{1/2} + 2\chi_1(z_1)z_2\right) + x^2\left(\chi_{\frac{3}{2}}(z_1)z_2^{1/2} + 2\chi_1(z_1)z_2 + \left(3\chi_{\frac{3}{2}}(z_1) + \chi_{\frac{1}{2}}(z_1)\right)z_2^{3/2}\right) + \cdots$$

$$I^{(-)}(x, z_1, z_2) = I^{(+)}(x, z_1, z_2^{-1})$$

The Superconformal Index is

$$I = 1 + x^{2/3}\left(\chi_{\frac{1}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + 2\right) + x^{4/3}\left(2\chi_1(z_1)\chi_1(z_2) + 3\chi_{\frac{1}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + 5\right) + x^2\Big(3\chi_{\frac{3}{2}}(z_1)\chi_{\frac{3}{2}}(z_2) + \chi_{\frac{3}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + \chi_{\frac{1}{2}}(z_1)\chi_{\frac{3}{2}}(z_2) + 6\chi_1(z_1)\chi_1(z_2) + 8\chi_{\frac{1}{2}}(z_1)\chi_{\frac{1}{2}}(z_2) + 10\Big) + \cdots$$

$$I = 1 + x^{2/3}\left(\chi^{SO(5)}_{(0,1)}(z_1, z_2) + 1\right) + x^{4/3}\left(2\chi^{SO(5)}_{(0,2)}(z_1, z_2) + \chi^{SO(5)}_{(0,1)}(z_1, z_2) + 2\right) + x^2\Big(3\chi^{SO(5)}_{(0,3)}(z_1, z_2) + \chi^{SO(5)}_{(2,1)}(z_1, z_2) + 2\chi^{SO(5)}_{(0,2)}(z_1, z_2) - \chi^{SO(5)}_{(2,0)}(z_1, z_2) + 3\chi^{SO(5)}_{(0,1)}(z_1, z_2) + 2\Big) + \cdots$$

Other examples

Background manifolds

$$Q^{1,1,1} = \frac{SU(2)\times SU(2)\times SU(2)}{U(1)\times U(1)}$$

$$SU(2)_1 \times SU(2)_2 \times SU(2)_3 \times SU(2)_4$$

$$Q^{2,2,2} = Q^{1,1,1}/\mathbb{Z}_2$$

$$SU(2)_1 \times SU(2)_2 \times SU(2)_3 \times SU(2)_4$$

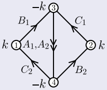
$$M^{1,1,1} = \frac{SU(3)\times SU(2)\times U(1)}{SU(2)\times U(1)\times U(1)}$$

$$SU(3) \times SU(2) \times U(1)$$

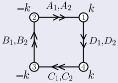
$$N^{0,1,0} = \frac{SU(3)}{U(1)}$$

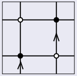
$$SU(3) \times SU(2)$$

Dual CFT's

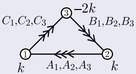


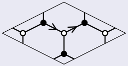
$$SU(2) \times U(1)_F \times U(1)_B \times U(1)_R$$



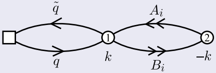


$$SU(2) \times SU(2) \times U(1)_B \times U(1)_R$$





$$SU(3) \times U(1)_B \times U(1)_R$$



$$SU(2) \times U(1)_B \times SU(2)_R$$

Conclusions

- We extended the formula for the Superconformal Index to the case of $\mathcal{N} = 2$ superconformal theory with large N $U(N)$ gauge group.
- We checked several examples of AdS/CFT.

Future Plan

- We should check Indices of the gravity side and if they agree. A few examples of the indecies have been checked by Cheon et al.[6]
- It is interesting to extend the index to finite N cases and check if they cause symmetry enhancement.

[6] arXiv:1102.4273 JHEP 1105 (2011) 027